Portfolio Construction Based on Markowitz Model and Index Model for Multiple Assets

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Abstract. Asset allocation has a long history, and is of vital importance. In retrospect, big events (e.g., the World War) gave rise to various shocks to the financial markets and brought large fluctuations. Contemporarily, COVID-19 has been greatly affected the daily life of human beings as well as the stock market for 3 years. On this basis, the research topic is about portfolio construction for multiple assets using Markowitz Model and Index Model in order to gain stable and risk-hedge return from the volatile market. The paper includes a deep analysis of 3-year data of 8 stocks from 3 types of companies in USA. Then it calculates the Minimal Variance Point, Maximum Sharpe Ratio, together with the efficient and inefficient portfolio construction with regard to 3 constraints. Based on the evaluations, this study provides an idea and method of asset allocation in a certain situation, and is appliable in various situations if is flexibly used. These results shed light on guiding further exploration of portfolio construction.

Keywords: Portfolio construction; Markowitz model; Index model.

1. Introduction

Asset allocation has a history of over 200 years. The 19th century witnessed the birth of institutional investing. These institutions embraced the illiquidity premium in private credit markets under falling government bond yields. The First World War marked the start of a new inflationary cycle. Managing the risks of inflation would shape investment strategy from now on. The taming of inflation marked the start of the modern era. For the providers of insurance and pensions, asset allocation strategy is increasingly focused on managing risk. For long-term savers, asset allocation strategy must continue to target returns [1]. While active management has about the same impact on performance as a fund's specific asset allocation policy, asset allocation is still very important. For a true market-neutral hedge fund that has hedged away all possible beta risk exposures. For a long-only passive index product, asset allocation policy dominates [2-4].

Nowadays, many predecessors have built their own unique asset allocations. For Bridgewater Associates 13F Sector Allocation, from 2019, the start of COVID-19, the proportion of finance allocation has been reduced to one half of its past scale, and the proportion of healthcare and consumer stables has been largely increased [5]. For Renaissance Technologies LLC, they have had virtually unchanged all types of asset allocation since 2019, except for a slight increase in the proportion of healthcare allocation in 2020 [6]. For Man Group PLC, they also almost have had unchanged all types of asset allocation since 2019. During the period, the proportion of asset allocation of each type are continuously floating slightly, with industrials sharing the largest proportion [7]. For Millennium Management LLC, the proportion of healthcare increased in the beginning of COVID-19, then in 2020, they increased the proportion of finance asset allocation [8]. For Citadel Advisors LLC, the proportions of healthcare and finance allocation have been increased in the past 3-year time, and the proportion of energy has been greatly reduced [9].

COVID-19 increased volatility to the stock market and led to panic trading in many major indices. US S&P 500 was halted many times during March 2020 when it fell by 7% or more [10]. Although the situation has become better now, the impact still exists. Therefore, the motivation for writing this article is to study the best way to construct portfolios for multiple assets, based on the US stock market data over the past three years under the influence of the new crown epidemic. The rets part of the paper is organized as follows. Primarily, this study showed the historical data of 8 stocks from 3 industries in the United States selected and the reasons for the selection. Then, this study calculated

by applying Markowitz Model and Index Model to plot the minimum variance point and the maximum Sharpe ratio point, Minimal Variance Frontier, Efficient Frontier and Inefficient Frontier under 3 assumptions. Finally, this paper analyzed the limitations of this study and its implications for the future, and drew conclusions.

2. Data & Method

2.1 Data

In this paper, 8 stocks from Technology companies, Financial Services and Healthcare companies have been chosen. In order to give a better understanding and prediction of the stocks in the big environment of COVID-19 pandemic, daily data from October 15, 2019 to October 14, 2022 has been gathered from the website investing.com.

2.1.1 Technology companies

Apple Inc (AAPL), Intel Corporation (INTC) and Microsoft Corporation (MSFT) have been chosen in technology companies. It can be seen from Fig. 1 that the share prices of Apple Inc and Microsoft corporation has similar trend, this study chose them because their stock price had had several small short-term gains and decreases in recent times, and they had just experienced a small sustained decline, so this study expects their stock price to rise soon. For the Intel Corporation, its stock price had experienced a sustained decline in the past one and a half years, and an extremely sharp decline especially this year. Due to the sharp decline in Intel Corporation's share price, its share price is expected to have bottomed out and would recover in the short term.

