Portfolio Optimization for US. Stock with Mean-variance Model, CAPM, Fama French Three-factor Model

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Abstract. Applying modern investment theories to construct investment portfolios is a crucial way for investors to reduce risks and obtain high returns in the investment market. This paper selects eight stocks from eight industries of the US stock market and constructs a portfolio based on their historical return data over the past five years. This study uses the CAPM model and the Fama French three-factor model to estimate the expected returns of these eight stocks, and then applies the mean-variance model to construct the optimal risky portfolio under both expectations. Under the CAPM model, the expected monthly returns for these eight stocks are BKNG (1.05%), EQR (0.80%), MPC (1.38%), NTDOY (0.64%), ON (1.44%), PFE (0.68%), SBUX (0.86%) and FCX (1.60%) respectively. Under Fama-French three factor model, the expected monthly returns for these eight stocks are BKNG (1.21%), EQR (0.90%), MPC (2.06%), NTDOY (0.55%), ON (1.82%), PFE (0.60%), SBUX (0.82%) and FCX (1.96%) respectively. The optimal portfolio weights obtained by using CAPM model estimation results are BKNG (7.97%), EQR (23.72%), MPC (6.02%), NTDOY (11.61%), ON (8.24%), PFE (18.66%), SBUX (10.61%) and FCX (13.16%) respectively. Under Fama French three factor model, the weight distribution of the portfolio is as follows BKNG (3.01%), EQR (27.04%), MPC (33.89%), NTDOY (2.40%), ON (15.85%), PFE (4.98%), SBUX (0.00%) and FCX (12.83%). These results can provide a reference for investors who are willing to invest in US stock market, which will help them reduce investment risks and increase returns.

Keywords: Mean-variance model; US stock market; optimal risky portfolio construction.

1. Introduction

Individual investors, fund managers and brokers invest billions of dollars in the stock market each year. They pursuit profits throughout all investment environments and minimize losses during market downturns [1]. Most investors in the financial market are risk averse, they always seek higher returns in a lower risk level [2]. Diversification is an important way to reduce investment risk. Diversification is to construct a portfolio by investing in multiple types of assets, which can decrease the total risk of the portfolio [3]. A portfolio is deemed efficient compared to other portfolios, if it provides the highest return for the same risk level or gives the smallest risk with the same return [4]. Therefore, how to make a balance between risk and return is vital to investors.

How to obtain higher returns under the same risk level is a vital issue faced by investors. In 1952, Markowitz introduced a new investment theory, which can provide a quantitative trade-off risk and return, namely the mean-variance model [5]. This model creatively uses the covariance between assets as a measure of risks which initiated a new era in today's investment theory. Although seventy years have passed since the model was created, it is still the most widely used model for portfolios construction now [6]. Balqis, Subiyanto and Supian studied building optimal portfolio in Indonesia stock market. They selected LQ-45 stock samples and constructed two portfolios. Research find that using Markowitz model can make investors to avoid high risk and obtain higher returns [7]. Xie's study applies Markowitz model in Chinese stock market. Xie selects five stocks listed on the Shanghai Stock Exchange and collects their historical return data to construct a portfolio. This study found that the return on portfolio constructed by Markowitz model was higher than that of Shanghai Composite Index on most of the trading days in the past month [8].

Since the market is constantly changing, the optimal investment portfolio should also be updated in time. Whilst in recent years, there were not many studies on the application of the mean variance model on the US stock market. Furthermore, when applying the mean-variance model, the traditional

method directly uses the arithmetic mean of the historical returns of stocks as the expected return of stocks, but this will have some potential issues. First, the arithmetic mean is susceptible to extreme values. If the return of a stock fluctuates greatly in a certain period, the forecast result of expected return will also have a great bias. Second, this forecasting method relies entirely on historical returns, which may have lost their value as future forecasts. To address these issues, this paper will focus on the US stock market and use mean-variance model to construct optimal portfolio. This paper also no longer adopts the traditional method of taking the arithmetic mean of stock historical returns as the expected stock return but uses regression model to estimate the stock's expected return.

This study selects eight stocks listed in the US stock market, which are from different industries. For the eight stocks, their expected returns were estimated by using the CAPM model and the Fama French three-factor model. Then two groups of expected returns are brought into the mean-variance model and obtained two largest Sharpe ratio portfolio. The next part of the paper will discuss the data selection and analysis methods of this study, and the third part will provide the results obtained from the analysis and give some discussion about the results. The last part of the paper is a summary of the full text and also provides the limitations and reference suggestions of the paper.

