Black-Scholes Model's application in rainbow option pricing

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Abstract. In this paper, we use excel as a tool to explore the pricing of rainbow options and their advantages based on the Black-Scholes Model. Two-color rainbow options are mainly explored in the paper, in which the underlying stocks are Apple and ExxonMobil. Simulating the price of two stocks is performed through Excel. Return on the corresponding European options and rainbow options is obtained after that. Next, the differences between the return on rainbow options and European options and pricing on rainbow option are analyzed. Finally, sensitivity analysis is carried out to further explore rainbow option pricing.

Keywords: option; Pricing; Black-Scholes Model

1. Introduction

An option is one of the most common and widely used financial products in the market that allows you to trade the future price of the market. Buying an option is a buyer buying a right to trade in the market at a fixed price on a particular date before or at expiry. Options trade on equities, bond futures, interest rate futures, commodity & currency futures. They are one of the most popular financial instruments in the market. The two dominant styles of options in the market are American option and European option. While European style option can only be exercised at expiry, American style option can be exercised prior (or at) expiration. Option pricing theory is one of the most fundamental and influential research topics in financial mathematics, and its development has directly promoted the progress of modern financial mathematics. Option pricing theory has become the basis of financial and economic theory today. and it started in 1900 when French mathematician Louis Bachelier derived an option pricing formula under the presumption that the values of the underlying assets move in a Brownian motion with no drift.

Rainbow options are first proposed by Margrabe [1], and belong to the ‘‘exotic option’’.[2] Rainbow options are dependent on both the price and quantity of a natural resource deposit, therefore it is widely utilized in valuing natural recourses.

Rainbow options are a type of multi-asset option, which can contain not only underlying assets such as stocks and commodities but also some vanilla options. Rainbow options are available in a variety of flexible formats, which are different from American options and European options. Rainbow options are a great financial tool to hedge risk if a buyer has multiple assets on hand.[2]

One of the forms is called the better-of option. Its value depends on the largest value or reward among several target assets of the target assets, expressed mathematically as:

C=Max (R1, R2 …R4,0)

The other form of the rainbow option is called an outperformance option, which depends on the difference in the performance of the two underlying assets and is expressed mathematically as:

C=Max (R1-R2,0)

In this essay, the study focuses on Black-Scholes Model's application in rainbow option pricing.

Black-Scholes Model aims to decompose financial assets and derivatives into a set of differential equations, and it was first proposed by American economists Myron Scholes and Fischer Black.[3] In the early 1970s, in the area of pricing of European stock options, a significant breakthrough was made by Fischer Black, Myron Scholes, and Robert Merton, which still reverberates through the financial world nowadays. [4] The prerequisites of establishing the classical B-S equation are some strict assumptions, [5] and it is worth mentioning that those assumptions are hard to realize in the real world. To weaken these assumptions, some models have been proposed, such as stochastic interest model [6], Jump-diffusion model [7], stochastic volatility model [8]. Merton developed the theoretical foundation for option pricing and provided examples to support the theory.

The Black-Scholes model is presently playing a significant role in financial derivative instruments. It is frequently utilized in day-to-day trading and has generated many novel options, trading methods, and hedging strategies. [9-12] It is practical to apply the Black-Scholes Model in real-life examples. Two sample stocks, Apple and ExxonMobil, will be picked and form a simple basket in the study so that we can apply Black-Scholes Model in the basket that the stocks form.

2. Data and Method

2.1 Data

In our study, the underlying assets of the rainbow option are Apple and ExxonMobil stock, which form a simple manifestation of the rainbow option.

Apple is the world's leading U.S. high-tech company dedicated to innovation and providing consumer electronics, software, and online services to consumers. ExxonMobil is an American multinational oil and gas corporation, it’s the world's largest non-governmental oil and gas producer and is also one of the largest oil refiners in the world. Both companies have a great impact on daily life and the regional and world economy.

The adjusted stock price data from yahoo finance with Python, excludes dividends, stock splits, etc., and is a more accurate measure when predicting the stock value in the future. The figure of the stocks’ information and the stocks’ trend is figure 1, 2, 3, and 4. The date is from 2019.12.01 to 2020.8.12, and there is 679 days’ stock price, covering the entire period of the pandemic. After getting the adjusted stock price, calculate the return of each stock. The formula here is the difference between the share price of the day and that of the previous day divided by the previous day’s share price. Correlation is calculated using the return of two stocks, which is 0.3182, and it’s moderate. This paper sets the risk-free rate from the 5-year treasury bill, which is 3.45%. The standard deviation of the two stocks’ returns is 0.0229 and 0.0259 for apple and ExxonMobil, respectively. T is set as 1. Therefore, the yearly sigma is calculated by multiplying each stock’s standard deviation by the square root of 252, which represents 252 trading days a year. Apple’s yearly sigma is 0.3635, and ExxonMobil's yearly sigma is 0.4114.