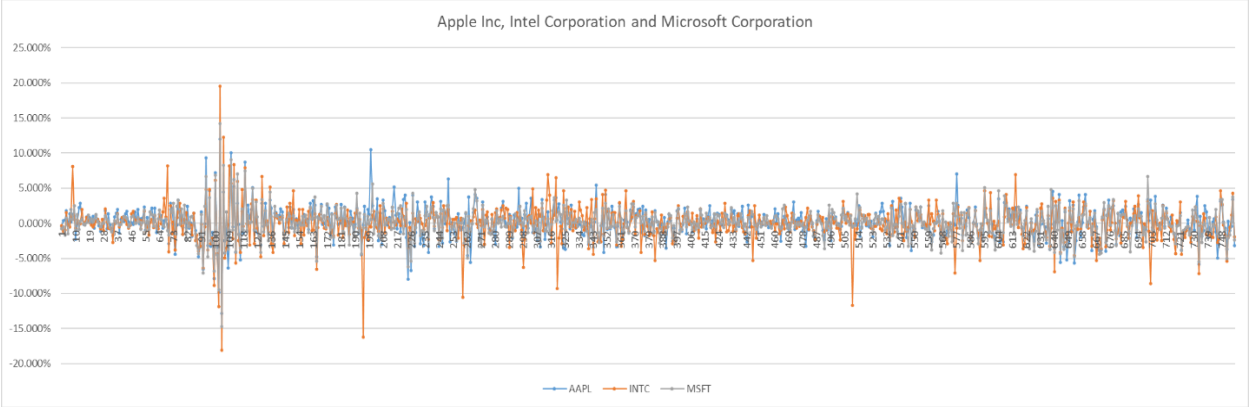

Figure 1. Apple Inc**,** Intel Corporation and Microsoft Corporation.

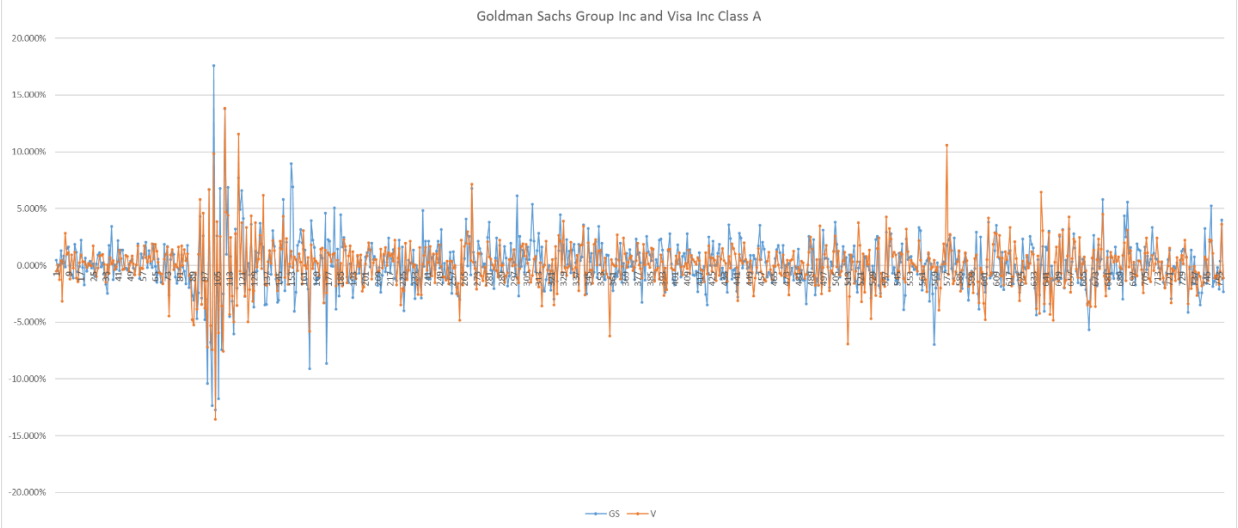

Figure 2. Goldman Sachs Group Inc and Visa Inc Class A.

