Portfolio Construction in Terms of Bitcoin, S&P 500, Gold Futures and American 10-year National Debt

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Abstract. Contemporarily, the financial sector occupies a vital part of the world, and asset allocation in it is the top priority. Portfolio construction is the main procedure of asset allocation. In order to gain better application of asset allocation, this paper selects two representative models from the most classic asset allocation models (i.e., Index model and Markowitz model), as well as processes and analyzes them with actual data and targets to obtain real model results. Specifically, this article compares the curves under five different constraints from two models. According to the analysis, the Index model has better performance and fit well in this case, while curves of the Markowitz model have more outliers, which may cause bad impacts in real cases. Based on the evaluations, the pros and cons of the two models and the different conditions for adaptation can be compared to a certain extent. These results shed light on guiding further exploration of portfolio designs.

Keywords: Markowitz model; Index model; return analysis; portfolio.

1. Introduction

Asset allocation refers to the allocation of funds to different uses. And this concept was proposed as early as 400 years ago. At that time, the Spaniard Cervantes once said "don't put all your eggs in one basket", and Shakespeare also conveyed the idea of "diversified investment" in "The Merchant of Venice". This is the historical concept and idea of asset allocation. In 1921, The Wall Street Journal proposed an optimal investment portfolio of 25% in sound bonds, 25% in sound preferred stocks, 25% in sound common stocks, and 25% in speculative securities. In 1952, Markowitz published the paper "Portfolio Selection" as the beginning of modern portfolio theory. In 1959, he published Portfolio Selection. This book was the birth of modern portfolio selection theory. Markowitz has also made outstanding contributions to asset allocation, for which he won the Nobel Prize. This was the first seminal contribution to the field of financial economics. A second important contribution came in the 1960s, when many researchers (of which William Sharp was a leading figure) used Markowitz's portfolio theory as the basis for developing a theory of financial asset price formation, known as the capital asset pricing model or CAPM. A third seminal contribution to financial economics involves the theory of corporate finance and the valuation of firms in the marketplace. This was achieved by Merton Miller in collaboration with Franco Modigliani. This theory explains the relationship between firms' capital asset structure and dividend policy on the one hand, and their market value on the other [1].

Nowadays, with the development of science and technology and the continuous progress of theoretical knowledge, there are a large number of new technologies to manage asset allocation, e.g., machine learning, Hierarchical Clustering [2], the coordinate descent, the alternating direction method of multipliers, the proximal gradient method. All of these different methods of asset allocation to construct portfolio are desire to improve return and reduce risk to the greatest extent possible, which can improve people’s happiness and wealthy with high efficiency safely. In recent future, machine learning may be a main trend to construct portfolio of asset allocation. Most portfolio building tasks can be done by machine learning, such as idea generation, alpha factor design, asset allocation, weight optimization, position sizing, and strategy testing [3].

In retrospect, Markowitz's contribution to economics is enormous. This mean-variance optimization framework marks the beginning of the financial allocation of the portfolio. Portfolio optimization appeared in the seminal paper by Markowitz. The original mean-variance framework is attractive because it is very efficient from a computational point of view. However, it also has a recognized failure, as it can lead to portfolios that are not financially optimal [4]. Although Markowitz's advice is intended to be practical and implementable. Ironically, the main results are normative and theoretical, while modern portfolio theory has rarely been implemented. There are three main reasons why Portfolio Theory is not implemented as follows [5]:

Difficulties in estimating the type of input data required (especially correlation matrices);

The time and cost required to generate an efficient portfolio (solving a quadratic programming problem);

Educating portfolio managers about the difficulties associated with risk-return trade-offs expressed in terms of covariance, return, and standard deviation.