2. Data

This paper selects eight stocks in the US stock market to construct investment portfolio. The eight stocks are BKNG, EQR, MPC, NTDOY, ON, PFE, SBUX and FCX. BKNG, the booking company, is mainly engaged in the business of travel and hotel reservations worldwide. Equity Residential (EQR) is a real estate company, mainly engaged in real estate acquisition management and rental business. Both of Marathon Petroleum Corporation (MPC) and Semiconductor Corporation (ON) are doing energy-related business, but the difference is that MPC is mainly engaged in traditional fossil energy, while ON is engaged in the semiconductor industry related to renewable energy. Pfizer (PFE) is a medical company that mainly provides drugs and vaccines. Nintendo (NTDYO), Starbucks (SBUX), and Freeport-McMoRan (FCX) are gaming company, food retail company, and mining company, respectively. The companies are diversified and are suitable for portfolio construction.

The data source for this paper are mainly from Kenneth French Data Library (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html) and Yahoo Finance (https://finance.yahoo.com/). The study used monthly adjusted stock prices for selected stocks over the last five years. The reason why this paper applied monthly data is that sample size of annually data is too small, and the frequency of daily data is too high. Both of these factors affect the predictive power of the data. This paper applied stock price data from last five years. Because short time period will reduce the sample size and cause prediction bias, and long time period contains too much old data that lose their predictive value. To sum up, five years is a suitable time period. According to the selection criteria above. Adjusted close price for a total of 61 months from 01/10/2017 to 01/10/2022 are selected. By using these 61-month adjusted close price, the monthly return of these stocks in recent five year are acquired. The formular is given below:

$$Total\ stock\ return = \frac{P_1 - P_0}{P_0} \tag{1}$$

Where:
$P_0 =$ Initial adjusted stock price
$P_1 =$ Ending adjusted stock price (Period 1)
After analyzing obtained return data, the basic information is shown in the table 1:

Table 1. Descriptive statistics of these selected stocks

	BKNG	EQR	MPC	NTDOY	ON	PFE	SBUX	FCX
Mean	0.20%	0.53%	2.21%	0.63%	2.73%	1.04%	1.19%	2.48%
Variance	0.0083	0.0046	0.0180	0.0076	0.0170	0.0054	0.0053	0.0218
Max	25.02%	23.29%	35.82%	26.65%	32.74%	22.84%	16.72%	36.21%
Min	-22.04%	-17.83%	-49.69%	-20.08%	-33.33%	-13.52%	-17.95%	-32.23%
Probability	55.00%	51.67%	63.33%	48.33%	63.33%	51.67%	58.33%	53.33%

Based on the table above, the average returns of selected stocks are all positive, among which MPC, ON and FCX have higher average returns than others. EQR has the least volatility in returns according to variance. Over the five years, MPC has the largest difference between its maximum monthly return and its minimum monthly return. The last row of the table gives the probability that each stock will have a positive monthly return over those five years. Combining the data, it can be seen that only the probability of Nintendo's monthly return being positive over last five years is less than 50%.

Table 2. Variance-Covariance Matrix

	BKNG	EQR	MPC	NTDOY	ON	PFE	SBUX	FCX
BKNG	1.0000							
EQR	0.5265	1.0000						
MPC	0.6393	0.4685	1.0000					
NTDOY	0.3511	0.0845	0.1233	1.0000				
ON	0.4939	0.4058	0.6385	0.2696	1.0000			
PFE	0.2259	0.2854	0.1890	-0.0356	0.2752	1.0000		
SBUX	0.5368	0.5058	0.3522	0.2936	0.5288	0.3891	1.0000	
FCX	0.5354	0.3712	0.6127	0.2131	0.6737	0.2048	0.4310	1.0000

According to the table 2, it can be seen that the correlation between some stocks is relative strong, such as BKNG and MPC, ON and FCX. The correlation coefficient between these assets is higher than 0.6. However, the correlation coefficient between some assets is relatively small, even Negative numbers, such as NTDOY and PFE. By adding such assets into portfolio can reduce total risk of portfolio.

The method of predicting stock returns in this paper is based on the CAPM model and the Fama-French three-factor model. The data required for these models are acquired from Kenneth French Data Library. The research selected all monthly three-factor model data from July 1926 to August 2022. Since these data are percentage values, they must be divided by one hundred before use. Among the selected data, data from November 2017 to August 2022 will be used for regression analysis along with the stock's return data. And all three-factor data from July 1926 to August 2022 are averaged as the parameters of the final prediction model. By doing this, the stability of the prediction model is enhanced, and it will make the model closer to the true model.

