A Study on the Applicability of Black-Scholes Option Pricing Model in Chinese Stock Index Option Market

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Abstract. Compared with the developed countries, Chinese stock index option market developed late and accumulated less historical data. At the same time, it has the characteristics that many developed countries' stock option markets do not have. Black-Scholes Model is an important model in the field of option pricing. It is not enough for a financial model to be reasonable in theory, but it needs to be tested by the actual market. Therefore, based on the main transaction data of the CSI 300 stock index option contracts from 2010 to 2020, this paper analyzes the compliance of the CSI 300 stock index option market to the basic assumptions of the B-S option pricing model, the main parameters of the CSI 300 stock index options in the B-S option pricing model, and the fitting degree of the B-S model's prediction and actual prices. The result shows that there is a certain deviation between the pricing conclusions of the B-S model and the actual option prices. This paper analyzes the reasons for the deviation from many aspects, and puts forward the corresponding improvement strategies that the users of the model should adopt, so as to improve the applicability of the B-S option pricing model in Chinese stock index option market.

Keywords: Black-Scholes Model; Fitting Degree; Deviation; Applicability.

1. Introduction

On December 23, 2019, the CSI 300 stock index options were officially listed in the limelight. As the risk control manager of a fund company, the author usually had business dealings with a number of securities and futures trading institutions. At that time, many friends in the industry told the author that 2019 was Chinese "first year of options". From the perspective of historical development, Chinese stock index option market has indeed developed and matured relatively late compared with developed countries, so that a representative and influential index like CSI 300 stock index will not have an option product corresponding to it until 2019 [1]. Soon after, the company where the author worked expanded the scope of investment in financial derivatives, and the author learned a lot of options pricing models in the process of obtaining FRM qualification, including the famous Black Scholes Model [2, 3].

If the B-S model, which was born in the United States in the 1970s, continues to shine in Chinese option market half a century later, in the author's opinion, it will at least go through the triple test of time, market and technology. Firstly, the financial market has experienced development of nearly 50 years from the first presentation of the B-S model in 1973 to today. During this period, old assumptions have been constantly questioned and overturned, and new theories and tools have emerged continuously. The first thing that the theoretical value of B-S itself faces is the test of time span. Secondly, Chinese financial market, especially the financial derivative market, has many characteristics that the United States does not have, such as the large influence of government policies, the high proportion of retail investors, the strong force of speculative psychology and irrational factors, etc. In addition, COVID-19 has an impact on the global economy, which makes the range of factors that affect the actual price far beyond what users of the B-S model can consider [4]. Thirdly, the progress of financial quantitative analysis technology supported by today's electronic transactions and big data technology has greatly reduced transaction costs and shortened the time for decision-making, negotiation and settlement of transactions [5]. The technical progress has enhanced the applicability of the B-S model to a certain extent, but it also makes the actual situation of the market deviate from the prediction of the B-S model from multiple perspectives [6].

Thus, a question arises: can Black Scholes Model become the pricing tool of CSI 300 stock index options? In other words, does the B-S option pricing model have strong applicability in Chinese stock index option market?

In order to answer the above question seriously, it's necessary to analyze the B-S model at different levels. Firstly, from its basic assumptions, the author analyzes the satisfaction of Chinese stock index option market to its basic assumptions. Of course, it is impossible for the Chinese market environment to perfectly meet the basic assumptions of the B-S model. For example, the reader can see at a glance that the assumption of "no tax and transaction costs" is not feasible in any country [7]. However, anyone cannot just deny the application prospect of the B-S model in the Chinese market, it's reasonable to deeply analyze the differences between the model assumptions and the actual market, and discuss the impact of differences on the model [8].

Secondly, starting from the historical data, the two key parameters, $N(d_1)$ and $N(d_2)$, are obtained, which are substituted into the model to obtain the prediction results of the model. Through the data from January 2020 to December 2020, we can obtain S, K, σ, T and other parameters, to obtain d_1 and d_2 quantiles, and then substitute them into the calculation formula of the model to obtain the prediction results.

