# Portfolio Optimization Analysis for Industries

Baizhou Li[1, †], Shaokun Tang[2, *, †]

[1]Admiral Farragut Academy Tianjin, Tianjin, China

[2]School of Social Science, McMaster University, Ontario, Canada

*Corresponding author: tangs72@mcmaster.ca

[†]These authors contributed equally

**Abstract.** This research is based on the existing financial models, which are placed in different markets to obtain high-quality investment portfolios. Based on the capital asset pricing model and the Fama-french three factor model, we maximize the sharp ratio. After analyzing the monthly data of some stocks in the banking, tourism, catering, technology and automotive industries in the past five years, and after analyzing a large number of stocks, we select them and then conduct mathematical modeling to obtain model conclusions that can provide investors with the optimal solution in the mathematical model. Through the analysis of the model, we can draw two different configurations for different investors, as well as the extreme value range of Sharp ratio. This result is highly relevant to investors. It is a very practical operation and practical method in the market. It allows investors to make simple analysis of historical data on fundamentals when selecting stocks, so that investors can be more rational.

**Keywords:** Optimization; risky and risk-free asset, factor model, asset portfolio.

## 1. Introduction

In the current stock market, the method of quantifying the historical trend of stocks with mathematical models is still relatively rare. Many people do not believe in the basic analysis and believe that stocks are impermanent. However, after generations of talents' continuous analysis and experiments on the market, they finally summarized a lot of useful mathematical models to choose the investment portfolio. At the same time, the research of this film involves several industries, which is the pillar of social stability and security and has a significant impact on the stability of the national economy and social development. Therefore, building a high-quality portfolio has become a research hotspot in the current field. In the current field, Markowitz is the founder of the quantitative fundamental information of mathematical models. He first proposed the mean-variance model, and then, it is quickly developed capital asset pricing model, Fama-french three factor model [1-4]. However, because investors in the market hold less capital, and investors are more risk averse and less risk tolerant, this study will give investors a clear indication of the relationship between risk and return, as well as where it is located, it can maximize the utility of investors, or in other words, how to obtain higher returns with the lowest possible risk is undoubtedly their most concern.

We find that some scholars have made research on asset portfolios under the current knowledge framework, but few have made analyses when they are involved in multiple industries. The stocks involved in our research are scattered in banking, travel, catering, science and technology, manufacturing, and other industries. At present, most of the existing papers analyze the whole industry. For example, Zhang analyzed the empirical research on the constituent stocks of the Shanghai Stock Exchange Index based on CAPM and Fama French's three-factor model [5]. Enkhbaatar studied the majority of company stocks in Mongolia's mining industry based on the CAPM model [6]. Dunbar also tested the applicability of the asset pricing model in the cryptocurrency market, and Fan conducted an empirical study on the behavior of traditional Chinese medicine based on the CAPM model [7-8]. The goal of this study is to select the stocks of five representative companies in five different industries, namely, the banking industry, tourism, catering, technology industry, and automobile manufacturing industry, and then analyze the ratio of risk and return to allocate assets to obtain an investment portfolio. In this paper, we use the historical monthly data of

five companies to calculate the variance, covariance, standard deviation and other basic data, and use these data to effectively build a model to give investors suggestions.

The structure of this paper is as follows. The second part introduces the stocks selected and the data used in the study. The third part introduces the principle of the model, how to predict the trend of the stock, and how to maximize the Sharp ratio. The fourth part will discuss the results and volatility of the stock forecast, and then the conclusions and discussions can be drawn in the fifth part.