Different investors have their own choices for asset allocation. For example, risk enthusiasts may be keen to choose high-risk and high-return targets including virtual currencies, futures options, etc.; while risk-averse people prefer low-risk and low-return targets (e.g., Treasuries, major ETFs in their countries). Among them, pension funds are a very classic low-risk asset allocation. As recently noticed by the UK pension fund, it is an asset allocation program mainly composed of British government bonds [6]. Although this asset allocation looks low-risk, it still has some risks, similar to the recent collapse of the British government bond has brought the market value of this asset allocation down. Therefore, there is no foolproof asset allocation plan. No matter which subject is chosen, it can only represent a certain possibility, not an absolute, for asset allocation.

Owing to the advancement of society and ideas, public demand for asset allocation is getting higher, and asset allocation is also developing rapidly. This article will compare the original most famous asset allocation model and index model, the same classic model, to draw the differences, limitations, and applications of the two. It will start by describing the background of asset allocation, and then elaborate the data processing process and the corresponding data results and chart content. Finally, it will systematically describe the differences between the operating results of the two models. At the end, the limitations and application prospects of these two asset allocation models will be presented.

2. Data & Method

2.1 Data

The paper constructs this portfolio consisted by bitcoin (BTC), S&P 500 (SPY), gold futures (GC) and American 10-year national debt (10NB) which have different profits and risks. In addition, it makes American 2-year national debt be risk free rate in this portfolio. All of the data is downloaded from investing.com. The data will use the closing price uniformly. Dates of data are from 10/15/2019 to 10/15/2022, which include 3-year data. Before carrying out the analysis, it is necessary to integrate data’s dates firstly, as opening day of these four investment subjects are different. Bitcoin’s opening day is year-round, while SPY’s opening day is weekday, especially there are some special days like Christmas and New Year for SPY are closing day. Therefore, the paper will continue data from last opening day to the next opening day. For example, SPY’s closing day is weekend, so the paper will make Friday’s data as Saturday and Sunday’s data. In addition, trading day of this paper is 365 days.

2.2 Models

The paper will refer and compare two financial models, i.e., Markowitz Model and Index Model. The Markowitz model proposed by Harry Markowitz in 1952 is a portfolio optimization model. This model helps select the most efficient portfolio by analyzing various possible portfolios for a given security. In other words, it selects portfolios for investors to form a portfolio that has the highest expected return at a given level of risk tolerance, the lowest expected return for a given level of risk tolerance risk tolerance [7]. The Markowitz model shows investors how to reduce risk by selecting different securities that do not change exactly together. Meanwhile, the Markowitz model is also known as the mean-variance model because it is based on the expected returns and standard deviations of various portfolios. It is the foundation of modern portfolio theory. The index model is also called the single index model. It is a simple asset pricing model developed by William Sharpe in 1963 to measure the risk and return of stocks. Single index models generalize linear regression and are applicable in various fields, such as discrete choice analysis in econometrics, where high-dimensional regression models are often used [8]. It is widely used in the financial industry today.

2.3 Procedure

2.3.1 Calculate major data indicators and correlation coefficient table

Firstly, risk free rate is used to calculate time value of assets, which is nominal risk-free rate (NRFR). Then, the paper divides the price of previous day by the price of today to get the return of each subject for every day. Subsequently, indicators are used to subtract nominal risk-free rate of same date to obtain excess return of four subjects. Then, one can get the most fundamental data indicators from excess return of four subjects: annualized average return, annualized standard deviation, sharp, minimum and maximum value, beta and annualized alpha (seen in Table. 1). In addition, calculate the residual returns with alpha and beta according to the formula and get the annualized residual standard deviation finally. In addition, this study uses excess return to calculate the correlation coefficient table for these four subjects as shown in Table. 2, which can show the correlation relationship of each subject clearly.

Table 1. Major indicators of four subjects.

	BTC	SPY	GC	10ND
Annualized Average Return	-0.00022	-0.03991	-0.03049	0.029498
Annualized StDev	0.787199	0.246328	0.1728	0.811877
Sharpe	-0.00028	-0.16204	-0.17644	0.036333
min	-0.16363	-0.08309	-0.05638	-0.30778
max	0.617523	0.122858	0.051175	0.539832
beta	1.216927	1	0.082767	0.813509
Annualized alpha	0.048355	0	-0.02719	0.061969
Annualized Residual StDev	0.727891	0	0.171593	0.786758

Table 2. Correlation coefficient table.