Finally, take all the collected real option price data as the verification interval, compare the model prediction results with the actual option prices, analyze the deviation between the prediction results and the actual prices, and analyze the reasons for the deviation from multiple perspectives such as the fat-tail effect and the impact of interest rates, and then point out the improvement strategies to improve the model and reduce the deviation, to improve the applicability of the B-S option pricing model in the Chinese market.

2. Method

2.1 Collection and Preliminary Processing of Historical Data

First of all, the author retrieved from the Shanghai Stock Exchange and Shenzhen Stock Exchange all the trading data of the CSI 300 stock index from January 2005 to September 2022, totaling more than 4300 pieces, and all the trading data of Huatai-PineBridge CSI 300 ETF options (including call and put options) since their listing at the end of 2019. The main information includes the opening price, closing price, daily maximum and minimum prices, etc. There are more than 25000 pieces of effective call option data and more than 2500 pieces of effective put option data. In this paper, unless otherwise specified, the asset price is based on the closing price.

The data of Huatai-PineBridge CSI 300 ETF options are more complicated than those of stock indexes. Firstly, the option data is divided into two parts of call and put options; Secondly, when the option contract is established, there are different exercise prices. According to common sense, when the option contract is established, the exercise price will not be far from the price of the underlying asset. Since the range of the CSI 300 stock index is about 3000-5000 points between 2020 and 2022, the exercise price of the option contract is mostly between 3000 and 7000.

In the EXCEL table, all option data are sorted by expiration time, exercise price and transaction date. Because the data of the last trading day of an option is the data of its expiration date, the backward compatibility of the LOOKUP function can be used to mark the expiration date for each data, and then the DATEDIF function can be used to calculate the date difference between the expiration date and the trading date and divide it by 365 to get (T-t), how long is the time before the expiration of the spot option.

2.2 Satisfaction Analysis of Basic Assumptions of B-S Option Pricing Model

The B-S option pricing model has six basic assumptions. Next, the author will analyze whether the nature of CSI 300 stock index options can meet these six assumptions.

2.2.1 Normal distribution assumption

The price of financial assets follows the logarithmic normal distribution, that is, the logarithmic yield of financial assets follows the normal distribution, which is the first basic assumption of the B-S option pricing model.

On this basis, the author draws the daily logarithmic yield of the CSI 300 stock index into a frequency distribution histogram as shown in the figure below. Seen intuitively from Figure 1 and Figure 2, the long-term CSI 300 stock index yield distribution between 2005 and 2022 is closer to the normal distribution than the short-term yield distribution in 2020.

