Optimal Portfolio with Pension under Three Models: Mean-Variance, CAPM and FF3F

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Abstract. In recent years, an increasing number of people are investing their money to increase their total wealth and maintain their quality of life after retirement. How to choose the optimal portfolio is a key part of financial management. This study chooses six assets which performs relative well in different industries together with one's pension which is regarded as an asset accounting for a solid weight. This paper uses three models: Mean-Variance analysis, CAPM and FF3F model to build the optimal portfolio to maximize sharp ratio and minimize variance respectively. The result shows that, in the mean-variance model, GLD takes up the largest weight both in the maximize sharp ratio portfolio and minimize variance portfolio. While under the CAPM and FF3F model, QQQ accounts for the largest weight in the maximize sharp ratio portfolio while GLD accounts for the largest weight in the minimize variance portfolio. The results in this paper may provide some recommendations for firefighters who wants to make an investment portfolio with pension.

Keywords: Portfolio optimization; mean-variance; CAPM; FF3F.

1. Introduction

In 1952, the born of portfolio theory based on quantitative economic is a breakthrough in finance. Since then, people can find the optimal portfolio which can balance his personal financial goals and risk appetite well [1]. Portfolio management is perhaps the greatest product, service, to investors [2]. And in recent years, with the development of society and economy, portfolio investment is widely accepted and used [3]. This has led to a growing awareness of the importance of portfolio theory and since the risks nowadays have been diversified and complicated, how to design an optimal portfolio is a hot topic in finance area.

In recent years, a great number of articles have focused on portfolio optimization in various areas. For instance, Fachrudin and Fachrudin analyzed the property portfolio in real assets and found that the portfolio is influenced by the company size and is related to the company performance [4]. Mills and MacGill studied the impact of electric vehicle and large-scale photovoltaic on future generation portfolio investment [5]. And Grant & Collins discussed the IT investment portfolio which can vastly create real business value [6]. Based on his own experience, Raymond discussed portfolio strategies of pharmaceutical products and make commercial perspectives [7]. According to the research of Wang, she takes Beijing as an example to study the portfolio of different property types and construct a portfolio optimization model which will benefit the launch and implementation of REITs in China [8]. Liu conducted research on the optimal portfolio of fund distribution companies by studying the HM company and got several recommendations [9]. During the epidemic, Xia and Shan studied the performance of different industries and tried to build portfolio which can resist the risks brought by the Covid-19 ad have a relatively good return [10].

However, seldom articles focused on optimal portfolios on accurate assets including certain stocks and mutual funds, and fewer articles consider people's pension as a part of the incoming asset. This paper seizes the issue and conducts research on it. In this paper, this paper assume that people choose to hold his pension as an asset which has a determined cash flow each year with a certain growth in terms of inflation effects. The pension itself accounts for 71% in the total portfolio. Apart from the pension, the paper selects four stocks: BYD, AAPL, SBUX, CVNA and an index fund QQQ and an ETFs GLD to build optimal portfolios. The steps are as follows. Firstly, this paper chooses these six assets in different industries to diversify the investment portfolio and then download their historical data in the past five years. After that, this paper adopts the mean-variance model to calculate the

optimal portfolio which maximizes sharp ratio. To satisfy investors with different risk preferences, this paper also build the minimize variance portfolio under the mean-variance model. However, the input data into the mean-variance model such as the expected return is easy to be inaccurate which may cause a big bias in the portfolio. So, this paper uses the CAPM to calculate the expected return and then build the maximize sharp ratio portfolio and the minimize variance portfolio under the CAPM. And then this paper finds that the CAPM faces empirical challenges. The CAPM is a single factor model which assumes so ideal that the asset is only influenced by one factor, MP. However, in the real market, two other factors, SMB and HML, also have great impact on assets. Therefore, this paper finally adopts the Fama-French three-factor model to build the maximize sharp ratio portfolio and the minimize variance portfolio respectively to get a more realistic portfolio.

The structure of this paper is shown below. Part 2 depicts data. Part 3 is the introduction of the three methods this paper use to build the portfolio. Part 4 presents the results and gives brief analysis of each portfolio. And Part 5 makes a conclusion.