## 2. Data

The data used in this paper are all from Yahoo Finance (https://finance.yahoo.com/). The five companies selected by this paper are Bank of Montreal, Rogers Communications, McDonald's, Apple and Bavarian Motor Works which have the stock codes BMO, RCI-B, MCD, APPL and BMW respectively. Their closing prices in last five years, which are from August 2017 to August 2022 are chosen for the calculation and analysis to get the optimal portfolio. The reason why these stocks are selected is that these five enterprises are all companies which are quite common in people's daily life and represents five typical domains of industry which are bank, telecommunication, catering, mobile phone and vehicle respectively, thus the diversity is ensured. Since they are brands closely related to people's daily life while each of them has a large quantity of customers, their stocks have attracted many investors to invest, which can be used as research cases. Some basic data of these five assets are illustrated by the Table 1 below:

**Table 1.** Descriptive statistics of the selected assets.

| | BMO | RCI-B | MCD | APPL | BMW |
|---|---|---|---|---|---|
| Mean | 0.01127 | 0.00211 | 0,01194 | 0.03055 | 0.0053 |
| Variance | 0.00577 | 0.00191 | 0.00252 | 0.00801 | 0.00695 |
| Max return | 0.23899 | 0.13193 | 0.13432 | 0.21438 | 0.24472 |
| Min return | -0.25958 | -0.07981 | -0.1431 | -0.18404 | -0.2005 |

As the table demonstrates above, it is obvious to find that APPL has the highest average return which is 0.03055 and the lowest value of average return is from RCI-B which is 0.00211. Then comparing these variances, APPL and RCI-B still have the highest and the lowest which are 0.00801 and 0.00191 respectively. For the maximum and minimum return, RCI-B has both lowest max return and highest min return at the same time. BMW has the highest value of max return while BMO owns the lowest value of min return.

## 3. Methodology

The methodology used in this essay is mean-variance model, Capital Asset Price Model and Fama-French three factor model.

### 3.1 Mean Variance Model

The modern portfolio optimization was established by Markowitz, and then Tobin proposed the "separation theorem of two funds" based on this theory. This theorem shows that: on the effective frontier of risk asset portfolio, if we defined two distinct points represent two different optimizations, it means that there are some points on the effective portfolio frontier, both are able to express linearly by the portfolio represented these two distinct points.

The model defined the risk as rate of return and volatility. It was the first time to apply mathematical statistics to study of portfolio optimization. It can take the balance of the weight of return and the weight of risk in this method. So, Markowitz established the effective frontier. The mean-variance model of asset portfolio is [9-10],

$$\sigma^2 = var\left(\sum_i x_i r_i\right) = \sum_{ij} x_i x_j cov(r_i, r_j) \tag{1}$$

$$E(r_p) = \sum_{i=1}^{n} \omega_i E(r_i) \tag{2}$$

The constraint conditions

$$\sum_i x_i E(r_i) \geq \mu \tag{3}$$

$$\sum_i x_i \leq 1, \ \ x_i \geq 0 \tag{4}$$

The asset i is allowed to short-selling, and the $x_i$ can be ignored. The $x_i$ indicates the proportion of all investment, and $\sum x_i \leq 1$represent the proportion of all investment isn't allowed to get out of the budget. The expectation of the return $r_i$ of the first stock is E (ri), and the covariance of the returns of the two stocks i and j is $cov(r_i, r_j)$ .The expected return of the required portfolio is $\sum x_i Er_i \geq \mu$. By adjusting the capital ratio $x_i$, it can calculate the balance between which minimizing the risk $\sigma^2$ and maximizing the expected benefits $\mu$.

### 3.2 Capital Asset Pricing Model

Capital Asset Pricing Model (CAPM) model is the pillar of modern financial market price theory, which is widely used in investment decision-making and corporate finance. The primary significance of this model is to establish the equivalent relationship between capital risk and return, clearly indicate that the expected return rate of securities is the sum of risk-free return rate and risk compensation rate and reveal the internal structure of risk-free return.