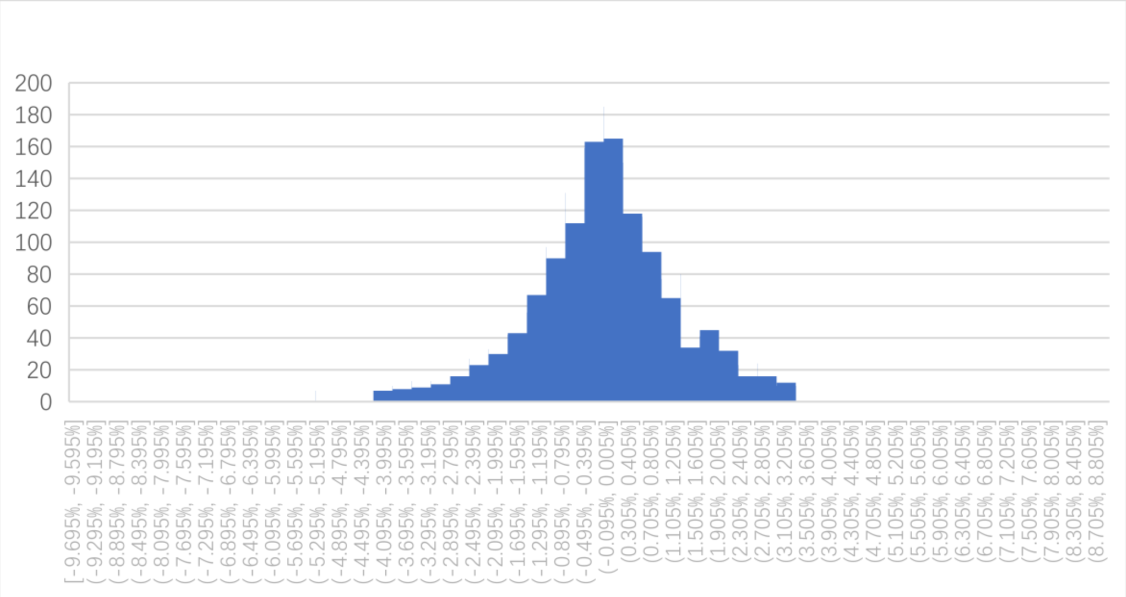

Figure 1. Distribution chart of daily logarithmic yield of CSI 300 stock index in 2005-2022 (Photo credit: Original)

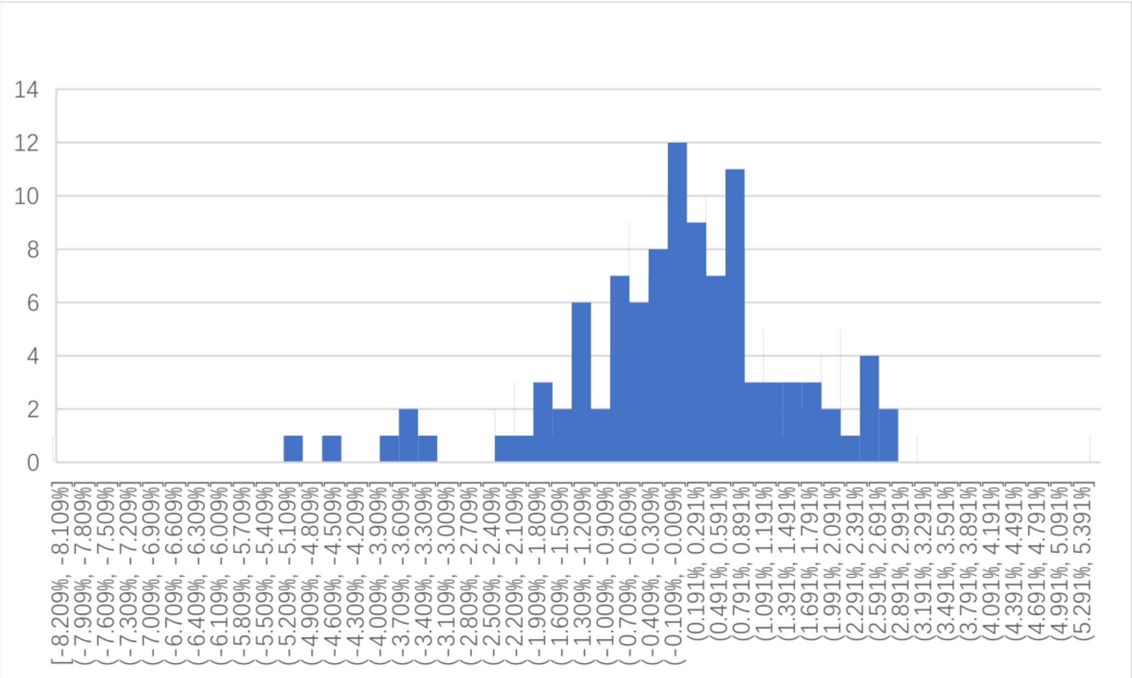

Figure 2. Distribution chart of daily logarithmic yield of CSI 300 stock index in 2020 (Photo credit: Original)

In addition to visual observation from Figure 1 and Figure 2, rigorous analysis from a quantitative perspective is also required. The author uses skewness, kurtosis, P-P Plot and other analysis tools to analyze the fitting degree of the logarithmic return rate of financial assets.