3. Methodology

3.1 Mean-Variance model

The mean-variance model is based on the theory that investors make investment decisions only based on two factors: the mean of portfolio returns, and the variance of portfolio returns. The mean is a proxy for portfolio’s expected return, and the variance is a proxy for investment risk. This paper does not directly use the historical returns arithmetic mean of each stock as the expected return of each stock. Instead, regression method is employed in this study to estimate the expected return of selected stocks in this portfolio. Mean-variance model assumes that investors are all risk averse, in

other words, given the same return on two assets, investors always prefer the less risky one. The relevant formulas of mean-variance model are as follows:

The expected return of the portfolio is calculated by the weighted average of the expected returns of each asset.

$$E(r_p) = \sum_{i=0}^{n} \omega_i r_i \tag{2}$$

$$\sum_{i=1}^{n} \omega_i = 1 \tag{3}$$

Where:
ω_i = The weight of asset i in the portfolio
r_i = The expected return of asset i
The variance of the portfolio is calculated as:

$$\sigma_p^2 = \sum_{ij} \omega_i \omega_j Cov(r_i r_j) \tag{4}$$

$$\sigma_p = \sqrt{\sigma_p^2} \tag{5}$$

Where:
ω_i = The weight of asset i in the portfolio
ω_j = The weight of asset j in the portfolio
$Cov(r_i r_j)$ = Covariance between the expected return of asset i and asset j
The Sharpe ratio measures how much excess return an investor can earn by taking per unit of risk. This paper is aimed to get the maximum Sharpe ratio portfolio.

$$Sharpe\ Ratio = \frac{E(r_p) - r_f}{\sigma_p} \tag{6}$$

Where:
$E(r_p)$ = Expected return of portfolio
σ_p = Variance of portfolio
r_f = Return of risk-free asset

3.2 CAPM Model

The Capital Asset Pricing Model (CAPM) is usually used to estimate expected return of risky assets. The formula below shows the linear relationship between the expected return and risk of a stock and indicates that risk-averse investors need higher returns than risk-free assets to compensate with more risk they take [9].

$$E(r_i) = r_f + \beta_i E(r_m - r_f) \tag{7}$$

Where:
$E(r_i) =$ Expected return of investment
r_f= Return of risk-free asset
β_i= Beta of stock
r_m= Market expected return

3.3 Fama French three-factor model

The three-factor model is derived from CAPM. The CAPM model only has one explanatory variable, the excess return of the market. Fama and French stated that some anomalies in CAPM

model can be better explained by their new model [10]. The Fama-French three-factor model adds two more independent variables, namely the size factor and the value factor. Due to the addition of new variables, the prediction results of the three-factor model are more accurate than that of the CAPM model. Several studies have also confirmed this claim, such as Sattar find that Fama French model can better predict variation in excess return than CAPM for the selected companies in the Cement industry over a certain period [11].

$$E(r_i - r_f) = \beta_{iM}E(r_m - r_f) + \beta_{iSMB}E(SMB) + \beta_{IHML}E(HML) \quad (8)$$

Where:
r_i= Expected return of the asset
r_f= Return of risk-free asset
r_m=Market expected return
$E(SMB)$=size factor (small minus big)
$E(HML)$=value factor (high minus low)
β= factor coefficients

4. Results

4.1 CAPM Model Results

First, this paper regress monthly excess returns of eight selected assets over the past five years on the corresponding market excess returns. The results obtained are summarized in the following table:

Table 3. CAPM Model Regression Results

	R-Square	CAPM Beta	P-value	Expected return
BKNG	0.4614	1.1679	0.0000	1.05%
EQR	0.3945	0.8001	0.0000	0.80%
MPC	0.4127	1.6525	0.0000	1.38%
NTDOY	0.1135	0.5620	0.0097	0.64%
ON	0.4976	1.7506	0.0000	1.44%
PFE	0.1915	0.6127	0.0006	0.68%
SBUX	0.4028	0.8854	0.0000	0.86%
FCX	0.4993	1.9830	0.0000	1.60%

As shown in the table 3, the p-values show that the beta parameters of all assets are significant at the 0.05 confidence level. The R-square indicated that CAPM model explains most of the assets well, except for Nintendo and Pfizer.

