Optimal Investment Portfolio under Different Models with Various Constraints Especially Considers COVID-19 Period

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Abstract. The outbreak of COVID-19 at the end of 2019 had a severe impact on global economic markets, with the U.S. stock market experiencing four circuit breakers in one month. As of July 2021, the real GDP of the United States has significantly outpaced the growth rate of the world's advanced economies. In order to study how investors invest in the stock market after the U.S. stock market experience circuit breaker, this paper selects six stocks as the research object using the monthly closing prices from May, 2001 to May, 2021 as sample data, and calculates the optimal portfolio by Markowitz model and Index model. Through calculation and constraint of different conditions, we obtain the variance and Sharpe ratio of the six selected sample stocks under the two conditions of minimum variance portfolio and maximum Sharpe ratio portfolio respectively. In the portfolio based on Markowitz model and Index model, we can draw a conclusion that Procter & Gamble Co. accounts for a larger proportion under different constraints. Different constraints will also lead to different results. In most cases, a board index is included, and the optimization constraints brought by it can make the portfolio return under Markowitz model and Index model reach the maximum. If the constraints of FINRA regulation T are taken into account, the Sharpe ratio values can be higher.

Keywords: stocks, portfolio, Markowitz model, Index model

1. Introduction

Since the end of 2019, the COVID-19 pandemic has spread widely across the globe, severely affecting all countries around the world. Financial crises have emerged repeatedly, and economic markets have collapsed, which has hit stock markets. As the United States went into lockdown to contain the outbreak, it suffered the worst economic shock since the great depression of the 1930s. But activity picked up in the third quarter of last year, based on the deep reserves of economic power and stimulus spending by the federal government.[1]

As of 2021, according to the World Economic Outlook released by the International Monetary Fund in July, the GDP of advanced economies is expected to grow by 5.6% in 2021, while the real GDP of the United States grew by 6.5% in the second quarter of 2021, according to the Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce. The recovery of the U.S. economy is remarkable, and it is in the forefront of major Western countries.

In particular, the stock market is closely related to the economic market. Since the outbreak of COVID-19, stock markets of all countries have been greatly affected and suffered a continuous slump. Subsequently, many stock markets have seen unprecedented intensive circuit breakers. The U.S. stock market alone started four circuit breakers, falling more than 30%, but then showed a "V-shaped" rebound and stock markets gradually recovered. In this special period, how investors invest in the financial market has become an important issue. According to Dowd [1], in the case of venture capital, one strategy that can reduce investment risk is to build a portfolio. Although the portfolio approach cannot eliminate the systemic risk of the financial market, it relatively reduces the non-systemic risk of the financial market and ensures the maximization of the interests of investors in the financial market [2].

Portfolio investing gives investors the opportunity to diversify their investments in stocks, which helps to achieve higher financial risk-adjusted returns [3]. Facing different venture capital opportunities, investors must be willing to take high risks while hoping for high returns. Therefore, it is very necessary to analyze the stock valuation, select the optimal portfolio of stocks, and measure the risk of investment capital market. The historical price and trend of a stock are important data we need to judge the future of a stock. The average, the expected return of a portfolio, is the weighted average of the expected return of individual securities, weighted by the corresponding investment proportion. The standard deviation of the return is often called volatility, which describes the risk of a portfolio [4]. At the same time, risk can be assessed more accurately by measuring the variance of portfolio returns, a general indicator widely applied to calculate risks in financial investment [5].