2.2.2 Rf constant assumption

During the term of the option, the risk-free interest rate is constant, which is the second basic assumption of the B-S model. However, in the Chinese financial market, as in other countries, there are officially published indicators that can be used as risk-free interest rates. There are many risk-free interest rate reference indicators available, such as the treasury bond yield over the same period, the fixed deposit interest rate of banks, the Shanghai interbank offered rate (SHIBOR), etc., but none of them is not in the process of change.

2.2.3 Fixed volatility assumption

In the B-S model, it is assumed that the price volatility of the underlying asset is a constant during the term of the option. From the perspective of economics, there are three main reasons for the emergence of volatility: the first is the impact of macroeconomic factors on an industrial sector, namely the so-called system risk; the second is the impact of a specific event on some enterprises, that is, the so-called unsystematic risk; The third is the impact of investors' psychological state or expected changes on the underlying asset price.

2.2.4 No tax or transaction cost assumptions

The third basic assumption of B-S model is that there is no tax or transaction cost in the market, that is, "frictionless" market. Obviously, this assumption is too idealistic. In Chinese financial market, futures companies or other market institutions will require a certain amount of commission when they accept customers' entrustment and enter the market for trading according to customers' instructions. In addition, exchanges and securities registration and settlement institutions will also charge transaction fees.

2.2.5 No dividend assumption

When the B-S model was first proposed, the non-dividend attribute of the underlying asset was one of the basic assumptions, but now this restriction has been relaxed, and the impact of dividend can be eliminated by revising the calculation formula.

2.2.6 European option hypothesis

The B-S model requires that the options to be valued are European options, which can only be exercised at maturity. The CSI 300 ETF option is a European option. The contract stipulates that the exercise date of the option is the expiration date of the contract, and the submission time of the exercise order is 9:15-9:25, 9:30-11:30, and 13:00-15:30.

2.3 Analysis of Fitting Degree

According to the calculation formula of B-S model, in order to get the price of call and put options, it’s necessary to calculate the two key parameters, d_1 and d_2 at first.

$$Price_{call} = SN(d_1) - Ke^{-rT}N(d_2) \tag{1}$$

$$Price_{put} = Ke^{-rT}N(-d_2) - SN(-d_1) \tag{2}$$

$$d_{1,2} = \frac{ln\,(\frac{S}{Ke^{-rT}})}{\sigma\sqrt{T}} \pm \frac{1}{2}\sigma\sqrt{T} \tag{3}$$

Step 1: Use VLOOKUP function to query the stock index on the trading day corresponding to the option in the database of CSI 300 stock index, that is, mark the underlying asset price S corresponding to each trading day. The exercise price corresponding to the option is K. For the risk-free interest rate

R, the author adopted the three-month lending rate of 2.994% reported by the Shanghai interbank lending market on January 2, 2020. As for Volatility σ, the fluctuation rate of the logarithmic yield of the CSI 300 stock index in 2020 is 0.0143952.

Step 2: With the above parameters, the author can obtain the values of d_1 and d_2 through the calculation formula of d_1 and d_2, and then obtain the corresponding cumulative probabilities $N(d_1)$, $N(d_2)$, $N(d_1)$ and $N(-d_2)$ through the NORM. S. DIST function. According to the calculation formula of call option and put option, the option price predicted by B-S model can be calculated.

Step 3: Compare the actual option prices in the market and the predicted option prices in B-S model, comprehensively consider the absolute error and relative error, and analyze the fitting degree between the B-S model's prediction and the real price of Huatai-PineBridge CSI 300ETF option price, so as to evaluate the applicability of the B-S model in the Chinese option market.

