Research on Capital Allocation Based on Markowitz Model

Zhenqi An*

School of Economics and Management, Inner Mongolia University, Huhhot, China

*Corresponding author: wangping@KSUA.EDU.KG

Abstract. This paper mainly uses Markowitz asset portfolio model to demonstrate the impact of different market rules (allowing and not allowing short selling) on investment returns and risks, and gives investment suggestions through this model, and judges the difference of returns and risks under different market conditions. In 10 companies known four industry under the premise of nearly 20 years of stock data, using data on behalf of the medium and long term monthly data and Excel Solver econometric analysis tools, respectively obtained the "short" are allowed and "empty" is not permitted in both cases the optimal portfolio and the minimum risk portfolio of a single investment subject matter weight and their earnings; Risk and Sharpe ratio. In both cases, the efficient frontier of the portfolio can be plotted and compared in conjunction with the optimal asset allocation line. Finally, through the comparative analysis of income and risk, some investment suggestions are given. It also analyzes the influence of market rules on investors' returns and risks.

Keywords: Return on capital; Markowitz portfolio model; Maximization of benefit.

1. Introduction

Harry Markowitz's 1952 article "Portfolio Selection" in the Journal of Finance was the first to use mathematical models to analyze portfolios, this revolutionary scientific approach has had a major impact on investment theory and the goal of asset selection analysis is to find the most efficient set of portfolios, the efficient frontier of investment. In this paper, the author puts forward the "mean-variance" analysis method and the effective boundary model of asset portfolio, and believes that the optimal asset portfolio follows the law of minimizing the risk under the premise of predetermined rate of return or maximizing the expected rate of return under the premise of certain risk. Furthermore, this paper emphasizes that the optimal portfolio should be selected on the efficient frontier of risky assets. Markowitz's portfolio theory not only has perfect theory, but also has been effectively used in practice (including China's securities market). Based on Markowitz portfolio theory, Shanmin Li and Jianli Chen selected sample stocks from Shanghai and Shenzhen Stock Exchange, to study the Chinese stock market [1]. The results show that the theory can effectively diversify the investment risk. In order to ensure the accuracy of the model, Cao Xing chose stocks with large capital stock as samples for empirical research [2]. Further, in the literature, Jingqin Su took the western region as the background and conducted empirical analysis on sample stocks covering various industries in the western region [3].

2. Methodology

2.1 Assumptions based on the Markowitz Model

The modelling assumptions in this paper are specified as below.

(1) The securities market is efficient and the price of assets can reflect their intrinsic value. Stock prices reflect all market information, investors know the expected return and standard deviation of various assets.

(2) All investors are rational and seek to minimize the variance of their portfolios.

(3) There is correlation between the returns of each asset, which can be expressed by correlation coefficient or covariance.

(4) Investors can borrow unlimited amounts from banks at the same deposit and lending rates.

(5) Transactions are frictionless, meaning there are no transaction fees or taxes [4].

2.2 Establishment of Markowitz Model

According to the above assumptions, Markowitz model established the calculation method of expected return and risk measurement and effective boundary theory, and established the mean-variance model for selecting the optimal asset portfolio [5]. The expected revenue of the portfolio is defined as the formula (1).

$$\mathrm{E}(\mathrm{r_p}) = \mathrm{w_1 E(r_1) + w_2 E(r_2) + L + w_{10} E(r_{10})} \tag{1}$$

The corresponding standard deviation is derived as the formula (2), with the restrictions $\sum_{i=0}^{10} w_i = 1$ (short selling allowed) or $\sum_{i=0}^{10} w_i = 1$ and $w_i \geq 0$ (Short selling is not allowed).

$$\sigma_\mathrm{p} = 1/2\left[\sum_{\mathrm{i}=1}^{10} \sum_{\mathrm{j}=1}^{10} w_i w_j \mathrm{Cov}(r_i, r_j)\right] \tag{2}$$

The Sharpe ratio is calculated by the formula (3).

$$\mathrm{S_p} = \mathrm{E(r_p)} - \mathrm{E(r_f)}/\sigma_\mathrm{p} \tag{3}$$

Where the expected return of the *i*th stock is $\mathrm{E(r_i)}$; the *i*th stock is weighted w_i; the expected rate of return on the whole is $\mathrm{E(r_p)}$; the standard deviation is σ_p; Sharpe ratio is $\mathrm{S_p}$ and $\mathrm{Cov}(r_i, r_j)$ is the correlation coefficient between two securities.