For optimal portfolio analysis, the risk-return expectation of sample stocks is generated first, which is expressed as the variance or standard deviation of expected rate of return and return. The expected return of an asset is the weighted average of the returns of individual stocks held in the portfolio. The variance and standard deviation of returns are another statistical method to measure investment risk [6]. In the calculation process, we focus on Sharpe ratio to measure the future development performance of stocks. Sharpe ratio is the most popular risk-adjusted performance measure in investment portfolios and investment funds. From an investor's point of view, the Sharpe ratio describes how the return on a portfolio compensates for the risk taken on by the investor [7]. Mean variance is one of the most important theories in financial research. Markowitz effective frontier refers to a group of portfolios that achieve the maximum expected return at a given risk level. Investors can obtain a lot of information, knowledge, how to invest, when to invest, and why to invest in a particular portfolio [8]. Markowitz's portfolio theory is based on the whole portfolio and focuses on the correlation between individuals and the balance between risk and equity of the portfolio as a whole [9]. On the one hand, non-systematic risks can be reduced by increasing low-correlation assets. On the other hand, it finds the maximum return and the lowest risk at a certain level of return. Recent research showed that Markowitz model is feasible for guiding portfolio selection and has important practical significance for portfolio risk diversification [10].

We select six stocks in three industries as cases, extract the closing price data at the end of each month from May 2001 to October 2021, establish the mean-variance model, and empirically-analyze how investors use known historical effective information such as mean returns and covariance. Under the condition of portfolio, stock investment is optimized by combining its own risk preference, acceptable average return or investment proportion, so that the selected stock portfolio can disperse investment risks and achieve the ideal goal of maximizing investment utility.

Through calculation, this paper finally comes to the conclusion that if the variance of the portfolio is minimized under the Markowitz model, Colgate-Palmolive Co. (CL) stocks account for the largest weight except under the benchmark constraint, and Procter & Gamble Co. (PG) stocks account for a larger weight in the remaining four constraints. At the same time, we find that if a broad index is included in the portfolio, the return, variance and Sharpe ratio of the portfolio are the maximum under the additional optimization constraints brought by this behavior. Under Index model, six stocks occupy the same proportion in the benchmark constraint, and the remaining four constraints are all PG stocks with a larger proportion. The return and variance of the portfolio are the highest under the benchmark constraint, and the Sharpe ratio is higher under the broad index constraint.

In the maximum Sharpe ratio portfolio, no matter Markowitz model or Index model, PG stocks eventually account for more weight. In the Markowitz model, the maximum return and variance can be obtained when the constraint conditions of the broad index are satisfied, and the maximum Sharpe ratio can be obtained if either of the constraint conditions of the broad index and the constraint conditions of the Regulation T of the Financial Industry Regulatory Authority (FINRA) of the United States are satisfied. Under the Index model, satisfying the constraint conditions of broad index can bring the highest return, and the variance also reaches the maximum at this time, while satisfying the constraint conditions of Regulation T of FINRA can get the highest Sharpe ratio.