2. Data

The data in this article is sourced from Flush Finance (https://www.10jqka.com.cn/). The article selected six assets: BYD, AAPL, CVNA, SBUX, GLD, QQQ together with the pension which is regarded as an asset to build a portfolio. BYD, AAPL, CVNA and SBUX are stocks in four different sectors: automotive, high-tech, used car dealership and restaurants. GLD is a gold ETFs and QQQ is an index fund. The article chose these six assets for two main reasons: firstly, this selection can diversify the assets, minimize correlation as much as possible and have both positive and negative correlations between assets, thereby reducing the overall risk of the portfolio; secondly, these selected assets performed relative well over the past five years and have a relative high market beta.

The article collected the monthly closing prices of 6 stocks for the last 5 years from August 2017 to August 2022, and then calculated the monthly returns from September 2017 to August 2022 using the formula:

$$R_x = (P_{tx} - P_{t(x-1)})/P_{tx} \quad (1)$$

Some statistical data for the seven assets, including the pension, are presented in Table 1 below.

Table 1. Descriptive statistics of the assets

	BYD	AAPL	CVNA	SBUX	GLD	QQQ	pension
Mean	0.0401	0.0114	0.0361	0.0099	0.0036	0.0138	0.0065
Max Return	0.4085	0.1962	0.5056	0.1672	0.1079	0.1497	0.0458
Min Return	-0.2013	-0.6964	-0.5141	-0.1795	-0.0940	-0.1360	-0.0128
Variance	0.0219	0.0158	0.0485	0.0054	0.0015	0.0035	0.0001

Looking through the above table, this paper can find that BYD has the highest average return, and the pension has the lowest average return. Besides, the lowest max return and the highest min return both appears in pension, while the highest max return appears in “CVNA”, and the lowest min return appears in “AAPL”. When it comes to the variance, this paper find that the lowest variance appears in pension and the highest variance appears in “CVNA”.

3. Methods

3.1 Mean-Variance Analysis

The mean-variance model is a risk investment model proposed by Markowitz to balance the return and risk of a portfolio. The mean is the average rate of return, and the variance represents the risk. This model is simple to construct and easy to understand, and investors can use this model to

maximize the return or minimize the risk of their portfolio. According to Levy, the mean-variance rule is possibly the most popular investment decision rule [11]. It is known to be the basis of modern portfolio analysis in general and the capital asset pricing model in particular [12].

The process of building the model are as follows.

Firstly, supposing that there are n kinds of risky assets in the market, the return of each asset is $r_{1,}r_{2,}\cdots\cdots r_n$, the weight of each asset is $w_{1,}w_{2,}\cdots\cdots w_n$ and $\sum_{i=1}^{n} w_i = 1$

And then this paper uses the following formula to calculate the expected return, variance and sharp ratio.

$$E(r_p) = \sum_{i=1}^{n} w_i \, E(r_i) \tag{2}$$

$E(r_p)$ means the expected return of the portfolio, $E(r_i)$ means the expected return for each asset.

$$Var(r_p) = \sum_{i=1}^{n} \quad \sum_{j=1}^{n} Cov(r_i, r_j) \tag{3}$$

$Var(r_p)$ means the varience of the portfolio, $Cov(r_i, r_j)$ means the correlation between the expected returns of asset i and the asset j. A positive correlation means the returns of the assets tend to move in the same direction while a negative correlation means the returns of the assets tend to move in the opposite direction.

$$Sharp\ ratio = \frac{E(r_p) - R_f}{\sigma_p} \quad (\sigma_p = \sqrt{Var(r_p)}) \tag{4}$$

σ_p means standard deviation, R_f is risk-free rate which means the return of a risk-free asset, $E(r_p)$ means the expected return of the portfolio. The sharp ratio is used to make a comparison between the return and risk. A high sharp ration means the relationship between the risk and return is favorable.