2.1.2 Financial services

Goldman Sachs Group Inc (GS) and Visa Inc Class A (V) have been chosen in Financial Services. Seen from Fig. 2, the share prices of them had fluctuated several times in recent years, with Visa Inc Class A floating more widely in its share prices. Since their share prices had all experienced small declines recently, it is speculated that they would rise soon.

2.1.3 Healthcare companies

Johnson & Johnson (JNJ), Pfizer Inc (PFE) and UnitedHealth Group Incorporated (UNH) have been chosen for healthcare companies. Owing to the troubles caused by the new crown epidemic in recent years, the stock price of the medical industry has generally increased in recent years as depicted in Fig. 3. Johnson & Johnson, Pfizer Inc and UnitedHealth Group Incorporated, whose stock prices have been growing overall and relatively stable since the second half of 2019, has been chosen. Since their share prices have declined slightly recently, it is inferred that their share prices will rise soon.

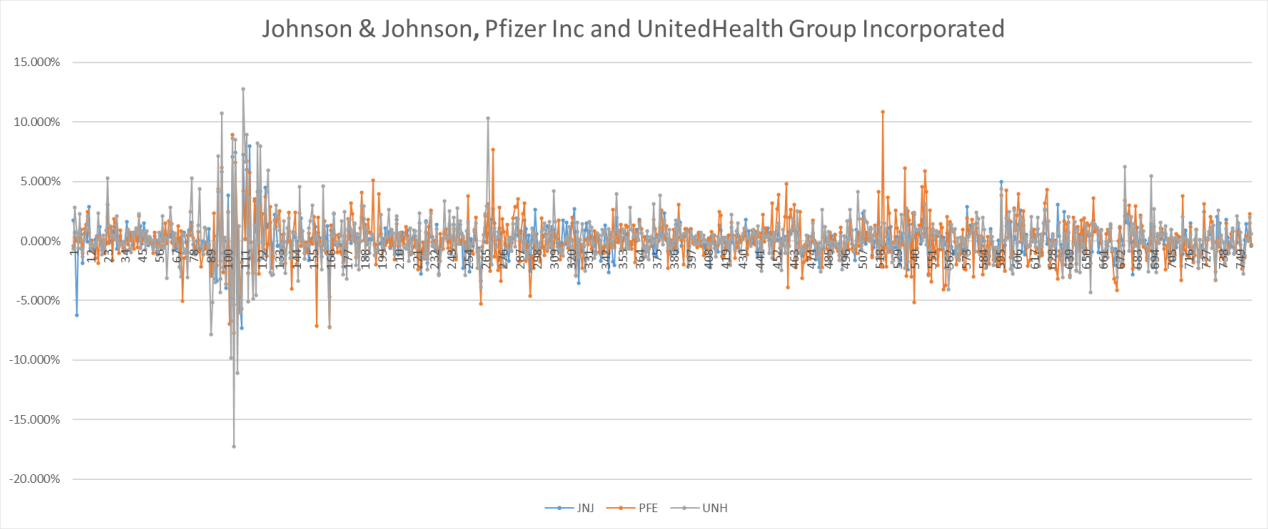

Figure 3. Johnson & Johnson**,** Pfizer Inc and UnitedHealth Group Incorporated.

Table 1. Different values calculated from the historical data

	SPX	AAPL	INTC	MSFT	GS	V	JNJ	PFE	UNH
Annualized Average Return	9.045%	34.861 %	-15.146%	21.520 %	19.427 %	6.091%	9.570%	11.461 %	30.944 %
Annualized StD	24.733 %	35.598 %	40.973%	33.296 %	37.383 %	32.850 %	22.177 %	29.490 %	32.829 %
Beta	1	1.175625	1.169779	1.154807	1.17805	1.080453	0.554927	0.572059	0.95414
Annualized Alpha	0.000%	24.227 %	-25.727%	11.074 %	8.771%	-3.682%	4.550%	6.286%	22.313 %
Residual StdDev	0.000%	20.537 %	29.013%	17.114 %	23.422 %	19.105 %	17.420 %	25.874 %	22.822 %