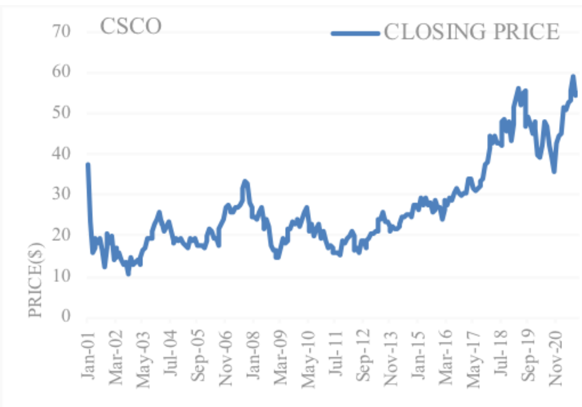
2. Data

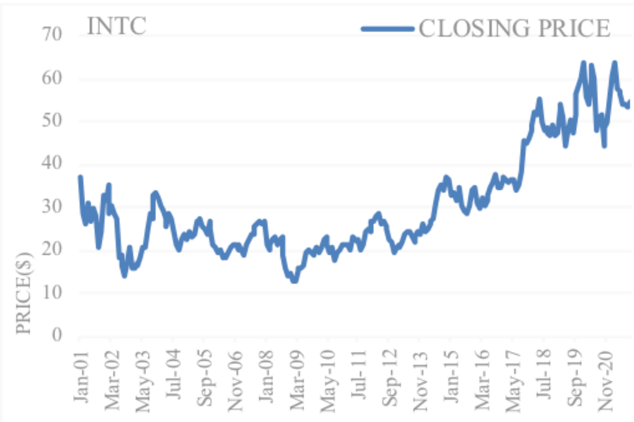
2.1 Stocks Introduction

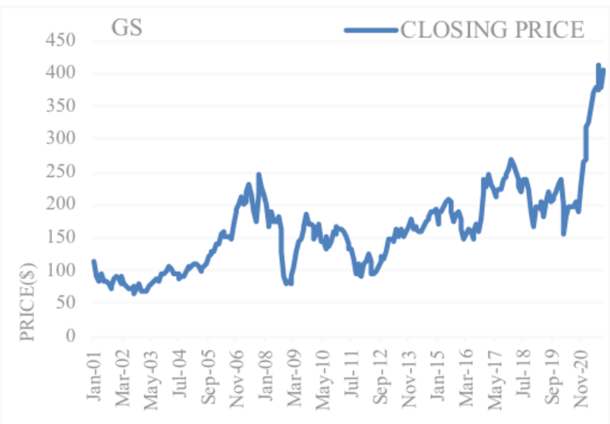
Cisco Systems, Inc. (CSCO) engages in the design, manufacture, and sale of Internet Protocol based networking products and services related to the communications and information technology industry. And another company, Intel Corp. (INTC) engages in the design, manufacture, and sale of computer products and technologies. It delivers computer, networking, data storage, and communications platforms. For the financial services section, Goldman Sachs Group, Inc. (GS) engages in global investment banking, securities, and investment management, which provides financial services. The Allstate Corp. (ALL) engages in the property and casualty insurance business and the sale of life, accident, and health insurance products through its subsidiaries. Finally, PG engages in the provision of branded consumer packaged goods. CL is engaged in the manufacturing and distribution of consumer products.

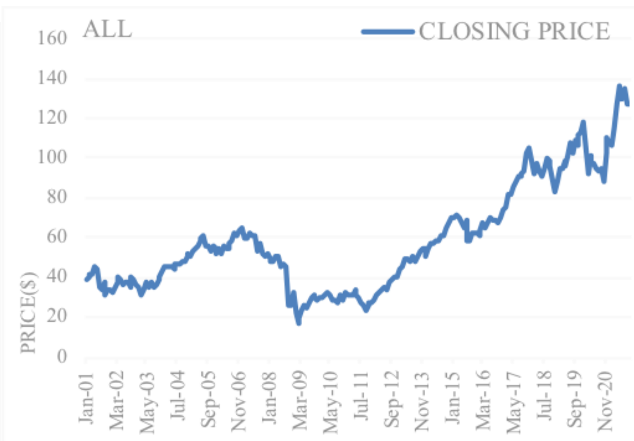
2.2 Data processing

It is important to collect raw data first. After collecting all the raw data, we need to process the data. In order to reduce the non-Gaussian effects, we change all daily data to monthly data. We pick the closing price of the last trading day of each month over the course of 20 years. And then convert these monthly figures into yields. The way it works is that the monthly yield for the current period is equal to this month's closing price divided by last month's closing price minus 1. With this step, we get the monthly returns including the S&P 500 index, six risky assets and the Fed Funds rate.