The first step, determine the rate of return opportunity faced by investors, obtain the minimum variance frontier (effective frontier), and then determine the minimum variance portfolio. Second, look for the asset allocation line with the highest reward-volatility ratio (Sharpe ratio). It is tangent to the efficient frontier, and its tangent point is the optimal risk portfolio. In order to find the minimum variance combination, formula (2) is taken as the objective function, $\sum_{i=0}^{10} w_i = 1$ as the constraint condition. The minimum standard deviation and rate of return can be obtained by using Excel solver. The point with the highest Sharpe ratio is found on the frontier, that is, the optimal risk portfolio on the efficient frontier. Take formula (3) as the objective function, $\sum_{i=0}^{10} w_i = 1$ as the constraint condition, the standard deviation of the optimal risk portfolio can be obtained by using Excel solver. In addition, according to whether assets can be short sold, this paper discusses the two situations of allowing short selling and not allowing short selling [6].

2.3 Data Statistics

2.3.1 Sample stock sourcing and selection

Ten stocks are selected in this paper, and their corresponding companies are NVIDIA Corporation; Cisco Systems, Inc; Intel Corporation; Goldman Sachs Group; U.S. Bancorp; Toronto-dominion Bank; The Allstate Corporation; Procter & Gamble; Johnson & Johnson and Colgate-Palmolive Company, which are divided into four industries; technology; financial services; consumer defensive and healthcare.

NVIDIA Corporation; Cisco Systems, Inc; Intel Corporation all went public in the National Association of Securities Dealers Automated Quotations. The Goldman Sachs Group, Inc; U.S. Bancorp; The Toronto-Dominion Bank; The Allstate Corporation; The Procter & Johnson; Colgate-Palmolive Company all went public in the National Association of Securities Dealers Automated Quotations.

This paper selects May 11, 2001 to May 12, 2021 as the sample time limit of the securities portfolio, a total of 5219 trading days in the period. The time interval used to calculate the rate of return can generally include year, month, week and day. Since this paper is mainly to analyze the risk and return of stock investment, so the monthly rate of return is used to represent the medium- and long-term investment rate [7].

2.3.2 Sample data processing

On the basis of the daily closing prices derived from the RESSET Financial database, the monthly returns are collated and the correlation coefficients between them are calculated. As Table 1, it sets the monthly risk-free return rate as a fixed value of 0.06%.

Table 1. Correlation coefficients of the sample stock

	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
SPX	1	0.526461	0.637777	0.576545	0.709495	0.609566	0.644479	0.63248	0.412681	0.54302	0.439737
NVDA	0.52646087	1	0.487827	0.523803	0.343333	0.15822	0.339009	0.15651	0.061198	0.167136	0.071157
CSCO	0.63777716	0.487827	1	0.615147	0.487132	0.32797	0.409547	0.296543	0.220304	0.237939	0.164655
INTC	0.5765453	0.523803	0.615147	1	0.411405	0.280761	0.41093	0.288926	0.134937	0.325829	0.108208
GS	0.70949514	0.343333	0.487132	0.411405	1	0.471208	0.493373	0.41636	0.173173	0.294484	0.202929
USB	0.60956562	0.15822	0.32797	0.280761	0.471208	1	0.540045	0.537738	0.338337	0.233674	0.219884
TD CN	0.64447912	0.339009	0.409547	0.41093	0.493373	0.540045	1	0.416329	0.229671	0.270138	0.209598
ALL	0.63248016	0.15651	0.296543	0.288926	0.41636	0.537738	0.416329	1	0.347378	0.449888	0.408233
PG	0.41268137	0.061198	0.220304	0.134937	0.173173	0.338337	0.229671	0.347378	1	0.493266	0.482593
JNJ	0.54301972	0.167136	0.237939	0.325829	0.294484	0.233674	0.270138	0.449888	0.493266	1	0.52577
CL	0.43973679	0.071157	0.164655	0.108208	0.202929	0.219884	0.209598	0.408233	0.482593	0.52577	1