Table 2. Correlation Matrix

	SPX	AAPL	INTC	MSFT	GS	V	JNJ	PFE	UNH
SPX	1	0.817	0.706	0.858	0.779	0.813	0.619	0.480	0.719
AAPL	0.817	1	0.587	0.803	0.538	0.630	0.459	0.335	0.541
INTC	0.706	0.587	1	0.641	0.532	0.527	0.410	0.335	0.485
MSFT	0.858	0.803	0.641	1	0.548	0.686	0.505	0.374	0.582
GS	0.779	0.538	0.532	0.548	1	0.643	0.435	0.339	0.564
V	0.813	0.630	0.527	0.686	0.643	1	0.501	0.400	0.589
JNJ	0.619	0.459	0.410	0.505	0.435	0.501	1	0.572	0.609
PFE	0.480	0.335	0.335	0.374	0.339	0.400	0.572	1	0.466
UNH	0.719	0.541	0.485	0.582	0.564	0.589	0.609	0.466	1

2.2 Models

Markowitz Model and Index Model have been applied to calculate the efficient frontier (Maximum return) and the inefficient frontier (Minimal Return) of the portfolio construction. After gathering the 3-year historical data of the 8 stocks plus the S&P 500 US, different values necessary for calculation, together with a correlation matrix has been made. The values and coefficients of correlation are shown in Table. 1 and Table. 2.

2.2.1 Markowitz model

Markowitz Model is put forward by Harry Markowitz in 1952. There are 3 steps in Markowitz Portfolio Selection:

- Determine the minimal variance frontier, allow risk-return combinations;
- Identify the optimal risky portfolio as the steepest CAL tangential to the opportunity set;
- Choose the appropriate complete Portfolio by mixing with the risk-free asset given risk-aversion [11].

2.2.2 Index model

The Single-Index Model was put forward by William Sharpe in 1963. The index model simplifies the estimation of covariance matrix problem. Moreover, it enhances the analysis of security expected returns. Additionally, it decomposes the risk into systematic and firm-specific components.

3. Results & Discussion

3.1 Three Constraints

This study gathered the three-year historical data, selected the weekly and monthly data counted by the frequency table and finally got the histogram of the 3-year data (seen from Fig. 4). In this case, this study doesn't put any constraints to the model. Namely, this study will see what the data outcome and the graph is going to be like if this study excludes any requirements in our investment. Firstly, this study uses the Markowitz Model (MM) for calculation.