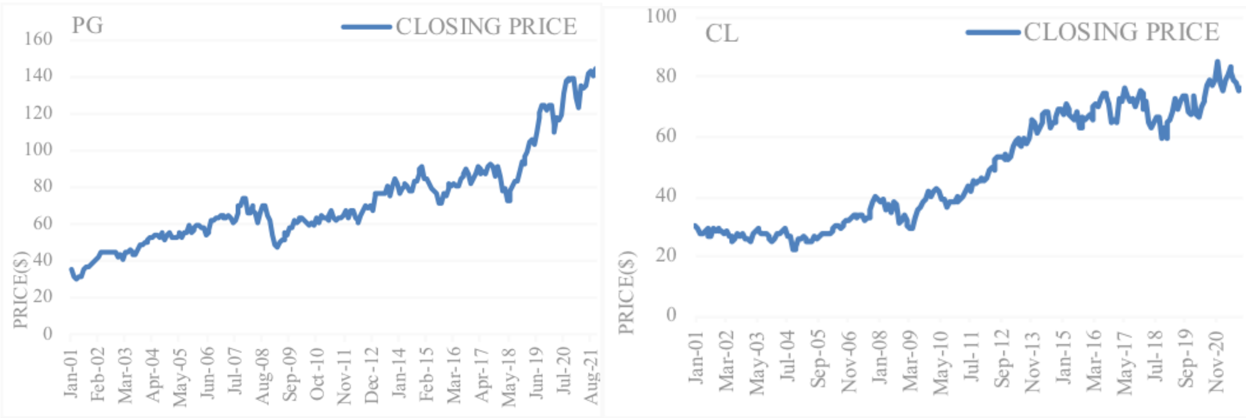


Fig 1. Closing price per month of six stocks

Source: Investing.com

From the monthly returns in the previous step, we can calculate the monthly excess returns of the six risky assets in the S&P 500 index. And we can find the excess return of the index and the stock is the difference between the return on the stock and the Fed Funds rate.

Then the monthly average excess return and standard deviation can be obtained through the functions of average and standard deviation in Excel. But these monthly numbers need to be converted to annually, where the annualized average excess return is equal to the average monthly excess return times 12, and the annualized standard deviation is equal to the monthly standard deviation times the square root of 12.

Through the calculation of the above three steps, we can calculate the Beta and Alpha values of the index and the stock respectively through excel function. Slope between stock and SPX is the beta value of a single stock. Alpha is measured as non-systemic risk, so the SPX 's Alpha is 0. The Alpha value of a stock is the excess return of intercept of stock and SPX. And the Alpha annualized data needs to be multiplied by 12.

Using calculated Annualized Average Residual Return and Residual Standard Deviation. From the results of Beta and Alpha in the previous step, we can get the monthly predicted return for each stock. The monthly residual value of the stock is the difference between the predicted and actual rate of return. Then repeat the operations in the third step to calculate Annualized Average Residual Return and Residual Standard Deviation.

In addition, in order to achieve the lowest risk of price volatility and the highest return, we limited the initial weight of the six stocks to 100%÷6≈16.67% and the expected monthly return of the portfolio to 1%. Therefore, we use these formulas to calculate the historical return rate data of sample stocks, and to calculate the mean and variance of return rate, use the obtained data to measure the expected return rate and risk of assets.

TABLE 1. THE RESULT OF DATA PROCESSING

	SPX	CSCO	INTC	GS	ALL	PG	CL
Annualized Average Return	0.075	0.097	0.089	0.108	0.101	0.094	0.071
Annualized Standard Deviation	0.149	0.308	0.305	0.296	0.249	0.146	0.154
Beta	1.000	1.321	1.188	1.410	1.056	0.405	0.454
Alpha	0.000	-0.003	-0.001	0.002	0.021	0.064	0.037
Residual Standard Deviation	0.000	0.238	0.249	0.209	0.193	0.133	0.138

TABLE 2: CORRELATION

	SPX	CSCO	INTC	GS	ALL	PG	CL
SPX	1	0.637	0.578	0.708	0.630	0.412	0.440
CSCO	0.637	1	0.614	0.488	0.297	0.220	0.165
INTC	0.578	0.614	1	0.411	0.286	0.136	0.110
GS	0.708	0.488	0.411	1	0.417	0.173	0.203
ALL	0.630	0.297	0.286	0.417	1	0.346	0.407
PG	0.412	0.220	0.136	0.173	0.346	1	0.483
CL	0.440	0.165	0.110	0.203	0.407	0.483	1