	BTC	SPY	GC	10ND
BTC	1	0.380797	0.119327	0.109756
SPY	0.380797	1	0.117985	0.246824
gold	0.119327	0.117985	1	-0.14665
ND	0.109756	0.246824	-0.14665	1

2.3.2 Calculate return, standard deviation and sharp of two models

The return, standard deviation and sharp of two models can be calculated by using these following formulae listed in Table. 3. In the initial portfolios of two models, the proportion of each subject is randomly set. For example, one sets up bitcoin, S&P 500 and gold futures at 20%, and the last subject is 1 subtract other subjects' weights. The obtained results are shown in Table. 4.

Table 3. Two models formulae.

MM Return=sum_i(weight_i*return_i)
MM StDev=sqrt (sum_ij(weight_i*standard deviation_i*correlation_ij*weight_j*standard deviation_j))
MM Sharpe=MM Return/MM StDev
IM Return=sum_i(beta_i*weight_i*return_market+alpha_i*weight_i+weight_i*residual_i)
IM StDev=sqrt (weight_i*beta_i*standard deviation_market) ^2+(weight_i*residual_i) ^2)
IM Sharpe=IM Return/IM StDev

Table 4. Two models formulas.

	BTC	SPY	GC	10ND	Return	StDev	Sharpe
MM	0.2	0.2	0.2	0.4	-0.233%	0.39662	-0.00586
IM	0.2	0.2	0.2	0.4	-0.233%	0.39854	-0.00583

Table 5. Markowitz model table.

MM (Constr1):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.2682	0.6975	0.0343	-3.096%	14.653%	-0.2113
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363
MM (Constr2):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.2682	0.6975	0.0343	-3.096%	14.653%	-0.2113
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363
MM (Constr3):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.2682	0.6975	0.0343	-3.096%	14.653%	-0.2113
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363
MM (Constr4):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.2682	0.6975	0.0343	-3.096%	14.653%	-0.2113
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363
MM (Constr5):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.2682	0.6975	0.0343	-3.096%	14.653%	-0.2113
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363

In these two typical models, their theories contain 5 constraints:

Constraint1: the absolute value of the sum of the proportions of all subjects is less than or equal to 2

Constraint2: the absolute value of the proportion of each subject is less than or equal to 1

Constraint3: no constraint

Constraint4: the proportion of each subject is greater than or equal to 0

Constraint5: the weight of the first subject is equal to 0

By applying such a constraint and using the solver function in excel, the paper can get different portfolios of the two models under these five constraints and a set minimum variance or maximum sharp, where the results are summarized in Table. 5 and Table. 6. Afterwards, this study can get minimal variance frontier, efficient and inefficient frontier of two different models of five constraints by using solver table function in excel. Finally, the paper will draw all of these frontiers into curves in two plots, which can show differences clearly.

Table 6. Index model table.

IM (Constr1):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.3028	0.6906	0.0067	-3.294%	14.889%	-0.2213
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363
IM (Constr2):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.3028	0.6906	0.0067	-3.294%	14.889%	-0.2213
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363
IM (Constr3):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.3028	0.6906	0.0067	-3.294%	14.889%	-0.2213
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363
IM (Constr4):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.3028	0.6906	0.0067	-3.294%	14.889%	-0.2213
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363
IM (Constr5):	**BTC**	**SPY**	**GC**	**10ND**	**Return**	**StDev**	**Sharpe**
MinVar	0.0000	0.3028	0.6906	0.0067	-3.294%	14.889%	-0.2213
MaxSharpe	0.0000	0.0000	0.0000	1.0000	2.950%	81.188%	0.0363