1. Researchers have several assumptions about the market,
2. In a public market, all investors can trade risky asset.
3. It must exist a risk-free asset, which means risky free asset can be bought infinitely
4. Each rational investor is risky-aversion, and everyone is maximizing the profit
5. The market is equilibrium.
6. Not exist transaction cost. Investors thinks that average minus variance as the standard of the qualify, and have the same expectation about expect return, variance, covariance

Since there is only one risk portfolio, all investors hold this risk portfolio and the portfolio of risk-free assets. Let r be the yield of the entire portfolio, σ Is the standard deviation of the entire portfolio, and rM is the yield of the market portfolio, σ M is the standard deviation of the market portfolio, h $\in$ R is the weight of the market portfolio, and 1 − h is the weight of risk-free assets. Then the expected return of the entire portfolio is,

$$E(\overline{r}) = (1-h)r_f + h\overline{r}m \tag{5}$$

Rewrite the above equation to get the capital market line

$$\overline{r} = r_f + \frac{\sigma}{\sigma_m^2}(\overline{r} - r_f) \tag{6}$$

Substitute β into the model

$$E(r_i) = r_f + \beta\big(E(r_m) - r_f\big) \tag{7}$$

E($r_i$): expect rate of return (P1r1+P2r2+...+Pnrn)
$r_f$: risk-free rate (no systematic risk)
$\beta$: systematic risk
$r_m - r_f$: market risk premium.

### 3.3 Fama-French Three Factor Model

Although the CAPM model is very useful for predicting the return of the portfolio, many empirical results show that the CAPM model cannot explain certain specific anomalies.

$$E\big(r_i - r_f\big) = \beta_i\big[E\big(r_m - r_f\big)\big] + s_i[E(SMB)] + h_i[E(HML)] \tag{8}$$

Although the CAPM model is a very useful model for predicting asset prices, problems will arise when many investors apply the model or when many scholars conduct empirical analysis, and the model cannot explain certain specific exceptions. For example, the empirical results found the market value scale effect. Specifically, the average stock return rate of small market value scale is higher than that of large market value scale (small market value scale stocks have higher risk, so the corresponding return rate should also be higher). On the other hand, the book P/E ratio represents the growth opportunities that enterprises have. Stocks with high book to market ratio will have higher returns than stocks with low book to market ratio.

### 3.4 Sharp Ratio

The core idea of Sharp ratio is that rational investors will choose and hold effective portfolios, which still means those portfolios that maximize the expected return at a given risk level or those portfolios that minimize the risk at a given expected return level. Sharp believes that investors should at least require a return rate greater than the risk-free return when building an effective portfolio.

$$sharp\ ratio = \frac{(E(r_p) - r_f)}{\sigma_p} \tag{9}$$

Therefore, Sharp ratio is a standardized data that can measure investment products, indicating how much return can be compensated for without adding more risk. In our research, in order to obtain portfolio for investors, we use lower risk to obtain higher return. Therefore, we can see that higher return is the expected return minus risk free return, and lower risk is the standard deviation p, which can maximize Sharp ratio.

## 4. Results

To get the CAMP and FF3F models, it is necessary to find the correlation between the stocks. After using the covariance matrix (showed by Table 2) and some related functions, the correlations can be calculated as the Table 3 below shows.

**Table 2.** Covariance matrix.

| | BMO | RCI-B | MCD | APPl | BMW |
|---|---|---|---|---|---|
| BMO | 0.00576 | 0.00105 | 0.00145 | 0.00295 | 0.00367 |
| RCI-B | 0.00105 | 0.00191 | 0.00059 | 0.00036 | 0.00054 |
| MCD | 0.00145 | 0.00059 | 0.00251 | 0.00145 | 0.00475 |
| APPL | 0.00295 | 0.00036 | 0.00145 | 0.00800 | 0.00735 |
| BMW | 0.00483 | 0.00549 | 0.00342 | 0.00976 | 0.0674 |

**Table 3.** The correlation between the equities.

| | BMO | RCI-B | MCD | APPL | BMW |
|---|---|---|---|---|---|
| BMO | 1 | | | | |
| RCI-B | 0.0603 | 1 | | | |
| MCD | 0.0839 | 0.0339 | 1 | | |
| APPL | 0.0171 | -0.0213 | 0.0833 | 1 | |
| BMW | 0.0277 | -0.3155 | -0.1966 | 0.0561 | 1 |

Generally speaking, the correlation between listed companies in the same industry is high, so the correlation of the company with itself is always one. The correlation between similar industries is the second and the correlation between companies belonging to different industries is the smallest. The nature of correlation illustrates that the higher the degree of correlation between stocks, the stronger the systematic risk of market composed on them and the worse effectiveness of the portfolio composed of stocks. As table 3 above shows, there is no correlation between equities has the value of -1 which means all these stocks are more or less correlated since correlation with the value of -1 means the two assets are totally not correlated.