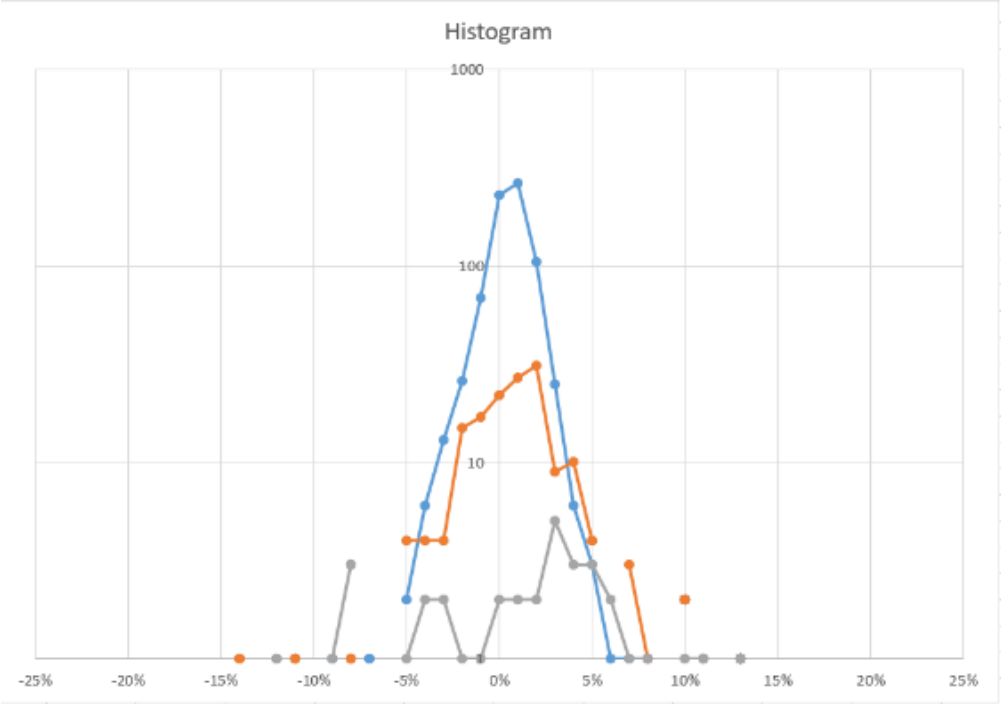

Figure 4. Histogram of the historical data

Table 3 shows the Minimal Variance, Maximum Sharpe and the Capital Line (CAL) of Markowitz Model for constraint 1. It can be read from the table that the minimal variance is 19.69% and the maximum Sharpe is equal to 2.146. The sharp ratio is exactly the slope of the Capital Line. Seen from Fig. 5, since the corresponding return and standard deviation when this study gest the maximum Sharpe is extremely large, the graph of the minimal variance point and the CAL seems to be a bit weird as the random points all gathered near the "return-axis". Therefore, this study adjusted the legend for better performance of the outcome. Besides, the horizontal axis is the "Standard deviation-axis", and the vertical axis is the "Return-axis".

In the Fig. 5, "MinVarFr" stands for "Minimal Variance Frontier", this frontier is rough, because it contains no more than 10 points including the Minimal Variance point. Conversely, the "MinVar1", which stands for "Minimal Variance Frontier under Constraint 1", is the precise curve. It has pointed every 0.5% difference in return. It is obvious that all the random points lie inside the Minimal Variance Frontier. "MaxRet1" is short for "Maximum Return under constraint 1", and is also known as the "Efficient Frontier under Constraint 1". As its name suggests, this curve shows the maximum return this study can get in the same standard deviation. Conversely, the "MinRet1", known as "Minimal return under Constraint 1" shows the minimum return that one can get in the same standard deviation. One can see that the combination of "MaxRet1" and "MinRet1" is approximately the same with the "MinVar1", except for the points near the minimal variance point. There are no some obvious outliers, which are much different from the value of "MinVar1" at some points. Table 4 shows the Minimal Variance, Maximum Sharpe, Capital Line (CAL) of Index Model under Constraint 1. While the Minimal Variance result is approximately the same as from Markowitz Model, the Maximum Sharpe from the two model are quite different. The points in Fig. 5 are well below the capital line, and this study adjusts the range of "Standard deviation-axis" for better demonstration of the outcome. Similar to what it is like using Markowitz Model, the Maximum return curve and the minimal return curve do not have many outliers compared to the Minimal Variance curve, and such phenomenon occurs near the minimal variance point.

3.1.1 Constraint 1: free of constraints

In this case, one doesn't put any constraints to the model. Namely, this study will investigate what the data outcome and the graph is going to be like if this study excludes any requirements in our investment.

3.1.2 Constraint 2: no sell short

Since weights can never go to plus or minus infinity, this study has to get prepared for different kinds of constraints. When the weights are limited, the returns and the standard deviations are also limited. this study assumed that no "Sell Short" are allowed. In formula, this study can write it as $w_i \geq 0$ for all $i = 0,1,2,\ldots,9$. To calculate in this constraint, this study should first find some limits. When one is not allowed to sell short, the curve should be closed and compact, and the smallest and biggest possible returns should be respectively the smallest and the biggest annualized average return of the 8 stocks plus the S&P 500 US. Known from Table 1m it makes no sense unless the returns are between -15.15% and 34.86%.

3.1.3 Constraint 3: exclusion of broad index

Now, one wants to see whether the inclusion of the broad index into our portfolio has positive or negative effect. Therefore, this study would like to consider an additional optimization constraint, which in formula is $w_1 = 0$. When this happens, the column of "SPX" should always be 0.

3.2 Discussion and Comparison of Different Models and Constraints

Table 3 and 4 show the Minimal Variance and Maximum Sharpe of Markowitz Model and Index Model under 3 constraints. For instance, for Markowitz Model under constraint 1, the minimal variance is 19.688% and the maximum Sharpe is equal to 2.146. The sharp ratio is exactly the slope of the Capital Line. Moreover, In the "MinVariance" Row of Markowitz Model under constraint 2, 5.134E-10 means 5.134×10^{-10}, which is a very small positive number.