Through calculation, and we can get the standard deviation of these 10 stocks from the monthly returns from 2001 to 2021 (see Table 2). In order to find out the minimum risk portfolio and optimal risk portfolio, this paper divided short selling allowed and not allowed to discuss two cases, then two Efficient frontier and most asset allocation line could be got.

Table 2. Standard deviation, average return and sharp ratio of sample stock

	standard deviation	average return	sharp ratio
SPX	14.85%	7.542%	50.3856%
NVDA	55.77%	32.802%	58.7050%
CSCO	30.81%	9.714%	31.3360%
INTC	30.50%	8.905%	28.9958%
GS	29.57%	10.825%	36.4033%
USB	23.68%	9.878%	41.4620%
TDCN	18.13%	11.010%	60.3823%
ALL	24.88%	10.080%	40.2677%
PG	14.59%	9.437%	64.2856%
JNJ	14.79%	8.464%	56.8410%
CL	15.35%	7.105%	45.8955%

3. Empirical Results

3.1 Results under Two Conditions

Planning to solve and Excel Solver are used to calculate the minimum mean portfolio and the optimal portfolio under the two conditions.

3.1.1 Short selling is allowed

In this case, the formula (2) is taken as the objective function, $\sum_{i=0}^{10} w_i = 1$ is the constraint, Excel solver was used to find the minimum variance combination. The minimum standard deviation is 10.9373%, The yield and Sharpe ratio are 7.4716 percent and 67.7645 percent, respectively. And you can get the risk portfolio. And draw the efficient boundary of the risk portfolio as shown in Figure 1.

On the basis of the efficient frontier, the combination with the highest Sharpe ratio on the frontier is found, that is, the optimal risk portfolio on the efficient frontier. Taking formula (3) as the objective function and $\sum_{i=0}^{10} w_i = 1$ as the constraints, using Excel solver, the standard deviation of the optimal risk portfolio can be obtained as 16.5945%. At this time, the yield and Sharpe ratio are

17.1216% and 102.8151% respectively, and the risk portfolio at this time can be obtained. Draw the optimal capital allocation line (see Figure 1) [8].

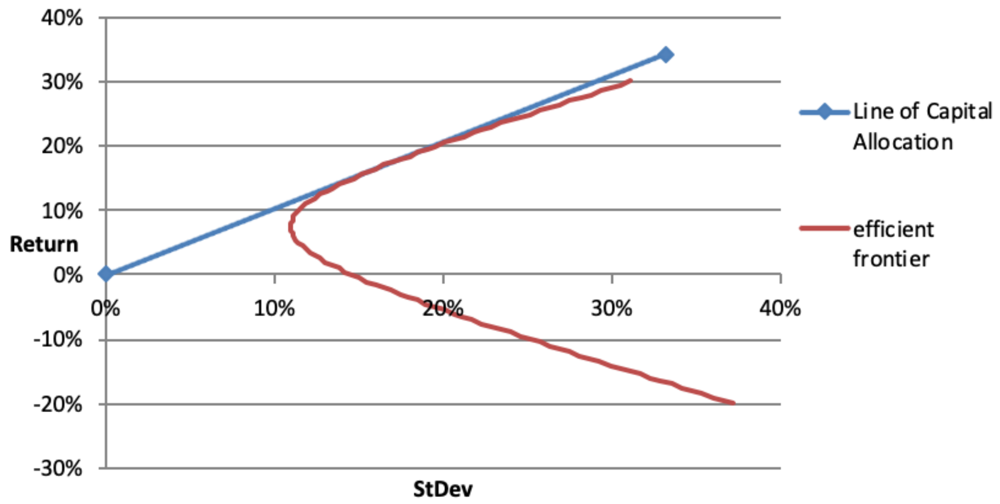

Figure 1. Efficient frontier and optimal allocation line of short selling
(Photo credit: Original)