3. Results & Discussion

3.1 Results

Due to space limitations, this paper will only show part of the running data in table 7 and table 8. The running data is the final result of these two models of this portfolio with different proportion. To better show the results of running the data, the paper plots the data into two different plots as illustrated in Fig. 1 and Fig. 2. In this way one can observe the result intuitively. These red scatters present the return and standard deviation of more than 3,000 random portfolio constructions simulated by the two models. In these two plots, one can find that almost all of scatters are included curves in two different models, but we can also clearly see that under normal operation, there is a curve in the Markowitz model that clearly has many outliers, which not appears in the Index model. These outliers can be eliminated by some further and more in-depth operations, but this also shows that Markowitz model has more uncertainty and higher complexity in operation normally. In addition, it can be roughly seen from the figure that compared with Markowitz model, the curve of the Index model fits better with the points of the generated portfolio, which also proves the higher reliability of the Index model's portfolio.

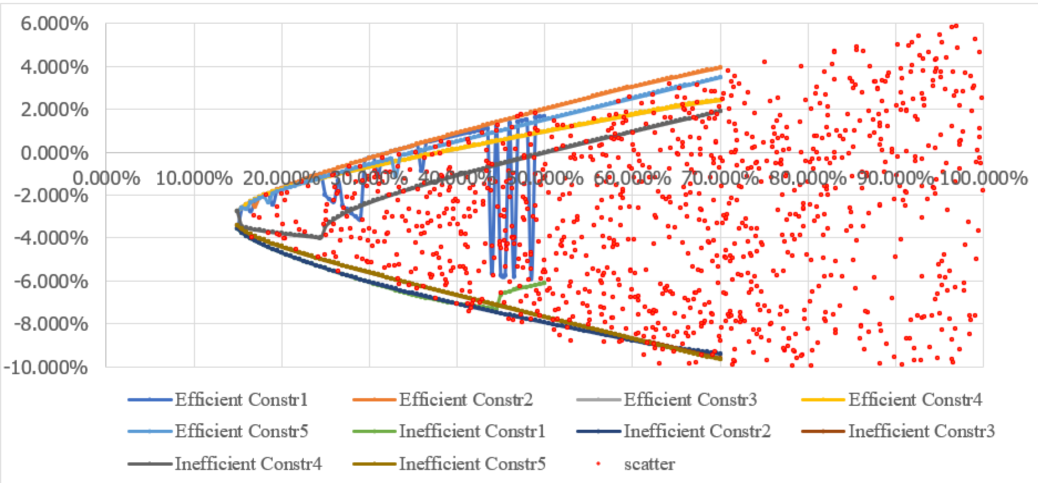

Fig. 1 Markowitz model.

Table 7. Markowitz model frontier.

MM efficient frontier						MM inefficient frontier					
StDev	**constr1**	**constr2**	**constr3**	**constr4**	**constr5**	**StDev**	**constr1**	**constr2**	**constr3**	**constr4**	**constr5**
15.000%	-3.583%	-2.787%	-2.787%	-2.787%	-2.787%	15.000%	-3.583%	-3.583%	-3.405%	-2.787%	-3.405%
15.500%	-2.599%	-2.599%	-2.599%	-2.599%	-2.609%	15.500%	-3.771%	-3.771%	-3.583%	-3.482%	-3.583%
16.000%	-2.454%	-2.454%	-2.454%	-2.454%	-2.672%	16.000%	-3.916%	-3.916%	-3.715%	-3.534%	-3.715%
16.500%	-2.767%	-2.329%	-2.329%	-2.329%	-2.366%	16.500%	-4.040%	-4.040%	-3.827%	-3.575%	-3.827%
17.000%	-2.218%	-2.537%	-2.218%	-2.218%	-2.266%	17.000%	-4.152%	-4.152%	-3.926%	-3.612%	-3.926%
17.500%	-2.115%	-2.115%	-2.115%	-2.115%	-2.175%	17.500%	-4.255%	-4.255%	-4.018%	-3.645%	-4.018%
18.000%	-2.018%	-2.018%	-2.022%	-2.022%	-2.089%	18.000%	-4.351%	-4.351%	-4.103%	-3.675%	-4.103%
18.500%	-1.927%	-1.927%	-1.939%	-1.939%	-2.352%	18.500%	-4.443%	-4.443%	-4.184%	-3.704%	-4.184%
19.000%	-2.461%	-1.840%	-1.864%	-1.864%	-1.931%	19.000%	-4.530%	-4.530%	-4.261%	-3.732%	-4.261%
19.500%	-1.755%	-1.755%	-1.793%	-1.793%	-1.857%	19.500%	-4.614%	-4.614%	-4.335%	-3.758%	-4.335%
20.000%	-1.674%	-1.674%	-1.727%	-1.727%	-1.785%	20.000%	-4.696%	-4.696%	-4.407%	-3.783%	-4.407%