Now it is convenient to find the CAMP models. Firstly, the paper shows the maximum sharp ratio portfolio under CAMP model which is showed by Table 4.

**Table 4.** Maximum Sharpe Ratio portfolio under CAMP model.

| | BMO | RCI-B | MCD | APPL | BMW |
|---|---|---|---|---|---|
| Weights | 0.0376 | 0.3177 | 0.3286 | 0.2964 | 0.0197 |
| Expected return | 0.0174 | | | | |
| Variance | 0.0031 | | | | |
| Standard deviation | 0.0559 | | | | |
| Sharp ratio | 0.3111 | | | | |

According to Table 4, it shows that MCD has the largest weight with the value of 0.3286 while the smallest value which is 0.0197 appears in BMW. The reason why the MCD got the highest weight is likely because it is one of the world's fast food business giants, there are many chain stores around the world and there are many customers every day. This is because in the fast-paced pace of contemporary life, fast food is the best way for people, especially office workers, to solve lunch. BMW received the lowest weight because car brands are traditionally one of the most competitive sectors, and BMW is relatively more expensive so not all the people can afford one. As well as the impact if the COVID-19 pandemic, which has limited people's mobility recent two years. The paper also finds the minimum volatility portfolio under CAMP model which showed by Table 5 below.

**Table 5.** Minimum Volatility Portfolio under CAMP model.

| | BMO | RCI-B | MCD | APPL | BMW |
|---|---|---|---|---|---|
| Weight | 0.115 | 0.2569 | 0.1465 | 0.4673 | 0.0143 |
| Expected return | 0.0078 | | | | |
| Variance | 0.0025 | | | | |
| Standard deviation | 0.0500 | | | | |
| Sharp ratio | 0.1559 | | | | |

From the Table 5, it is obvious that APPL has the largest weight with the value of 0. 4673. It has this weight probably because APPL is a big global company and the products introduced by it, such as mobile phones, laptops and tablets, are essential items in people's daily life, study and work, almost can be one for everyone. Moreover, relative to other electronic products brands, it has many fans all over the world and other such as Huawei could be only popular among Chinese. In this situation,

BMW still has the lowest value of weight which is 0.0143 which can get that it has a high volatility. One of the reasonable reasons for this situation is that there are a lot of new high-tech cars are invented recent years, like Tesla, and they may influence the old car brands. Now move to the FF3F model part. Firstly, it is important to get the estimation results of Fama French 3 factor model which are showed by Table 6 and Table 7 below.

**Table 6.** Estimations of FF3F parameters.

| | $\beta_i$ | $s_i$ | $h_i$ |
|---|---|---|---|
| BMO | 1.7438 | 0.6957 | 0.935 |
| RCI-B | 1.1043 | 0.5498 | -0.2018 |
| MCD | 1.275 | -0.0396 | 0.0161 |
| APPL | 0.896 | -0.0483 | 0.4374 |
| BMW | 0.9231 | -0.1558 | 0.0684 |

**Table 7.** Standard error and P-value.

| | $\beta_i$ | $s_i$ | $h_i$ |
|---|---|---|---|
| Standard error | 0.546 | 0.7896 | 0.5022 |
| P-value | 0.0006 | 0.4568 | 0.0796 |

After having these data, it is easy to find the maximum Sharpe ratio portfolio and minimum volatility portfolio under FF3F model. The maximum Sharp ratio portfolio is showed firstly by Table 8 below.