3. Results

3.1 Distribution of Return on Assets

It can be seen from Table 1 that the kurtosis and skewness of the CSI 300 stock index yield distribution are close to the normal distribution. The P-P analysis results (see Figure 3 and Figure 4) also show that the data distribution is close to the normal distribution, so the yield distribution can basically meet the first hypothesis.

Table 1. Analysis results of kurtosis and skewness of daily logarithmic yield of CSI 300 stock index in 2005-2022 and 2020

Statistics	2005-2022	2020
Mean	0.00030860	0.00099042
Std. Deviation	0.016688365	0.014424926
Skewness	-0.514	-0.982
Std. Error of Skewness	0.037	0.156
Kurtosis	4.025	5.581
Std. Error of Kurtosis	0.075	0.311
Range	0.186259	0.137219

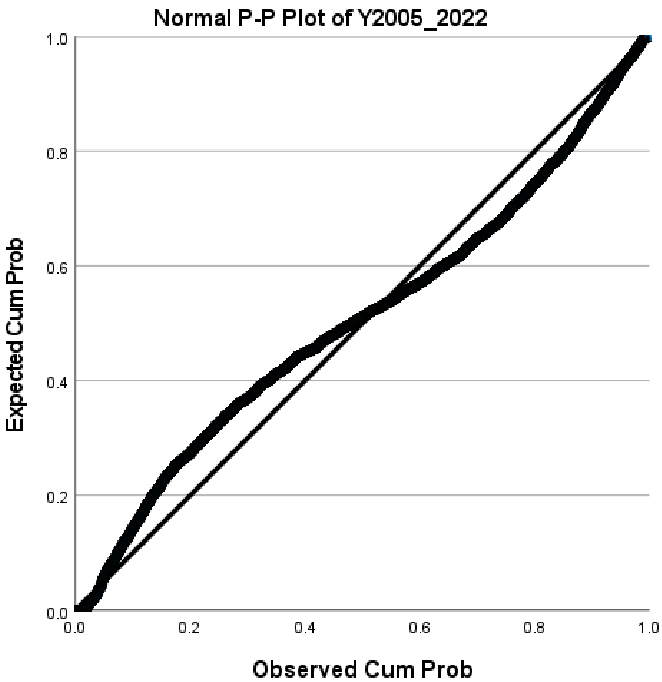

Figure 3. P-P Plot of daily logarithmic yield of CSI 300 stock index in 2005-2022
(Photo credit: Original)

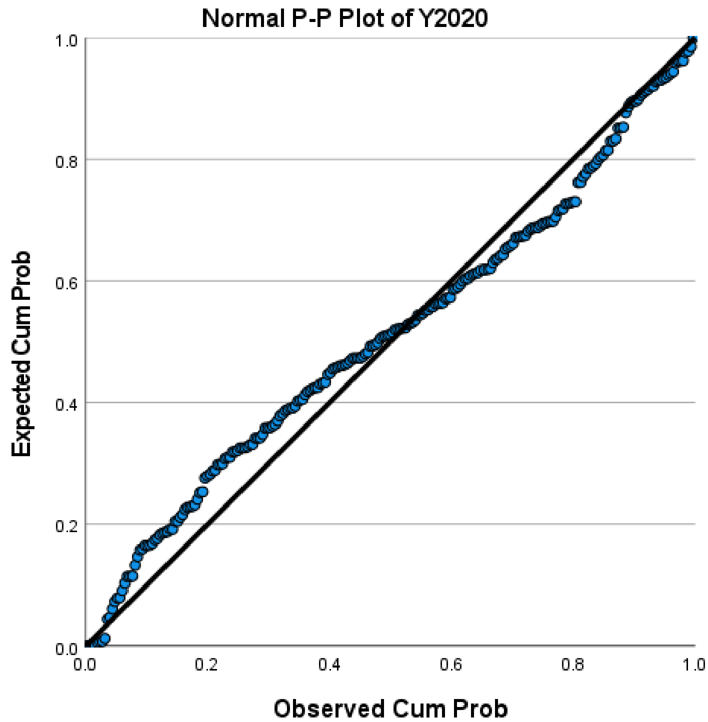

Figure 4. P-P plot of daily logarithmic yield of CSI 300 stock index in 2020
(Photo credit: Original)