Table 8. Index model frontier.

IM efficient frontier						IM inefficient frontier					
StDev	**constr 1**	**constr 2**	**constr 3**	**constr 4**	**constr 5**	**StDev**	**constr 1**	**constr 2**	**constr 3**	**constr 4**	**constr 5**
15.000%	-3.121%	-3.121%	-3.542%	-3.121%	-3.125%	15.000%	-3.542%	-3.542%	-3.542%	-3.282%	-3.463%
15.500%	-2.858%	-2.858%	-2.858%	-2.858%	-2.894%	15.500%	-3.805%	-3.805%	-3.805%	-3.482%	-3.694%
16.000%	-2.691%	-2.691%	-2.691%	-2.691%	-2.751%	16.000%	-3.972%	-3.972%	-3.972%	-3.534%	-3.838%
16.500%	-2.556%	-2.556%	-2.556%	-2.556%	-2.634%	16.500%	-4.108%	-4.108%	-4.108%	-3.575%	-3.954%
17.000%	-2.437%	-2.437%	-2.437%	-2.437%	-2.533%	17.000%	-4.226%	-4.226%	-4.226%	-3.612%	-4.055%
17.500%	-2.330%	-2.330%	-2.855%	-2.330%	-2.441%	17.500%	-4.333%	-4.333%	-4.333%	-3.645%	-4.147%
18.000%	-2.230%	-2.230%	-2.230%	-2.230%	-2.356%	18.000%	-4.433%	-4.433%	-4.433%	-3.675%	-4.233%
18.500%	-2.136%	-2.136%	-2.136%	-2.136%	-2.275%	18.500%	-4.527%	-4.527%	-4.527%	-3.704%	-4.313%
19.000%	-2.047%	-2.047%	-2.047%	-2.048%	-2.199%	19.000%	-4.616%	-4.616%	-4.616%	-3.732%	-4.389%
19.500%	-1.961%	-1.961%	-1.961%	-1.968%	-2.126%	19.500%	-4.702%	-4.702%	-4.702%	-3.758%	-4.463%
20.000%	-1.879%	-1.879%	-1.879%	-1.893%	-2.055%	20.000%	-4.784%	-4.784%	-4.784%	-3.783%	-4.533%