3. Method

The approach of using the historical stock prices and trend in evaluating future stock performance is referred to as technical analysis. In this study we use the results of the statistical analysis conducted on the historical data to create insights about the future prospects of the respective stocks under analysis. The statistical parameters used in the comparative analysis over the period are; the average return over the period, the variance which represents volatility and risk, the standard deviation which can be used as a proxy for the extent of deviations from the mean and lastly the Sharpe ratio which is considered as a more reliable measure of stock performance since it indicates the risk adjusted returns.

Investors can choose a passive or active investment portfolio strategy; in the case of a passive investment portfolio strategy all the fundamental and technical analysis plus other techniques are employed prior to investing, then once the stocks are purchased, they are held in the same proportion in the long term until the investors decides to dispose of their shareholding. However, in the case of an active investment portfolio strategy; the financial analyst or investor conducts a regular review of the status of the investment portfolio, identifying any opportunities that may allow for profits through portfolio restructuring or opportunities to save the investment from losses through portfolio restructuring. In this case we will assume an active portfolio management strategy, the study will therefore evaluate the performance of the stocks in the past period.

3.1 Two models

In this paper, the weight of each portfolio is constructed by the function of solver table from excel, which is primarily based on the following two standards: (1) Setting goals as minimum variance or maximum Sharpe ratio; (2) Setting constraints respectively according to the five ones which will be introduced later.

The portfolio is analyzed under two different models, which are IM and MM.And the portfolio is formed as: $weight_{spx} \times spx + weight_{CSCO} \times CSCO + weight_{INTC} \times INTC + weight_{GS} \times GS + weight_{ALL} \times ALL + weight_{PG} \times PG + weight_{CL} \times CL$. Where stands for the weights allocated to each stocks.

The index model (IM), first suggested by Sharpe, offers insight into portfolio diversification. Suppose that we choose an equally weighted portfolio of n securities.

Markowitz model (MM) assumes an investor has 2 considerations when constructing an investment portfolio: expected return and variance (risk) in return. This model was developed Harry Markowitz, who derived the expected rate of return for a portfolio of assets and an expected risk measure. Markowitz showed that the variance of the rate of return was a meaningful measure of portfolio risk under a reasonable set of assumptions, and he derived the formulas for computing the variance of a portfolio.

We now show that, as the number of stocks included in this portfolio increases, the part of the portfolio risk attributable to non-market factors becomes ever smaller. This part of the risk is diversified away, and therefore will be of little concern to investors. In contrast, market risk remains, regardless of the number of firms combined into the portfolio.

To make more portfolios, we give 5 constraints as following.

3.2 Constraints Introduction

1) Benchmark

A "free" problem, without any additional optimization constraints, to illustrate how the area of permissible portfolios in general and the efficient frontier in particular look like if you have no constraints.

2) Constraint1: $\sum_{i=1}^{7}|w_i| \leq 2$

This additional optimization constraint is designed to simulate the Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity.

3) Constraint2: $|w_i| \leq 1, for \forall i$

This additional optimization constraint is designed to simulate some arbitrary "box" constraints on weights, which may be provided by the client.

4) Constraint3: $w_i \geq 0, for \forall i$

This additional optimization constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions.

5) Constraint4: $w_1 = 0$

Lastly, if the inclusion of the broad index into our portfolio has positive or negative effect, for that we would like to consider an additional optimization constraint.

In the judgment of the optimal portfolio of these stocks, we draw Markowitz model and Index model as the basis to find the best investment point. The use of the optimization inputs is targeted at realizing the permissible portfolios which will include the minimum variance portfolio and the maximum Sharpe portfolio.

