Portfolio Optimization of Five Stocks Based on the Mean-Variance Model

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Abstract. Portfolio optimization, as an essential part of asset allocation, has become a core issue in the financial investment field in recent years. This paper focuses on the diversification of assets in semiconductor and integrated circuits, commodities, fast-moving consumer goods, air transportation, and streaming services industries, aiming to provide investors with a portfolio that optimizes risk and return. The mean-variance analysis, capital asset pricing model, and Fama-French three-factor model are applied to construct the portfolio. According to the asset weight allocation, the results are analyzed by maximizing the Sharpe ratio and minimizing variance. The results demonstrate that UL contributes the most proportion to the maximum Sharpe ratio and minimum variance under the capital asset pricing model. In the Fama-French three-factor model, DAL and UL provide the highest weight for the maximum Sharpe Ratio and minimum variance, respectively. Since KLIC is negatively correlated with UL and DAL has a negative correlation with NFLX, comprising these assets in the portfolio can function as a diversification. Therefore, the results are helpful for investors interested in portfolios in related industries.

Keywords: Mean-variance; capital market pricing model; Fama-French-Three-factor model; portfolio optimization.

1. Introduction

As a part of modern portfolio theory, the mean-variance model was first raised by Markowitz in 1952 [1]. It assumes that investors make rational investment decisions given complete information: the highest possible expected return and the lowest possible uncertainty. To do so, investors can utilize mean-variance analysis to create data sets and build portfolios correspondingly. Therefore, around the multi-objective optimization of risk and return, mathematical statistics has developed into an essential tool in the field of modern portfolio research [2]. To diversify unsystematic risk, the risk-adjusted return of investing in a low or negative correlation portfolio will be greater than that of the individual asset, so investors can enjoy ‘safer’ portfolios by combining the weight proportion of different assets.

Researchers have provided new perspectives and concepts for portfolio optimization in recent years. For example, Olivares et al. studied the portfolio problem with transaction fees [3]. They proposed a data-driven portfolio optimization method that can balance the estimation error between the acquisition of data and excessive trading. Zhi et al. applied the copula model to implement a scheme for inventory financing providers to optimize collateral portfolios and reduce default risk [4]. Meade et al. posed a portfolio selection based on density prediction, which effectively improved the accuracy and sensitivity of data processing and proved the superior performance of this strategy [5]. Dai et al. developed a mean-variance model with sparseness, which reduces the impact of parameter uncertainty and assessment error on the model [6]. Some studies also involve hot topics in the financial investment community, such as ESG. Davide et al. incorporated digital ESG into dynamic pricing theory and introduced ESG affinity parameters to achieve the purpose of portfolio optimization [7].

Despite the previous studies on portfolio optimization, investment research on certain assets is relatively scarce, drawing the author’s interest in further exploration. Kulicke and Soffa Industries, Inc. (KLIC), Albemarle Corporation (ALB), Unilever PLC (UL), Delta Air Lines, Inc. (DAL), and Netflix, Inc. (NFLX) are five stocks as investment and research objects.

The research process can be summarized as follows: first, select five companies in different industries as the investment target, then collect and sort the monthly adjusted closing prices for the entire five years from May 2017 to May 2022 as the sample set. Secondly, this paper conducts risk and return analysis based on the mean-variance statistical method. In particular, to compare and analyze the capital asset pricing model (CAPM) and the Fama-French three-factor model. The third step locates the portfolio weight under the maximum Sharpe ratio and minimum variance by applying the CAPM method. Nalini & Paldon points out that since CAPM has a single risk factor that depends on the efficient market hypothesis, the explanatory power of the relationship between portfolio risk and return could be inefficient [8]. Therefore, the fourth step recalculates the portfolio with the maximum Sharpe ratio and the minimum variance under the Fama-French three-factor model per the same process. According to Akhtar, this paper argues that the Fama-French three-factor model compensates for CAPM's limitations and reflects both market and firm-specific risk, so it is more accurate and feasible than CAPM in this case [9].