Table 3. Minimum Variance, Maximum Sharpe, CAL of Markowitz Model for 3 Constraint.

MM Constraint 1	SPX	AAPL	INTC	MSFT	GS	V	JNJ	PFE	UNH	Return	StdDev	Sharpe
MinVariance	1.20	-0.12	-0.07	-0.21	-0.17	-0.11	0.50	0.11	-0.13	0.01	0.20	0.06
MaxSharpe	-56.93	21.70	-10.39	14.85	14.24	-2.86	1.26	3.54	15.61	15.13	7.05	2.15
MM Constraint 2	**SPX**	**AAPL**	**INTC**	**MSFT**	**GS**	**V**	**JNJ**	**PFE**	**UNH**	**Return**	**StdDev**	**Sharpe**
MinVariance	0.32	0.00	0.00	0.00	0.00	0.00	0.55	0.12	0.00	0.10	0.21	0.47
MaxSharpe	0.00	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.49	0.33	0.30	1.10
MM Constraint 3	**SPX**	**AAPL**	**INTC**	**MSFT**	**GS**	**V**	**JNJ**	**PFE**	**UNH**	**Return**	**StdDev**	**Sharpe**
MinVariance	0.00	0.05	0.00	0.04	0.05	0.06	0.70	0.16	-0.05	0.10	0.21	0.49
MaxSharpe	0.00	1.85	-1.81	0.48	0.57	-1.40	-0.43	0.25	1.50	1.50	0.80	1.86

In the same model under different constraints, it can be seen that the Maximal Sharpe point are a lot different from each other for both models, and the Minimal Variance point under constraint 2 and 3 are similar, and are quite different to that under constraint 1 in both model Under the same constraint in different models, it can be seen that the Minimal variance points of Markowitz Model are closer to the origin than those of Index Model, and the Maximum Sharpe points of Markowitz Model are farther to the origin than those of Index Model. Additionally, for both models, the Minimal Variance when there is constraint is higher than that when there is no constraint, and the Maximum Sharpe ratio when there is constraint is lower than that when there is no constraint.

Table 4. Minimum Variance, Maximum Sharpe, CAL of Index Model for 3 Constraints.

IM Constraint 1	**SPX**	**AAPL**	**INTC**	**MSFT**	**GS**	**V**	**JNJ**	**PFE**	**UNH**	**Return**	**StdDev**	**Sharpe**
MinVariance	0.14	0.00	0.00	0.00	0.00	0.00	0.57	0.25	0.03	0.11	0.20	0.54
MaxSharpe	-8.40	3.92	-2.09	2.58	1.09	-0.69	1.02	0.64	2.92	2.72	1.36	2.00
IM Constraint 2	**SPX**	**AAPL**	**INTC**	**MSFT**	**GS**	**V**	**JNJ**	**PFE**	**UNH**	**Return**	**StdDev**	**Sharpe**
MinVariance	0.14	0.00	0.00	0.00	0.00	0.00	0.57	0.25	0.03	0.11	0.20	0.54
MaxSharpe	0.00	0.52	0.00	0.00	0.00	0.00	0.00	0.00	0.48	0.33	0.31	1.08
IM Constraint 3	**SPX**	**AAPL**	**INTC**	**MSFT**	**GS**	**V**	**JNJ**	**PFE**	**UNH**	**Return**	**StdDev**	**Sharpe**
MinVariance	0.00	-0.04	-0.02	-0.03	-0.03	0.05	0.66	0.29	0.12	0.11	0.20	0.57
MaxSharpe	0.00	1.39	-1.35	0.47	0.10	-0.98	0.13	0.14	1.09	1.12	0.62	1.80