3.2 Model Prediction Results

According to the analysis method of fitting degree introduced above, the author calculated more than 50000 model forecast prices, and calculated the absolute difference and relative difference between the predicted prices and the actual prices. The average absolute difference of call options is -46.49￥, the average relative difference is -51.72%, and the average absolute difference of put options is -74.96￥, the average relative difference is -64.95%.

3.3 Qualification Rate

Only focusing on the mean value of the difference, the error of the model prediction is large. However, such evaluation is one-sided, because when the option price is very low, the absolute error is very small, but because the denominator value is small, the relative error is large, and vice versa. Therefore, the author introduces the qualification rate to comprehensively evaluate the prediction performance of B-S model.

Using IF and AND functions, the author sets corresponding thresholds for absolute and relative differences. For any prediction result, if both the absolute difference and the relative difference exceed the threshold, the prediction result is judged as unqualified and marked as 0, otherwise it is qualified and marked as 1.

Table 2. Example of qualified rate sheet

No.	Trade Date	Closing price	N(d1)	N(d2)	Model Price	Absolute Difference	Relative Difference	Pass or not
1	2019-12-23	0.3825	1.00	1.00	375.94	-6.5557	-1.7139%	1
2	2019-12-24	0.4059	1.00	1.00	401.51	-4.3884	-1.0812%	1
3	2019-12-25	0.4029	1.00	1.00	399.12	-3.7740	-0.9367%	1
4	2019-12-26	0.4392	1.00	1.00	433.95	-5.2478	-1.1949%	1
5	2019-12-27	0.4387	1.00	1.00	429.69	-9.0026	-2.0521%	1
…	…	…	…	…	…	…	…	…

Calculate the ratio of qualified prediction results to the total number of data to get the qualified rate of the model. Table 2 is an example. If the absolute difference does not exceed 40￥ or the relative difference does not exceed 80%, the model is considered qualified, and the qualified rate of the model is 77.54%.

The author draws the qualification rate performance of the model corresponding to different judgment thresholds, as shown in the Table 3 and Table 4.

Table 3. Pass rate of forecasting call option price

threshold	≤10￥	≤20￥	≤30￥	≤40￥	≤50￥
≤20%	54.55%	58.88%	61.95%	64.59%	66.97%
≤40%	61.44%	65.71%	68.70%	71.20%	73.32%
≤60%	65.32%	69.58%	72.55%	75.00%	77.07%
≤80%	67.88%	72.13%	75.10%	77.55%	79.58%
≤100%	98.51%	98.51%	98.51%	98.51%	98.51%

Table 4. Pass rate of forecasting put option price

threshold	≤10￥	≤20￥	≤30￥	≤40￥	≤50￥
≤20%	42.97%	48.84%	53.45%	56.61%	59.72%
≤40%	50.58%	56.38%	60.42%	63.77%	66.60%
≤60%	54.81%	60.60%	64.60%	68.88%	70.63%
≤80%	57.90%	63.67%	67.64%	70.90%	73.61%
≤100%	99.62%	99.63%	99.65%	99.65%	99.66%

As Table 3 and Table 4 shown, with the increase of the threshold value, the qualification rate of the model also increases. Under the strict condition that the absolute difference does not exceed 10 ￥ or the relative difference does not exceed 20%, the model still maintains a qualification rate of more than 50% for the prediction of call options. The prediction qualification rate of the model for the put option is lower than that of the call option as a whole, but under most conditions, the prediction qualification rate of the model for the put option can reach more than 50%.