The remaining parts are organized as follows: Part 2 displays the data, Part 3 explains the model and methodology, and Part 4 and Part 5 present results and conclusions correspondingly.

2. Data

The data employed in this paper came from Yahoo Finance (https://ca.finance.yahoo.com/) and Kenneth French Data Library (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The five stocks selected are KLIC, ALB, UL, DAL, and NFLX, with monthly closing prices from May 2017 to May 2022, giving an entire 5-year period. This paper considers the above five companies because they provide similar volatility to the broader market, with acceptable performance in the past five years. In addition, the chosen companies are distributed in different industries to diversify risk, with correlation coefficients less than 0.5, indicating a low correlation. Since the collected stock data in this paper is the monthly adjusted closing price, which needs to be converted into monthly return form, the formula is shown below:

$$R = \frac{P_n - P_{n-1}}{P_{n-1}} \tag{1}$$

Where R represents the monthly return, P_n is the month-end stock price, and P_{n-1} is the month-beginning stock price. Basic details of R is shown in Table 1:

Table 1. Descriptive Statistics of the Selected Assets

	'KLIC'	'ALB'	'UL'	'DAL'	'NFLX'
Mean	0.0221	0.0234	0.0028	0.0053	0.0126
Variance	0.0132	0.0163	0.0022	0.0106	0.0150
Max	0.3978	0.4587	0.1020	0.3136	0.4081
Min	-0.1697	-0.3113	-0.1076	-0.3773	-0.4918

It can be observed from Table 1 that ALB has the highest average return and variance, which are 0.0234 and 0.0163. In contrast, UL has the lowest average return and variance, with 0.0028 and 0.0022. Besides, the max average return also comes from ALB, but NFLX contributes the lowest min return. Noticeably, the lowest max return and highest min return are both in UL.

3. Methods

3.1 Mean-Variance Analysis

As a statistical analysis tool, mean-variance analysis helps weigh the risk and return of a portfolio, allowing investors to identify the topmost return for a given risk or the undermost risk for a set return based on risk appetite [2]. Curtis states that according to modern portfolio theory (MPT), portfolios on the efficient frontier contain multiple stocks and assets to achieve diversification and provide the highest expected return and the lowest risk [10]. The relevant formulas are listed as follows:

$$\sum_{i=1}^{n} w_i = 1 \quad (2)$$

Where w_i represents the weight of each individual asset in the portfolio.

$$E_{(r_p)} = \sum_{i=1}^{n} w_i r_i \quad (3)$$

$E_{(r_p)}$ stands for the portfolio' expected yield, w_i is the proportion of each single asset, and r_i is the expected return of each individual asset.

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) \quad (4)$$

Where σ_P^2 means the variance of the portfolio, w_i and w_j represent the weight of $asset_i$ and $asset_j$, and $Cov(r_i, r_j)$ is covariance between expected return of $asset_i$ and $asset_j$.

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p} \quad (5)$$

Where R_p indicates the portfolio's expected return, R_f stand for the risk-free rate, and σ_p is the standard deviation of the portfolio's extra return. Sharpe ratio describes how much return an investor should receive per unit of risk and measures the portfolio performance of the investment strategy [11].

3.2 Capital Asset Pricing Model

The capital asset pricing model (CAPM) interprets the link between risk and return on the invested asset [8]. It quantifies the impact of systematic risk on asset value, which helps investors find the optimal portfolio on the boundary of the efficient frontier and judge whether the asset's current price is consistent with the future return [12]. With simple and practical, it is the cornerstone of price theory in modern financial markets and is widely applied in investment decision-making and risk management [8]. CAPM can be described by following equation:

$$E_{(R_i)} = R_f + \beta_i\left(E(R_m) - R_f\right) \quad (6)$$

Where $E_{(R_i)}$ denotes the portfolio expected return, R_f is the riskless rate of return, β_i represents the sensitivity of risky assets to the broader market, and $E(R_m) - R_f$ expresses the market risk premium, which is the part the market yield exceeds the riskless rate of return. Moreover, a positive β_i greater than 1 indicates the asset is riskier than the market portfolio, and a positive β_i less than 1 shows the asset has a positive relationship but less risk compared to the market portfolio [13]. However, a negative β_i gives an opposite direction to market movement.