3.1.2 Short selling is not allowed

Similarly, in order to find the smallest variance combination, taking equation (2) as the objective function, $\sum_{i=0}^{10} w_i = 1$ and $w_i \geq 0$ as the constraint conditions, the minimum standard deviation can be obtained by using Excel solver. The return and Sharpe ratio are 10.3766% and 91.5935%, respectively. Meanwhile, the risk portfolio at this time can be obtained (as shown in Table 1). The valid boundary can be seen in Figure 2 [9].

Similarly, in this case, given the efficient frontier, find the optimal risk portfolio on the efficient frontier. Taking equation (4) as the objective function and$\sum_{i=0}^{10} w_i = 1$ and $wi \geq 0$ as the constraint condition, Excel solver can be used to solve the optimal risk portfolio. The standard deviation of the optimal risk portfolio is 12.7011%, and the return and Sharpe ratio are 13.1524% and 103.0806%, respectively. And you can get the risk portfolio. The optimal capital allocation line can be seen in Figure 2.

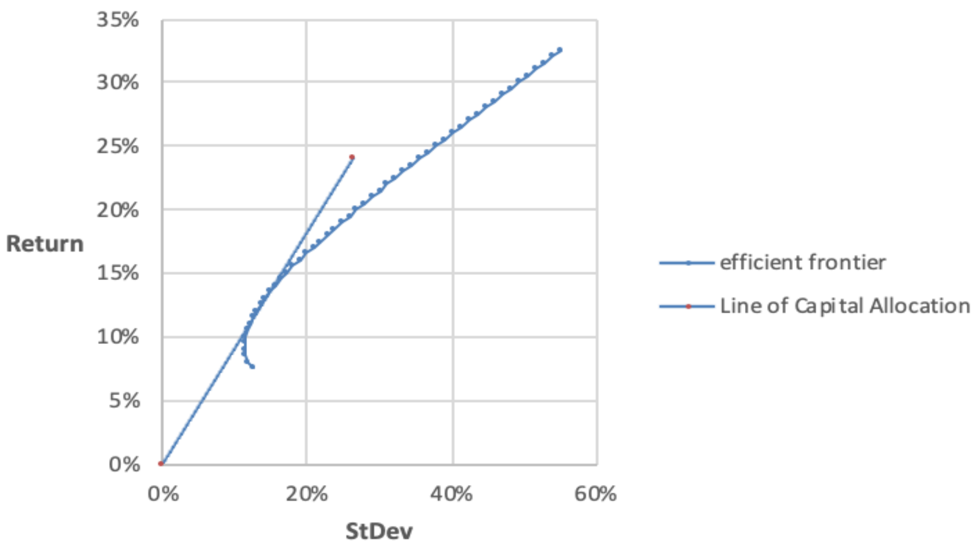

Figure 2. Efficient frontier and optimal allocation line of no short selling
(Photo credit: Original)

3.2 Comparison

By using Markowitz mean-variance model, monthly data and covariance obtained from historical data can be used to calculate the minimum variance set. The optimal asset portfolio is obtained by means of finite bounds and Sharpe ratio. The above calculation results show that under the short-selling mechanism, the minimum standard deviation is 10.9373% and the yield is 7.4716%; The minimum standard deviation and yield of the optimal portfolio are 16.5945% and 17.1216%, respectively (in Table 3). It shows that blindly pursuing the minimum risk, that is, the minimum standard deviation, the Sharpe ratio of the portfolio obtained is relatively low, which indicates that the investment activity is meaningless, and the return is lower considering the transaction costs. Secondly, compared with the minimum standard deviation portfolio, the rate of return of the optimal portfolio increases significantly more than that of the standard deviation, and the risk borne by the unit return decreases. But it shows that the optimal configuration of the model has obvious effect. Due to the restriction of the trading mechanism, the minimum standard deviation is 11.2635% and the yield is 10.3766% under the short selling mechanism. The minimum standard deviation and yield of the optimal portfolio are 12.7011% and 13.1524%, respectively. It can also be seen that the optimal portfolio is better than the minimum variance portfolio in return rate and Sharpe ratio. According to the Sharpe ratio in the case of "short selling is allowed" and "short selling is not allowed", it can be seen that the Sharpe ratio in the case of minimum standard deviation and optimal portfolio is relatively high under the influence of the trading mechanism of "short selling is not allowed", and the risk of unit return is small [10].