**Table 8.** Maximum Sharp ratio.

| portfolio under FF3F model | BMO | RCI-B | MCD | APPL | BMW |
|---|---|---|---|---|---|
| Weights | -0.0128 | -0.0236 | 0.4629 | 0.4952 | 0.0783 |
| Expected return | 0.0089 | | | | |
| Variance | 0.00242 | | | | |
| Standard deviation | 0.04918 | | | | |
| Sharp ratio | 0.18098 | | | | |

As Table 8 demonstrates, APPL has the highest value of weight which is 0.4952 and the lowest value of weight appears in RCI-B. APPL produce electronic devices and RCI-B is a telecommunication company which means actually the goods they produce are both closely related to people's daily life. The reason why APPL has the highest weight is that it has a higher popularity and it has clients all over the world. On the contrary, telecommunication companies do not have a high popularity because they usually only serve people in their own country and are only known by these people. For instance, RCI-B serves in Canada so it is mostly known by Canadians. That is why it has the lowest weight. The paper finds the minimum volatility portfolio under FF3F model either which is showed by Table 9 below.

**Table 9.** Minimum volatility portfolio under FF3F model.

| | BMO | RCI-B | MCD | APPL | BMW |
|---|---|---|---|---|---|
| Weights | 0.3737 | -0.1735 | 0.3843 | 0.5429 | -0.1274 |
| Expected return | 0.0081 | | | | |
| Variance | 0.00352 | | | | |
| Standard deviation | 0.05936 | | | | |
| Sharp ratio | 0.13646 | | | | |

Table 9 shows similar result with Table 8 which shows that APPL and RCI-B own the highest and lowest value of weight respectively. The reason is similar either, APPL has a higher popularity among the world and attracts more customers, so it always has the smaller volatility compared with other equities. Whatever how the factors change outside, there are always many people all over the world willing to purchase the products from APPL.

## 5. Conclusion

Overall, according to the current statistics, the research for the investment portfolio is generally based on analysis on the popular industries that used by people frequently. The main purpose of the paper is to help potential investors to apply the investment portfolio optimization by do portfolio analysis of the domain of bank, telecommunication, catering, electronic devices and vehicles. The paper list three methods to find the optimal investment portfolio which are mean-variance model, CAMP model and FF3F model. First, using the covariance matrix and correlation between equities to find out the forecast situation of asset allocation while the mean-variance model is used to optimize the portfolio, then the maximum Sharp ratio portfolio and minimum volatility portfolio under CAMP model and FF3F model can be constructed. Finally, using the Sharp ratio to find the optimized choice of investment portfolio. According to the research above, the business of catering and electronics are heavily weighted in both investment portfolios while according to the FF3F model, the maximum Sharp ratio portfolio is more stable than the minimum volatility portfolio relatively. However, there is no denying that this paper also includes some demerits. For example, it does not include the graph of efficient frontier which is especially helpful to those people who are risk averse and prefer yield.

## References

[1] Markowitz H. Portfolio Selection. The Journal of Finance, 1952, 7(01): 77-91.

[2] Fama E F., French K R. The cross- section of expected stock returns. The Journal of Finance, 1992, 47(02): 427-465.

[3] Fama E F., French K R. Common risk factors in the returns on stocks and bonds. Journal of financial economics, 1993, 33(01): 3-56.

[4] Fama E F., French K R. Value versus growth: The international evidence. The Journal of Finance, 1998, 53(06): 1975-1999.

[5] Zhang S. Empirical Research on China's 50 Index of Shanghai Stock Exchange -- Based on capm model and fama french three factor model, empirical research on China's 50 Index of Shanghai Stock Exchange, 2022.

[6] Enkhbaatar N. Impact of Macroeconomic Policies on Stock Market in Mongolia. China-USA Business Review, 2019, 1.

[7] Dunbar K., Owusu-Amoako J. Cryptocurrency returns under empirical asset pricing. International Review of Financial Analysis, 2022: 102216.

[8] Fan S. Based on the empirical research of CAPM model in the Chinese medicine industry, 2020.

[9] Basak S., Chabakauri G. Dynamic mean-variance asset allocation. The Review of Financial Studies, 2010, 23(8): 2970-3016.

[10] Markowitz H. Mean–variance approximations to expected utility. European Journal of Operational Research, 2014, 234(2): 346-355.