3.3 Fama-French Three-Factor Model

The Fama-French three-factor model was first put out in the 1990s, obtained from historical stock returns through regression, adding a size component and a value component to the framework of CAPM to form a three-factor asset pricing model. Since this model holds that small-cap stocks surpass large-cap stocks and value stocks exceed growth stocks in the long run, it is considered a conducive tool for evaluating portfolio performance. The model is shown as follows:

$$E(r_i - r_f) = \beta_i[E(r_m - r_f)] + S_i[E(SMB)] + h_i[E(HML)] \tag{7}$$

$E(r_i - r_f)$ is the expected extra return of the portfolio, r_f shows risk-free rate, r_m represents the return of market portfolio, $(r_m - r_f)$ signifies the factor of market excess return, and SMB stands for market capitalization, which is the size premium factor. HML stands for book-to-market values, which is the value premium factor [9]. Besides, β_i, S_i, and h_i are coefficient factors, which demonstrate the sensitivity of each factor to market volatility.

4. Results

The correlation coefficient gauges the degree of linear correlation between two assets, ranging from -1 to 1 [14]. A correlation coefficient more prominent than 0 implies a positive linkage, while less than 0 shows a negative relationship. A correlation coefficient equal to one is a perfect positive correlation, and a value of 0 implies no correlation between the two assets. Therefore, a negative correlation or low correlation coefficient means a diversified portfolio can better offset the impact of volatility. Choi and Shin explain that when the correlation coefficient is less than 0.5, it is generally considered a low correlation between variables [14]. It is worth noting that the correlation coefficients of KLIC and UL, as well as DAL and NFLX, are less than 0, which plays the role of diversifying portfolio risk. The correlation coefficient of each asset in the following table 2 is nonzero and below 0.5, indicating a weak or adverse correlation.

Table 2. The Correlation Between the Equities

	'KLIC'	'ALB'	'UL'	'DAL'	'NFLX'
'KLIC'	1				
'ALB'	0.4229	1			
'UL'	-0.0242	0.1184	1		
'DAL'	0.2884	0.2637	0.1804	1	
'NFLX'	0.2825	0.1944	0.1059	-0.0699	1

Through CAPM, this paper obtains portfolio expected return, the weight of each asset under the maximum Sharpe ratio and minimum variance, the outcomes are exhibited as follows in Table 3:

Table 3. Results for Maximum Sharpe Ratio Portfolio Optimization under CAPM

	'KLIC'	'ALB'	'UL'	'DAL'	'NFLX'
Weight	0.1194	0.1541	0.3030	0.2136	0.2099
Expected return	0.0093				
Variance	0.0031				
Standard deviation	0.0559				
Sharpe ratio	0.1190				

Under the condition of maximizing Sharpe ratio, UL contributes the highest weight of 0.3030, and KLIC has the lowest weight of 0.1194. Since UL is a fast-moving consumer goods (FMCG) company, its business scope covers a wide range of food, beverage, daily necessities, and other areas, with low vulnerability to market fluctuation. KLIC, on the other hand, is a high-tech company focusing on semiconductors and electronics assembly, relying on cyclical and seasonal dynamics within the industry and macroeconomic factors. This paper also works out the consequence of minimum variance under CAPM, as shown in the table 4 below:

Table 4. Results for minimum variance portfolio under CAPM model

	'KLIC'	'ALB'	'UL'	'DAL'	'NFLX'
Weight	0.0979	0.0110	0.7494	0.0769	0.0649
Expected return	0.0065				
Variance	0.0018				
Standard deviation	0.0423				
Sharpe ratio	0.0900				