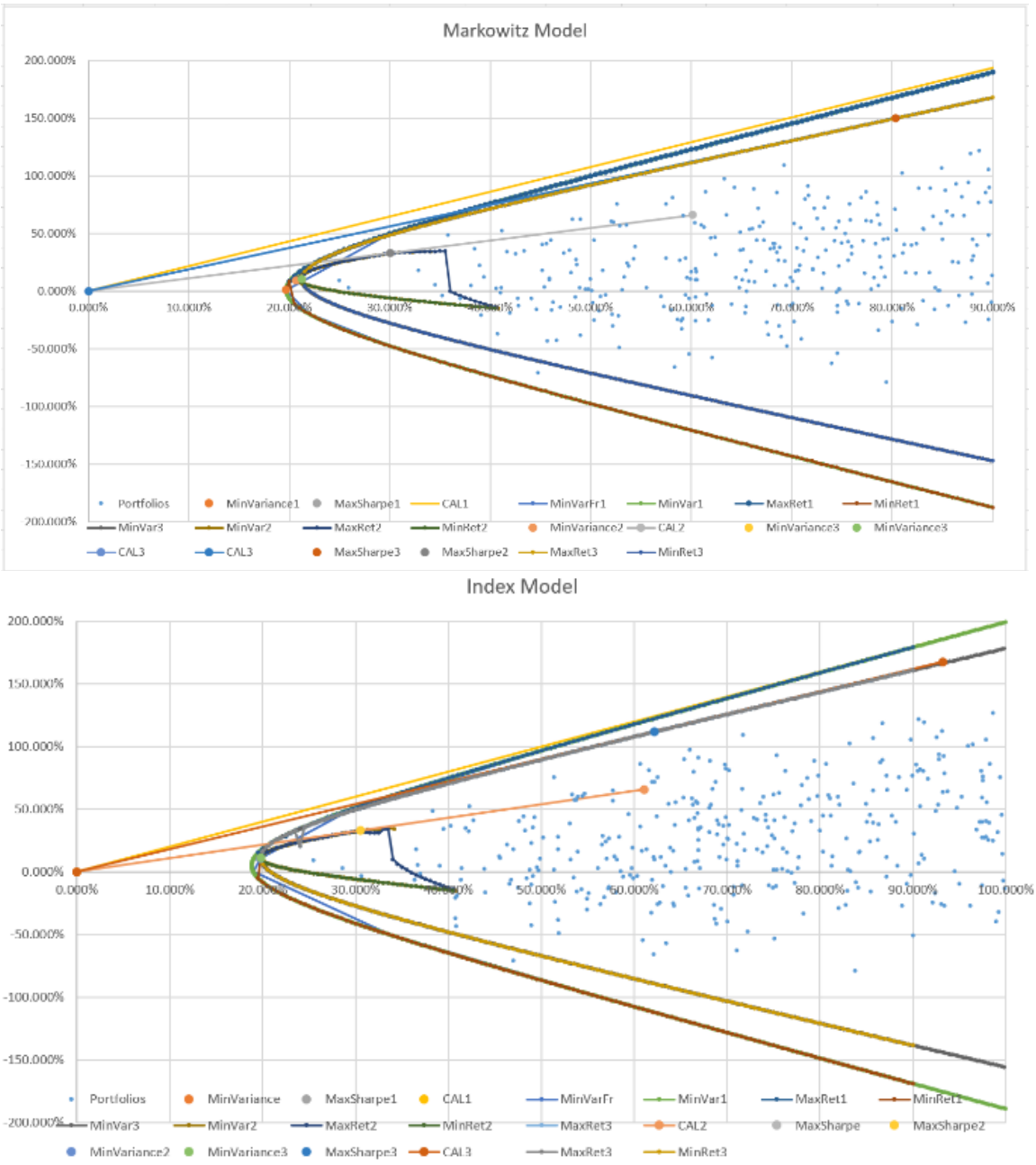

Figure 5. All Values, Points and Frontiers under 3 constraints

Fig. 5 shows all values, points and frontiers under 3 constraints using Markowitz Model and Index Model. The horizontal axis is the "Standard deviation-axis", and the vertical axis is the "Return-axis". In the graph, "MinVarFr" stands for "Minimal Variance Frontier", this frontier is rough, because it

contains no more than 10 points including the Minimal Variance point. Conversely, the "MinVar1", which stands for "Minimal Variance Frontier under Constraint 1", is the precise curve. It has pointed every 0.5% difference in return. It is obvious that all the random points lie inside the Minimal Variance Frontier. For instance, "MaxRet1" is short for "Maximum Return under constraint 1", and is also known as the "Efficient Frontier under Constraint 1". As its name suggests, this curve shows the maximum return one can obtain in the same standard deviation. Conversely, the "MinRet1", known as "Minimal return under Constraint 1" shows the minimum return that one can get in the same standard deviation. The combination of "MaxRet1" and "MinRet1" is approximately the same with the "MinVar1", except for the points near the minimal variance point. One doesn't not have some obvious outliers, which are much different from the value of "MinVar1" at some points.