The spot price of asset apple and ExxonMobil stock is the share price of 08/12/2022, and the set cash flow is $1,000,000.

2.2 Method

Black-Scholes Model is a differential equation proposed by Fischer Black and Myron Scholes.

For the B-S model, it cannot provide a closed-form solution to most exotic options American style, but it can solve the European vanilla option. The option discussed in the study is the European option. There are several basic assumptions about the model: the underlying assets, such as stock, follow a geometric Brownian motion; There are no risk-free arbitrage opportunities; An option is a European-style option; No dividends and other income during the option period; No taxes and transaction costs, and all securities are fully divisible; Security trading is continuous; The risk-free rate and financial asset return variables are constant before an option expires.

3. Results

Firstly, after finishing the preliminary work, this study conducts the simulation first. This research generates 1000 random numbers for Apple stock by using the formula “NORMSINV (RAND ())” and uses the Black-Scholes model to simulate the stock price 1000 times for Apple. Then the study generates 1000 random numbers for ExxonMobil stock based on the formula:

Correlation between two stocks*corresponding random number of apple stock + NORMSINV (RAND ()) *SQRT (1- Correlation between two stocks ^2). After that, use the same formula to get a simulated stock price for ExxonMobil. Then get a max return and max loss over the simulated share price. To obtain the max return and max loss over each simulation, the formula is MAX/Min ((simulated apple share price/Apple spot price)/Apple spot price, (simulated Exxon Mobil share price/ Exxon Mobil spot price)/ Exxon Mobil spot price,0).

Secondly, compare the basket with the European put option and the European call option. To get these, the formulas applied are Max/Min ((simulated share price of AAPL or XOM - spot price of AAPL or XOM)/ spot price of AAPL or XOM,0). This provides the groundwork for our later comparisons. The average return of call on the rainbow option is 0.2913, while the average return of put on the rainbow option is 0.2131. It is obvious that the rainbow option can always provide a greater return than the corresponding European options. It can be found that the European call option and European put option are always more expensive than the rainbow option since the rainbow option always chooses the best/worst performers among the underlying assets. The result is in figure 5. Then, the study obtains the rainbow option price by using the formula: Cash *EXP (-5-year treasury bill rate*T *1) *return on basket call/put.

Thirdly, change the correlation between two underlying assets to see how the rainbow call price would fluctuate. Chart 1 and 7 shows the result. There is a trend that as the correlation increases, the rainbow call price will fall.

Fourthly, change the volatility to see how the rainbow call price would fluctuate. The results show that with the volatility increases, call and put option prices go up. When volatility decreases, option prices go down. Figures 8, 9, 10, and 11 demonstrate that phenomenon. Volatility is positively correlated with the option's price. More specifically, it is a key to option pricing. Traders can infer the implied volatility (IV) that traders expect the underlying to move from an option's price. Implied volatility is likely to increase when options markets are on a downward trend. The relationship between the option price and time is not a simple linear relationship.

Fifthly, change the T to see how the rainbow call price would fluctuate. The result shown in figure 9, indicates that when T increases, the option price decreases. The figure is like a half downward parabola.

	AAPL
count	678.000000
mean	124.750935
std	32.945134
min	55.174358
25%	106.114052
50%	129.555382
75%	148.473022
max	181.259933

Figure 1. Statistical data of Apple

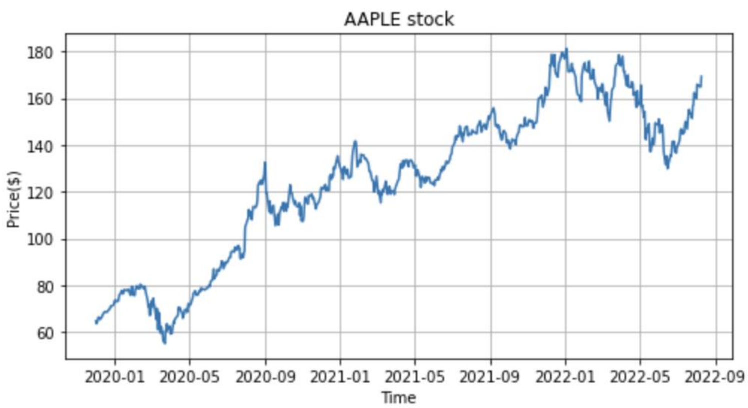

Figure 2. Apple stock trend

	XOM
count	563.000000
mean	56.201873
std	18.231871
min	28.157316
25%	39.328377
50%	54.530972
75%	63.265528
max	103.583549