Under the requirement of minimum variance, the highest weight appears in UL, which is 0.7494, implying that UL has the lowest variance (shown in Table 1). As mentioned above, the reason is that UL, an FMCG company with comprehensive business coverage, has superior resistance to volatility. The lowest weight offered by ALB is 0.0110. It may have such a weight because ALB treats lithium battery research and development as the primary business involving oil refining and crop production, which are vulnerable to market demand. Hence its highest variance leads to the lowest weight. The following Table 6 displays the results under the Fama-French three-factor model. Table 5 demonstrates the coefficient factors under the Fama-French three-factor model:

Table 5. Estimation of Fama-French Three-Factor Model Parameters

	$\boldsymbol{\beta_i}$	$\boldsymbol{S_i}$	$\boldsymbol{h_i}$
'KLIC'	1.1315	0.7520	0.4078
'ALB'	1.4337	0.4416	0.4536
'UL'	0.4132	-0.3798	-0.0691
'DAL'	0.9439	0.4403	1.0078
'NFLX'	1.1929	0.4653	-1.1164

According to the above coefficient factors, this paper attains the maximum Sharpe ratio in line with the Fama-French three-factor model:

Table 6. Results for Maximum Sharpe Ratio Portfolio under Fama-French Three-Factor Model

	'KLIC'	'ALB'	'UL'	'DAL'	'NFLX'
Weight	0.1735	0.1912	0.1764	0.3658	0.0931
Expected return	0.0116				
Variance	0.0043				
Standard deviation	0.0655				
Sharpe ratio	0.1360				

The highest weight under the Fama-French three-factor model is DAL, which is 0.3658. A possible explanation might be that DAL is a sizeable international air transport company with two independent business units: an airline and a refinery, allowing it to enjoy high operating income with low fuel costs. The lowest weight derives from NFLX, with 0.0931. The main logic for this weight is that the Internet and streaming media services industry is highly volatile and depends on the external economic cycle. This paper also derives the weights under the minimum variance condition, the results are presented in Table 7:

Table 7. Results for minimum variance portfolio under Fama-French Three-Factor Model

	'KLIC'	'ALB'	'UL'	'DAL'	'NFLX'
Weight	0.0979	0.0110	0.7494	0.0769	0.0649
Expected return	0.0064				
Variance	0.0018				
Standard deviation	0.0423				
Sharpe ratio	0.0878				

The table above denotes that UL dominates the portfolio weight, contributing 0.7494. The main reason for this weight is that UL has the lowest variance in the portfolio, with the solid anti-volatility ability of the industry at a macro level. On the contrary, the high volatility of the lithium battery manufacturing industry and the weak demand caused by the COVID-19 epidemic might be the motive for the lowest weight of ALB, with 0.0110.

5. Conclusion

The modern portfolio investment theory created by Markowitz quantifies the impact of return and risk. Investors can construct a portfolio with the highest Sharpe ratio or the lowest risk based on the mean-variance model. This paper aims to assist investors in making optimal investment choices from semiconductors and integrated circuits, commodities, FMCG, air transportation, and streaming media services by constructing a portfolio of assets in the five industries. As per the mean-variance model, this paper adopts two portfolio optimization methods: the capital asset pricing model and the Fama-French three-factor model, to work out the portfolios with maximum Sharpe ratio and minimum variance, respectively. Concerning maximizing the Sharpe ratio, this paper finds that the FMCG industry accounts for the most significant proportion under the CAPM, followed by air transport and streaming services. By contrast, the air transport industry dominates beneath the Fama-French three-factor model, followed by commodities, FMCG, and semiconductors. This result may be because the Fama-French three-factor model considers further risk factors, thus with the more substantial explanatory power of risk and return. However, the result may not represent the performance of all stocks due to the limitation of data scope and the model application.

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