3.2 Capital Asset Pricing Model

CAPM is based on H.M. Markowitz theory, and it is the backbone of modern financial market price theory. According to Kim, since its introduction, the CAPM has been a benchmark model in the asset pricing research due to its ease-of-use and efficiency [13]. The CAMP is a single factor model which only considers market portfolio, and it allows investors to evaluate and select assets only according to the systematic risks. The model is quite powerful to measure the relation between risk and expected return. The CAPM is still widely used in applications, such as estimating the cost of capital for firms and evaluating the performance of managed portfolios [14]. The following is the core formula.

$$E(r_i) = r_f + \beta_i E(r_m - r_f) \tag{5}$$

r_m is the return of market portfolio, which is the average rate of returns of all the assets in the market, r_f is the return of risk-free asset,$E(r_i)$ is the expected return for asset i. $E(r_m - r_f)$ is the expected excess return. β_i represents the systematic risk of asset i which can measure the fluctuation of the asset's price compared to the whole market. Different assets probably have different β and in most cases β is positive.

3.3 Fama-French Three-factor Model

Fama-French three-factor model is a multifactor model based on the CAPM. The model considers three risk factors: MP (market portfolio), SMB (a size factor) and HML (a book-to-market equity factor). SMB and HML are two firm-related factors adding to the traditional CAPM. The model is

mainly used to study the correlation between stock returns and market risk in the equity market. The model is highly applicable and is now widely used in the fields of asset portfolio returns, fund stock selection and so on [15]. The following is the core formula of this model.

$$E(r_i - r_f) = \beta_{iM}E(r_m - r_f) + \beta_{iSMB}E(SMB) + \beta_{iHML}E(HML) \quad (6)$$

$E(r_i - r_f)$ means the expected excess return. SMB is the return on a portfolio of small stocks minus that of big stocks and HML is the return on a portfolio of value stocks minus that of growth stocks in terms of B/M [16]. The data for SMB and HML are downloaded from French's homepage. β_{iM}, $\beta_{iSMB}, \beta_{iHML}$are the coefficient of MP, SMB, HML.

4. Result

This paper uses three models: Mean-Variance Analysis, the CAPM, the Fama-French Three-Factor Model to build portfolios for the firefighter. Since the pension has a solid weight 0.71, the following discussions are developed among the six other assets. In this paper, this paper does not take short selling into consideration. Via Mean-Variance model, the paper builds the portfolios to maximize sharp ratio and minimize variance respectively. The results are shown in the Table 2 & Table 3 respectively.

Table 2. Results for the portfolio maximize sharp ratio under Mean-Variance model

	BYD	AAPL	CVNA	SBUX	GLD	QQQ	pension
Weight	0.0699	0.0004	0.0040	0.0061	0.1335	0.0761	0.71
Expected return	0.0091						
Variance	0.0002						
Standard deviation	0.0145						
Sharp ratio	0.4450						

Among the six assets whose weight can be changed, the highest weight 0.1335 appears in 'GLD' while the lowest weight 0.0004 appears in 'AAPL'. GLD has the highest weight may because it is a gold ETF which has high mobility and low transaction costs. AAPL has the lowest weight probably because in recent years the price of the stock is quite unstable because of the emerge of 5G, its own lack of innovation, the epidemic and the fierce competition in the industry.

Table 3. Results for the portfolio minimize variance under Mean-Variance model

	BYD	AAPL	CVNA	SBUX	GLD	QQQ	pension
Weight	0.0245	0	0	0.0192	0.2170	0.0293	0.71
Expected return	0.0069						
Variance	0.0001						
Standard deviation	0.0118						
Sharp ratio	0.3643						

The highest weight 0.2170 appears in 'GLD'. GLD has the lowest variance among the 6 assets because the gold trading market is already mature and relative stable. Besides, GLD is backed by real gold, and it is a gold ETF which follows the gold price timely and has low bid ask spread. The lowest weight appears in AAPL and CVNA which means they have relative high variance. AAPL has a high risk perhaps because its decreasing competitiveness causing by the appearance of 5G, the increasing security vulnerability of products, the lack of innovation, which may lead to uncertainty of its stock price. The reason for CVNA is that perhaps the new energy vehicles has given a blow to the used car trade industry.

Using the CAPM, the paper builds the portfolios to maximize sharp ratio and minimize variance respectively. The results are shown in the Table 4 & Table 5 respectively.