Figure 3. statistical data of Exxonmobil

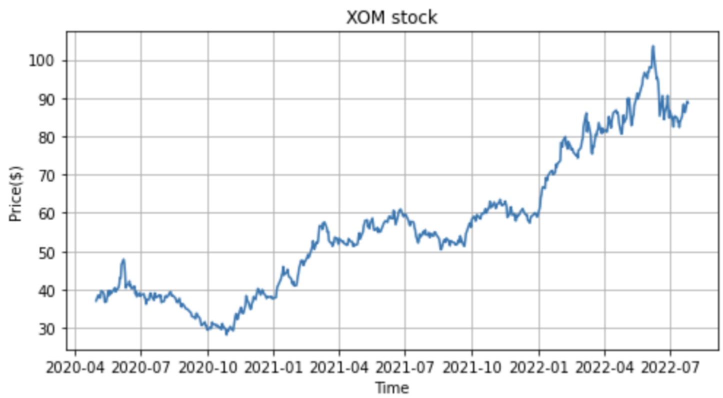

Figure 4. Exxonmobil stock trend

1				
0.298902403	Euro call option of TSLA	0.18629	Euro put option of TSLA	0.11713
0.205342926	Euro call option of TEVA	0.18593	Euro put option of TEVA	0.14618

Figure 5. return on rainbow option and European option

Table 1. Sensitivity Analysis: changes in correlation

correlation	rainbow call price
	281399.9176
0.1	288134.5632
0.15	282488.3608
0.2	302855.4647
0.25	300456.6366
0.3	269545.2279
0.35	261030.5297
0.4	275048.5154
0.45	231571.2942
0.5	232923.3984
0.55	258606.6906
0.6	249600.0306
0.65	244708.0353
0.7	230674.0953
0.75	246608.9091
0.8	209240.1903
0.85	225601.031
0.9	209396.8137

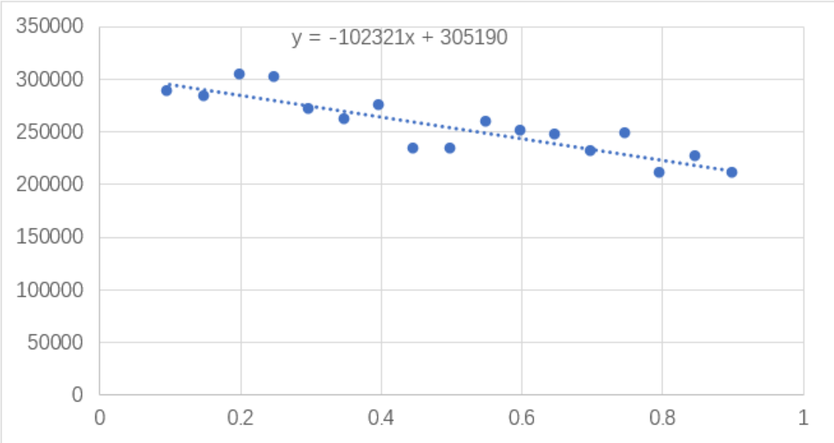

Figure 6. corresponding graph of table 1's data

Table 2. how rainbow option price is affected based on changes in Apple volatility

Apple Volatility	rainbow call price
	281399.9176
0.1	201603.2668
0.15	224246.1743
0.2	222609.402
0.25	240454.2193
0.3	260882.2112
0.35	277115.0625
0.4	298559.7011
0.45	317646.6083
0.5	331265.8483
0.55	351699.748
0.6	364880.939
0.65	377161.4998
0.7	421571.8528
0.75	392771.9023
0.8	423125.3067
0.85	437624.0603
0.9	520138.7042
0.95	472866.8754

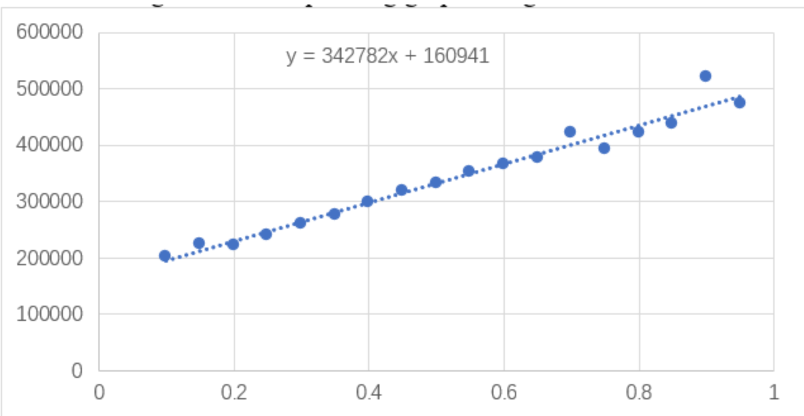

Figure 7. corresponding graph of figure 10's data

Table 3. how rainbow option price is affected based on changes in ExxonMobil volatility