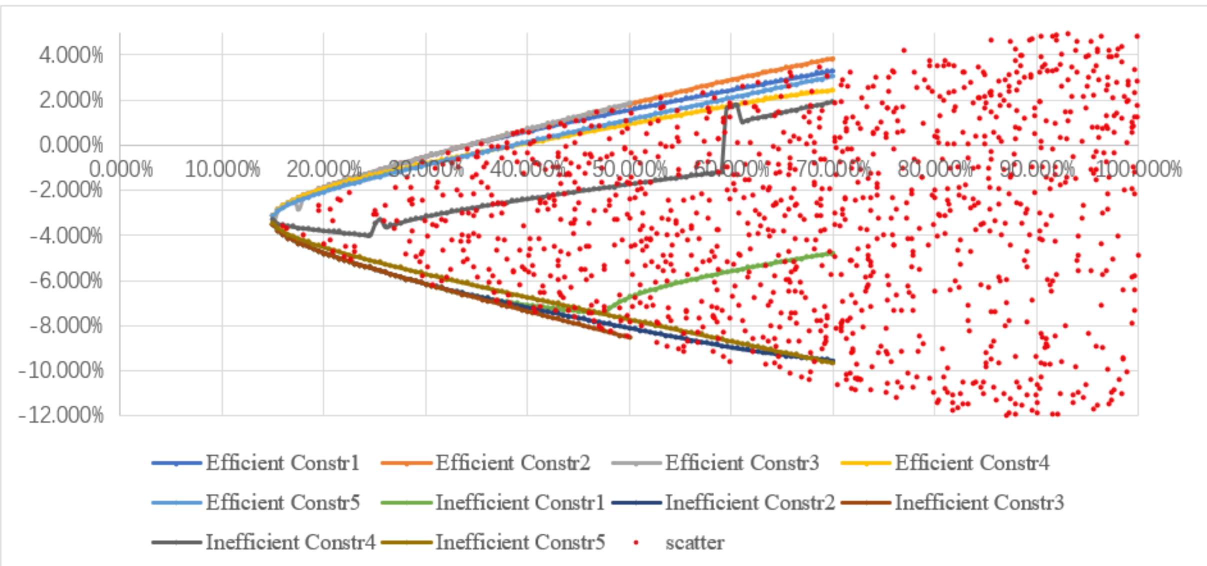

Fig. 2 Index model.

3.2 Discussion

Markowitz models require extensive estimates of expected returns, variance, and covariance. This model did not explain how these estimates are generated, and the only guess is to use historical sample averages, which may be unreliable in past returns to some extent. As for the index model, it enhances the analysis of the expected return on security (risk premium) and explicitly decomposes risk into systemic and firm-specific components [9]. Therefore, it shows more of the power and limitations of diversity. However, if stocks with correlated residuals have high alpha, then the exponential model may produce a much worse portfolio than the Markowitz model. To be specific, if the excess returns of BP and RDS/A were correlated, the index model would ignore this. In addition, the Markowitz model will take this correlation into account [10]. Nevertheless, if its radical reduction in the number of measurables, the exponential model is the model of choice in practical portfolio management. Therefore, choosing different models according to different asset allocations is the most sensible choice.

4. Limitation & Future outlooks

This article is only based on two classic models, the introduction of asset allocation is very limited, and only a small part of the knowledge and application of asset allocation can be shown. In addition, this paper selects four subjects, bitcoin (BTC), S&P 500 (SPY), gold futures (GC) and American 10-year national debt, which have a certain degree of subjective judgment, which is also a part of the factors that affect the results of the model. Moreover, the data covered in this article only covers the past three years, the data range is not large enough, and the impact of the covid-19 may lead to certain deviations in the model results. All of these factors can cause some bad impacts in this article’s result. This article is very optimistic about the future outlook for asset allocation. With the progress of the times and the broadening of people's minds, asset allocation is becoming one of the hottest topics in the world, and it will also have an extremely important impact on people's lives in the future. Due to the higher demand for asset allocation, the technical and theoretical knowledge of asset allocation must also continue to improve. Coupled with the continuous improvement of computer technology, asset allocation may develop to an extremely advanced level in the future. Machine learning is becoming one of the important application technologies for asset allocation, which can construct portfolio efficiently.

5. Conclusion

In summary, this paper selects these four indicators, bitcoin (BTC), S&P 500 (SPY), gold futures (GC) and American 10-year national debt, constructs Markowitz model and Index model and compares and analyzes the data results generated by these two models. Through the data processing and analysis results in this paper, it is known that in this case, the Index model performs better than the Markowitz model in constructing portfolios. The results show that Markowitz model is more prone to outliers, which may have a more serious impact on portfolio construction. At the same time, the curves of different constraints of the Index model have a better fitting effect on the portfolio. Nevertheless, this study has some subjectivity and contingency, which is the limitation of this paper. In the future, it is believed that asset allocation will have a better development in terms of computers. Overall, these results offer a guideline for portfolio designs.

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