Table 4. Results for the portfolio maximize sharp ratio under the CAPM

	BYD	AAPL	CVNA	SBUX	GLD	QQQ	pension
Weight	0.0076	0.0011	0.0009	0.0314	0	0.2489	0.71
Expected return	0.0050						
Variance	0.0004						
Standard deviation	0.0198						
Sharp ratio	0.1169						

The highest weight 0.2489 appears in 'QQQ' while the lowest appears in 'GLD'. QQQ has the highest weight perhaps because it is an ETF which contains several technology stocks with highest market value listed on Nasdaq, which remains relatively good development in the past five years. GLD has the lowest weight perhaps because it is a gold ETF which performances well as a short-term investment but not very suitable for the long-term investment.

Table 5. Results for the portfolio minimize variance under the CAPM

	BYD	AAPL	CVNA	SBUX	GLD	QQQ	pension
Weight	0.0245	0	0	0.0192	0.2170	0.0029	0.71
Expected return	0.0033						
Variance	0.0001						
Standard deviation	0.0118						
Sharp ratio	0.0556						

The weight distributions of the assets are the same with that of the Mean-Variance Model. The highest weight also appears in 'GLD', and the lowest weights appear in 'AAPL' and 'CVNA' as well. Under the Fama-French Three-Factor model, the paper builds the portfolios to maximize sharp ratio and minimize variance respectively. The results are shown in the Table 6 & Table 7 respectively.

Table 6. Portfolios maximize sharp ratio under the Fama-French Three-factor Model

	BYD	AAPL	CVNA	SBUX	GLD	QQQ	pension
Weight	0.0154	0.0041	0	0.0635	0	0.2069	0.71
Expected return	0.0046						
Variance	0.0004						
Standard deviation	0.0190						
Sharp ratio	0.1023						

The highest weight also appears in 'QQQ' while the lowest weight appears in 'QQQ' and 'CVNA'. CVNA has the lowest weight perhaps because the new energy vehicles have sprung out in recent years. Due to the lower price and the trend of environmentally friendly, the new energy vehicle industry is becoming a strong competitor to the used car trading industry.

Table 7. Portfolios minimize risk under the Fama-French Three-factor Model

	BYD	AAPL	CVNA	SBUX	GLD	QQQ	pension
Weight	0.0245	0	0	0.0192	0.2170	0.0293	0.71
Expected return	0.0069						
Variance	0.0001						
Standard deviation	0.0118						
Sharp ratio	0.0440						

The weight distributions of the assets are the same with that of the Mean-Variance Model. The highest weight also appears in 'GLD' and the lowest weights appear in 'AAPL' and 'CVNA' as well.

5. Conclusion

Nowadays, many articles on asset portfolios focus only on a single sector or a few sectors that are correlated and are studied from a sector-wide perspective. This article includes stocks and funds in different sectors which have low correlations. What's more, this article considers people's pension as an asset to build the portfolio which is seldom seen in other similar papers. This paper chooses four stocks in automotive, high-tech, used car dealership, restaurants industry, an ETFs, an index fund together with pension as the total assets, which can diversify and disperse the risk of the whole portfolio. The paper adopts three models to build optimal portfolios. Since the weight of the pension in the portfolio is unchangeable, the following conclusions don't take the pension's weight into consideration. In the Mean-Variance model, the paper finds that the two funds account for a relatively large proportion in both the maximize sharp ratio portfolio and the minimize risk portfolio. Under the CAPM, the index fund QQQ and SBUX takes up the two higher weights in the maximize sharp ratio portfolio while the ETFs GLD and BYD takes up the two higher weights in the minimize risk portfolio. When it comes to the Fama-French Three Factor model, QQQ and SBUX are the two main assets in the maximize sharp ratio portfolio while in the minimize risk portfolio, it is GLD and QQQ that are the prominent assets. The aim of our research is to help people who wants to invest with pension to make optimal portfolio in diversified fields. However, it is certain that there are shortcomings exist in the methods this paper use. The three analysis methods only use the variance to measure risks but there are many other indicators which can measure variance as well but are not used in the models. Besides, in the real market, there are more factors than MP, SMB, HML that will influence the investment portfolio which are not considered in the passage.

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