For Markowitz model, it requires a large number of estimates of expected returns, variances, and co-variances. In the long run, the efficient portfolios should beat the ones with less reliable input lists, and therefore inferior reward-to-risk.

Markowitz model quantifies investment risk through the expected return rate and the standard deviation between the returns. We assume that every investor in risk averse, so we need to find the point of least risk under the constraint of constant returns. Under the constraint risk, the maximum return point should be found. Similarly, under certain constraints, the decision made by investors are related to expected rate of return and variance, and the risks generated by portfolios are related to the risks of individual assets and the co-variance between assets.

The sample selected in this paper has a long time period, from May, 2001 to May, 2021. Since Markowitz's portfolio model focuses on the comprehensive analysis of stock investment risks and returns, long-term returns should be adopted, so we use monthly returns to measure.

For Index model, it can exhibit the power and limits of diversification, simplifies the estimation of co-variance matrix problem and enhances the analysis of security expected returns. The cost of index-model simplification is in restrictions it places on statistics of assets returns.

In investment markets, when conditions improve, the price of individual stocks rises, and vice versa. A single index model can be used to value stocks in an investment. Under the condition of venture capital, one strategy that can reduce investment risk is to build a portfolio. At the same time, in order to measure the size of risk, variance can be used to estimate.

The CAPM is a statement about the expected returns. We can replace the theoretical market portfolio of the CAPM with the well-specified and observable market index.

4. Results and discussion

An investment portfolio consists of a group of stocks chosen by an individual investor, institutional investor or financial analysts for the purpose of a combined investment. The main motivation for choosing a variety of stocks is to improve the overall return while simultaneously reducing the extent of risk exposure. By choosing a mix of high return, high risk stock and medium return low risk stocks the investor will be able to gain from an upward trend in the high return stocks while at the same time protecting their overall investment given the low-risk stocks.

In determining the companies that are likely to post good returns into the future we consider the risk versus return profile, the Sharpe ratio is a risk adjusted return and is thus the best parameter to be used in determining the stocks that are likely to have a continued good performance into the future based on their past trends. The risk adjusted nature of the Sharpe ratio makes it a superior measure of stock performance. In determining the optimal portfolio, the study choose the portfolio that results in the highest Sharpe ratio since it constitutes the highest risk adjusted return. In that case this becomes the optimally risky portfolio. However, it is important to note that different investors have different preferences in regards to their risk appetite, for this reason certain investors may prefer low risk portfolios and, in this case, will choose the minimum portfolio variance.

TABLE 3. WEIGHTS UNDER MINIMUM VARIANCE PORTFOLIO

MM	SPX	CSCO	INTC	GS	ALL	PG	CL
Benchmark	55.3	-7.0	2.7	-5.5	-4.6	29.2	30.1
Constraint 1	57.1	-6.0	2.7	-5.5	-9.6	35.2	26.2
Constraint 2	59.0	-6.0	2.9	-6.5	-9.6	32.0	28.1
Constraint 3	34.1	0.0	1.5	0.0	0.0	36.9	27.5
Constraint 4	0.0	0.0	8.9	5.5	0.0	47.1	38.5
IM	SPX	CSCO	INTC	GS	ALL	PG	CL
Benchmark	14.3	14.3	14.3	14.3	14.3	14.3	14.3
Constraint 1	51.1	-6.5	-3.5	-10.8	-1.7	38.6	32.8
Constraint 2	51.0	-6.5	-3.5	-10.8	-1.7	38.6	32.9
Constraint 3	23.0	0.0	0.0	0.0	0.0	41.6	35.4
Constraint 4	0.0	-0.4	1.8	-2.6	6.5	50.5	44.3

Note: 1. All the data in the table need to multiply 1%
2. MM stands for the Markowitz model; IM stands for the Index model.