Table 3. Standard deviation, rate of return and Sharpe ratio in both cases

Allow short-selling	Minimum standard deviation	10.9373%	rate of return	7.4716%	Share Ratio	67.7645%
	Standard deviation of the optimal combination	16.5945%	rate of return	17.1216%	Share Ratio	102.8151%
No short-selling	Minimum standard deviation	11.2635%	rate of return	10.3766%	Share Ratio	91.5935%
	Standard deviation of the optimal combination	12.7011%	rate of return	13.1524%	Share Ratio	103.0806%

4. Conclusion

First, the return on the optimal portfolio is much higher than the return on any individual underlying asset, and the risk of the portfolio is lower than the risk of the individual investment subject. According to the comparison between Sheet 2 and Sheet 3, under the same market rules and the same risk or risk premium, the expected return and risk of effective portfolio are significantly better than that of single underlying securities. Portfolio can diversify investment to a certain extent, reduce risk, improve returns and make profits more stable. This suggests that it is better to choose a portfolio consisting of multiple investment subjects than a single investment subject.

Second, the market mechanism has a great impact on the risk and return of the optimal portfolio. Through the comparative analysis of two market mechanisms, short selling is allowed and short selling is not allowed, it is found that the relatively strict market mechanism (short selling is not allowed) will reduce the investment rate of return of investors to a certain extent, and has no significant impact on the risk. This indicates that more flexible financial instruments or market rules in the financial market have a positive effect on improving investors' investment returns and stimulating investors' investment enthusiasm, which means that relatively flexible market rules can promote market prosperity to a certain extent.

Finally, the distribution of the investment subject weight in the optimal portfolio is concentrated. According to the empirical study, under the current stock market does not allow short selling mechanism, the optimal portfolio includes three stocks, and mainly concentrates on two of them. In this case, investors can approach the most rational and optimal portfolio by focusing their energy and capital on a few stocks in the optimal portfolio instead of dispersing many stocks, so as to conduct efficient investment management and seek the highest Sharpe ratio as possible.

References

[1] Li Shanmin, Xu Pei. Research on the application of portfolio theory Markowitz model [J]. Economic science, 2000 (1): 42 ~ 51.

[2] Cao Xing, Peng Di. The application of Markowitz Portfolio Theory in Chinese securities market [J]. Journal of Central South University: Social Science Edition, 2003 (12): 785 ~ 788.

[3] Su Jingqin, Chen Dongxiao. Markowitz Portfolio Theory and Empirical research [J]. Journal of Dalian University of Technology: Social Science Edition, 2002 (6): 31 ~ 32.

[4] H.M. Markowitz. Portfolio Selection [J]. Journal of Finance, 1952, 7 (1).

[5] Zhao Yanfen. Research on the application of modified Markowitz Portfolio model in financial markets [D]. Dalian: Dalian University of Technology, 2013 (6): 7 - 10.

[6] William F. Sharpe. A Simplified Model for Portfolio Analysis [J]. Management Science, 1963, 9 (2).

[7] Xiang Junwen. Stock investment strategy based on Markowitz portfolio theoryc [J]. China Securities Futures, 2012 (08).

[8] Ouyang Guangzhong, Li Jinghu. Portfolio and investment analysis [M]. Beijing: Higher Education Press, 1997.

[9] Zhou Yansen. Analysis of stock portfolio based on mean-variance modelc [J]. Economic Research Guide, 2022 (23).

[10] Du Jinmin. On the development and limitation of modern securities portfolio theoryc [J]. Academic research, 1999, (6).