XOM volatility	rainbow call price
	281399.9176
0.1	191245.6926
0.15	194548.2314
0.2	201269.8579
0.25	238370.8036
0.3	260284.5384
0.35	244114.5024
0.4	270255.5069
0.45	291070.5087
0.5	316523.349
0.55	338093.9744
0.6	310937.5639
0.65	346210.1337
0.7	381621.6617
0.75	398857.3435
0.8	370483.4052
0.85	459768.7083
0.9	420322.2357
0.95	483968.463

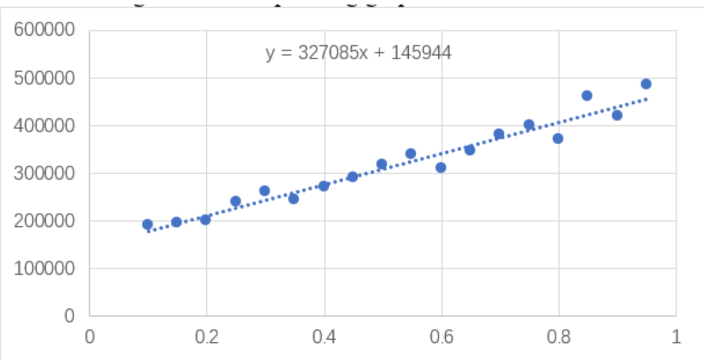

Figure 8. corresponding graph of table 3's data

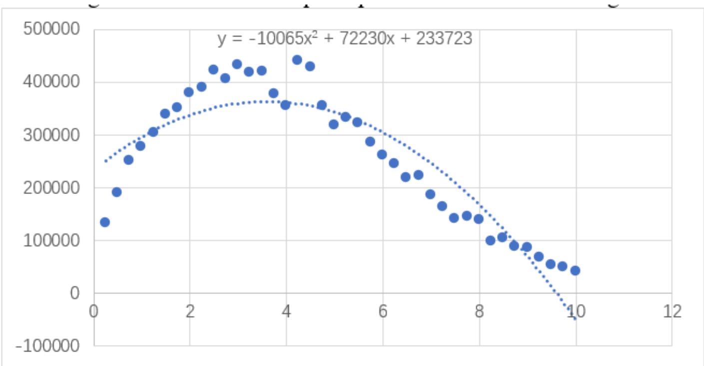

Figure 9. how rainbow option price is affected as T changes

4. Discussion

Volatility is commonly used to describe the uncertainty of futures prices over a period. 15%-60% is usually reasonable volatility for a stock. It is generally expressed as an annual standard deviation and is divided into historical volatility (HV), implied volatility (IV), and realized volatility(RV).[14] Historical volatility is statistical data calculated from past data. It is the annualized standard deviation of returns calculated based on historical price data of the underlying asset, which reflects historical price fluctuations. Implied volatility is an important "correction" in options trading, which reflects the market's forecast price volatility forecasts. Essentially, implied volatility is the options market's prediction of the impending statistical volatility of the underlying assets over an option period. Volatility is the core element of option pricing and hedging. Usually, Options with high volatility are worth more than options with low volatility. It can be found in graphs 9 and 11. Usually, 30-90 days is the most common time period for most options trading strategies. The loss of time value is not uniformly linear; the closer time is to expiration, the faster the loss.

5. Conclusion

The study focus on the application of the Black-Scholes model in the rainbow option’s pricing. The study verifies the rainbow option’s validity and the B-S model’s practical application significance

In this paper, by using the last three years of stock data of Apple and ExxonMobil, a basket consisting of these underlying assets is priced using the B-S model and compared with European-style options. The essay also studies rainbow option’s underlying assets volatility, correlation, and options maturity time. Intrinsic value plus its extrinsic value equals the premium of any option. The longer an option maturity time is, the higher the option premium is. This paper theoretically enriches the methods of option pricing. Practically, it is helpful for the improvement of options market pricing for investors' decision to invest in options.

This paper mainly uses excel. In the aspect of research technology, it needs to be further developed, and the depth of this paper also needs to be further improved.

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