TABLE 4. RETURN, STANDARD DEVIATION AND SHARPE RATIO UNDER MINIMUM VARIANCE PORTFOLIO

MM	Return	Standard deviation	Sharpe ratio
Benchmark	0.0756	0.1165	0.6489
Constraint 1	0.0758	0.1156	0.6554
Constraint 2	0.0747	0.1155	0.6464
Constraint 3	0.0814	0.1182	0.6889
Constraint 4	0.0857	0.1230	0.6962
IM	Return	Standard deviation	Sharpe ratio
Benchmark	0.0909	0.1607	0.5653
Constraint 1	0.0754	0.1070	0.7051
Constraint 2	0.0754	0.1070	0.7052
Constraint 3	0.0818	0.1111	0.7360
Constraint 4	0.0840	0.1138	0.7378

TABLE 5. WEIGHTS UNDER SHARPE RATIO PORTFOLIO

MM	SPX	CSCO	INTC	GS	ALL	PG	CL
Benchmark	1.5	0.2	6.4	9.0	3.4	61.4	18.2
Constraint 1	1.5	0.2	6.4	10.0	4.4	61.4	16.1
Constraint 2	1.5	0.2	2.4	11.0	4.4	61.4	19.0
Constraint 3	2.5	0.2	6.4	10.0	6.4	59.0	15.0
Constraint 4	0.0	0.4	6.6	10.3	4.6	61.8	16.3
IM	SPX	CSCO	INTC	GS	ALL	PG	CL
Benchmark	5.3	-0.7	-0.1	1.3	8.6	56.9	29.1
Constraint 1	7.4	-0.7	-0.1	0.7	8.6	54.8	29.4
Constraint 2	10.5	-0.7	-0.1	0.7	7.2	54.8	28.0
Constraint 3	9.3	0.0	0.0	0.7	8.6	53.7	27.9
Constraint 4	0.0	0.2	0.6	1.9	9.8	56.7	30.8

Note: 1. All the data in the table need to multiply 1%

TABLE 6. RETURN, STANDARD DEVIATION AND SHARPE RATIO UNDER MAXIMUM SHARPE PORTFOLIO

MM	Return	Standard deviation	Sharpe ratio
Benchmark	0.0912	0.1265	0.7204
Constraint 1	0.0917	0.1271	0.7209
Constraint 2	0.0913	0.1272	0.7177
Constraint 3	0.0914	0.1269	0.7202
Constraint 4	0.0919	0.1275	0.7209
IM	Return	Standard deviation	Sharpe ratio
Benchmark	0.0877	0.1160	0.7560
Constraint 1	0.0868	0.1147	0.7562
Constraint 2	0.0868	0.1148	0.7559
Constraint 3	0.0869	0.1150	0.7558
Constraint 4	0.0881	0.1166	0.7554

5. Conclusion

Due to the COVID-19 pandemic which has spread world-widely since the end of 2019, the stock markets have been negatively affected. As a result, we built both minimum variance and maximum Sharpe ratio portfolios under Index model and Markowitz model limited by 5 different constraints. Using relevant data of 6 stocks in 3 industries from May, 2001 to October, 2021, we aim to find out which combination is suggested to the investors that can best reduce potential risks as to achieve the ultimate goal of returns.

Portfolio that results in the highest Sharpe ratio should be chosen; Generally speaking, from table 3 to table 6, we can see that no matter which portfolio or model it is under, the constraint requiring the weight of SPX to be zero always generates the biggest Sharpe ratio. Such as 0.7378 in the case of minimum variance portfolio under Index model, and 0.7209 as for maximum Sharpe ratio portfolio under Markowitz model. Also, when taking the values of return into account, we found that basically the same constraint produces the highest ones as well. For example, 0.0857 in minimum variance portfolio constructed by Markowitz model and 0.0919 under maximum Sharpe ratio portfolio with Index model. So we drew the conclusion that to invest in combination with constraint that makes the weight of SPX zero is relatively the best method for investors to obtain the maximum return.

As according to the results, we advise investors to change the weight of SPX to zero in order to attain optimal return.

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