4. Discussion

Through data analysis, it can be seen that the B-S model has certain applicability in Chinese option market, but the prediction bias still exists. In the author’s opinion, the following three sources are the important causes of forecast deviation.

4.1 Fat-tail Effect

In the real market, the distribution of financial asset returns tends to deviate from the normal distribution to the left, to more low frequency high loss or high frequency low loss events, and to result in a higher left tail of the distribution figure. This effect is generally known as the fat tail effect [9]. It is not difficult to see from the daily logarithmic yield distribution chart of the Shanghai and Shenzhen 300 stock indexes from January 2020 to December 2020 that extreme loss values deviating from the symmetry of the normal distribution appeared at -0.082, -0.050, etc. The fat tail effect caused by extreme losses has a very negative impact on the prediction accuracy of B-S model. People's mentality of preventing extreme losses will reduce the estimated value of assets in people's minds, thus affecting the actual price of options [10].

4.2 Interest Rate Impact

Although the B-S model assumes that the risk-free interest rate within the duration of the option is constant, the reality is that different authorities will issue dynamically adjusted reference values of risk-free interest rates every day, and the price of call options will increase with the rise of risk-free interest rates, while the put options will do the opposite. Therefore, the real risk-free interest rate in the market has changed. However, when the risk-free interest rate standard adopted by B-S model fails to catch such change in real time, it will cause delay, which will lead to forecast deviation [11].

4.3 Impact of Risk Control Strategies of Large Option Trading Institutions

Options market and stock market are very different in the composition of trading participants. A large proportion of retail investors participate in Chinese stock market, but the future and option trading have the characteristics of high risk, high leverage and high requirements for capital reserves, which makes the proportion of retail investors in the future and option market lower than that in the stock market and the proportion of institutional investors is higher. Because the short selling of options, especially the short selling of call options, is of great risk, the exchange also has very high requirements on the quality of investors' margin and credit [12].

Institutional investors have thorough risk prevention and control strategies. When the price signal in the market touches the critical point, institutional investors will use the risk control measures under the corresponding situation to prevent losses. In this case, the state of the market will not only force an institution to adopt coping strategies, but also trigger group effects and generate selling panic. For example, in the middle of March 2020, the CSI 300 stock index fell sharply. At that time, the author asked colleagues who were engaged in trading in several securities companies and futures companies, and learned that most institutions had adopted stop loss strategies to varying degrees, such as increasing the amount of put option positions in the CSI 300 stock index, or compulsory closing of option positions with losses exceeding the limit. Although the CSI 300 stock index gradually recovered in the following month, these actions have deeply affected the actual prices of the options market, making the prediction of the B-S model deviate.

5. Conclusion

Through data analysis, we can know that the B-S model has certain applicability to the CSI 300 stock index options. When using different thresholds to evaluate the B-S model, the qualified rate will be different. In order to improve the prediction eligibility of B-S model in Chinese option market and the applicability of the model, the author deems that the current B-S model can be improved from the following two aspects.

The d_1 and d_2 parameters in the B-S model determine the weight of the underlying asset price and fixed income bonds on the option price through the standard normal distribution. Although the use of standard normal distribution has logical self-consistency in mathematics, if the standard normal distribution is replaced by a specific empirical distribution, that is to fine tune the standard normal distribution, it can make up for the inherent shortcomings of the standard normal distribution to some extent, such as its symmetry does not conform to the Fat tail effect in the real market.

In the discussion, the author mentioned that the users of the B-S model have a lag in catching the latest market information when modeling, so the timeliness of the model can be enhanced by dynamically adjusting the interest rate and other parameters in the model according to the market situation. Technically, this dynamic update requirement is not difficult to achieve. Model users only need to access the Internet and update and use the latest data from the data source. The real difficulty lies in the decision makers' choice: there are many optional interest rate indicators and many optional volatility indicators. Which is the best one for prediction? Users of the model can select the data source with the best performance through a period of tracking and evaluation.

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