Seen from the results, the "extreme points" calculated by the two models have certain difference, while the "extreme values" are roughly the same. For Minimal Variance 2, since it makes no sense unless the returns are between -15.15% and 34.86%, the graph is much "narrower" that others. The graph under constraint 1 and 3 are of similar shape. Moreover, the outcome of the two models is roughly the same. Regarding to the maximum and minimal return comparison under 3 Constraints. The graph under constraint 1 shows the range of all possible points in investments. No point is going above the efficient frontier or going below the inefficient frontier. The graph under constraint 2 is a lot different from that under Constraint 1. It is also easy to explain. When all indexes are nonnegative, the range of possible numbers of each index approximately 1/2 of that under free constraints. Therefore, the total range of the 9 indices should be approximately $1/2^9$ of the overall range under free constraints. Since the range of every index under constraint 2 is bounded, the combination of the Maximum and Minimum Return again forms a closed and convex area. The graph under Constraint 3 is of similar shape with the one under Constraint 1 instead of Constraint 2. This is because only 1 index is restricted, and all the other points can be random numbers, thus the total influence is tiny. This accounts for the fact that only 1 point is out of the frontier among all 1,000 random points. While the outcomes of the two models are mainly the same, compared to the graphs of Markowitz Model, the graphs of Index Model under 3 constraints have more outliers, especially under constraint 1 and 3, which is significantly different from the corresponding point in the Minimal Variance curve. But, what is similar is that the outliers are all near the Minimal Variance point.

4. Limitations & Future Outlooks

The Markowitz Model and the Index Model had good abilities in demonstrating the exact points and graphs that this study required, and the results are clear and are easily to understand. However, this study still hasn't considered enough many conditions that could happen in the market, which will lead to different constraints in the model. Moreover, this study lacked the concrete policy of the government under COVID-19 period, or data supporting such policies. Additionally, the selection of stocks is based on our inference under the big environment of COVID-19. Nevertheless, the impact of COVID-19 on every aspect of life tends to decline to nothing, meantime the policy both nationwide and worldwide will certainly be different. It is equal to say that after certain period of time, the specific type of companies which has a promising perspective with regard to their share prices will be different. therefore, the practical period of the results of this study will not be long. Last but never the least, since the investors hold different funds invested in the stock market and have different identities, their corresponding optimal portfolio construction plans will also be different. The impact of the constraints on the eventual efficient frontier and inefficient frontier is very large, with small investments leading to a significant reduction in expected yields and large investments significantly increasing their instability if one pursues high yields. It is a pity that this study didn't go on to discuss the different plans for different populations. Although the research result is efficient only a short period, the research provides a general method of portfolio construction under multiple assets. Combining the method with the changing situation in the stock market, it is not difficult to refresh the result and stick to the maximum return when investing.

5. Conclusion

To cope with portfolio construction under multiple assets, the paper researched into 8 stocks from 3 types of US industries. Markowitz Model and Index Model has been used to carry out the result. The graph outputs of the two models are roughly the same. The minimal variance tends to be higher under stricter constraints, and the maximum Sharpe ratio tends to be smaller in that case. When every index has a bounded range, the graph of the combination of the efficient frontier and the inefficient frontier should be a closed and convex region. There may be some outliers in the graph, which means significant difference from the corresponding point in the minimal variance curve. The research didn't cover a lot of constraint assumptions or consider different identities of investors, and was not efficient in the long time. However, the idea from the research was not old-fashioned. Once the current situation of the stock market was grasped, new results would come out using the same method. The meaning of conducting this research was to provide a feasible method in order to get the most out of what one put in, and that would never be out of time.

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