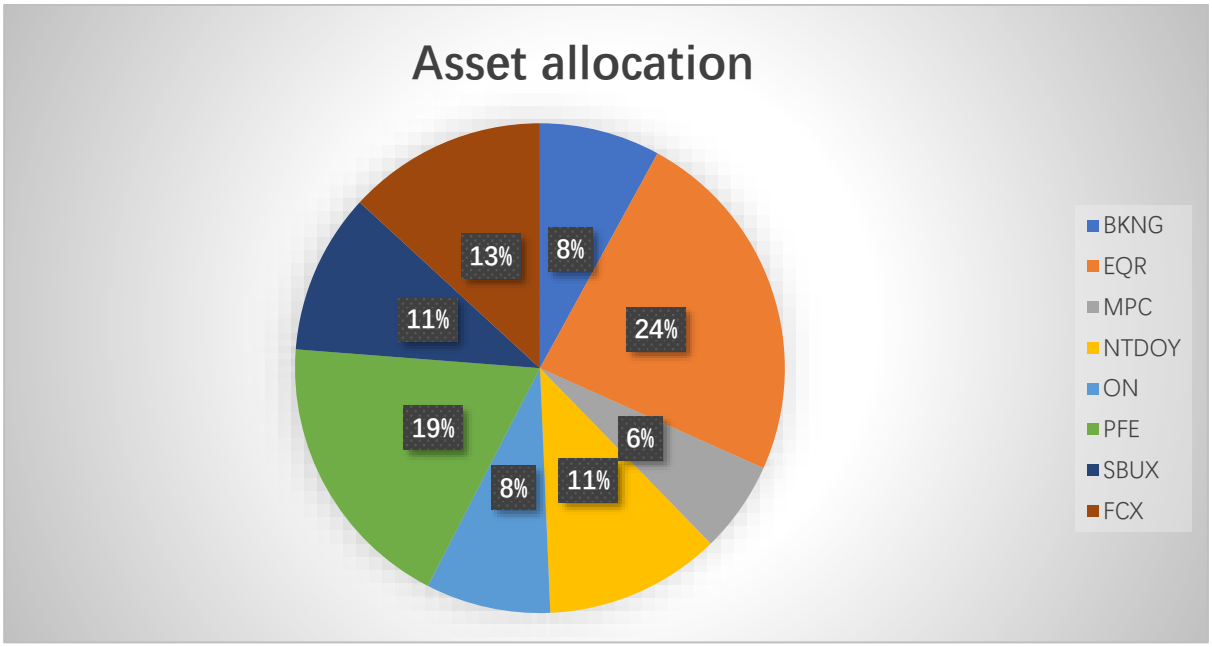

Figure 1. Optimal risky portfolio based on CAPM model

As mentioned before, this paper is aimed to find the largest Sharpe ratio portfolio, which is also known as the optimal portfolio. The figure 1 above provides the optimal portfolio asset allocation under the CAPM model. The figure is showing that the asset distribution of the optimal portfolio is relatively even, which is conducive to risk diversification.

Table 4. Descriptive statistics of portfolio

Expected monthly return	Portfolio Variance	Standard Deviation	Sharpe ratio	Annual return
0.0098	0.0039	0.0627	0.1139	11.77%

The Sharpe ratio of optimal risky portfolio based on the CAPM model is 0.1139, and the annualized rate of return that can be obtained is 11.77% (See Table 4).

4.2 Fama French three-factor model results

The study also used Fama French three-factor model to analyze the data in the same period, and the regression results obtained are as follows:

Table 5. Fama French three-factor model regression results

	R-Square	MKT beta	P-value	SMB beta	P-value	HML Beta	P-value	Expected return
BKNG	0.5369	1.2135	0.0000	-0.3080	0.3463	0.5388	0.0080	1.21%
EQR	0.4245	0.7869	0.0000	0.0669	0.8035	0.2696	0.1009	0.90%
MPC	0.7354	1.4701	0.0000	1.0331	0.0067	1.7140	2.5E-10	2.06%
NTDOY	0.1371	0.6379	0.0058	-0.4578	0.2912	-0.1666	0.5229	0.55%
ON	0.6125	1.5619	0.0000	1.1185	0.0116	0.8176	0.0025	1.82%
PFE	0.2555	0.7300	0.0001	-0.7185	0.0363	-0.0614	0.7622	0.60%
SBUX	0.4090	0.9174	0.0000	-0.1926	0.5198	-0.0751	0.6767	0.82%
FCX	0.5813	1.8037	0.0000	1.0620	0.0395	0.7844	0.0125	1.96%

Comparing the R-square Column of Table 5 and R-square Column of Table 3, it is evident that the explanatory power of Fama French three-factor model is stronger than that of the CAPM model. In Fama French three-factor model, the market factor betas for all assets are significant at the 0.05 confidence level. However, the size factor beta and value factor beta of some assets are not significant, such as NTDOY and SBUX.

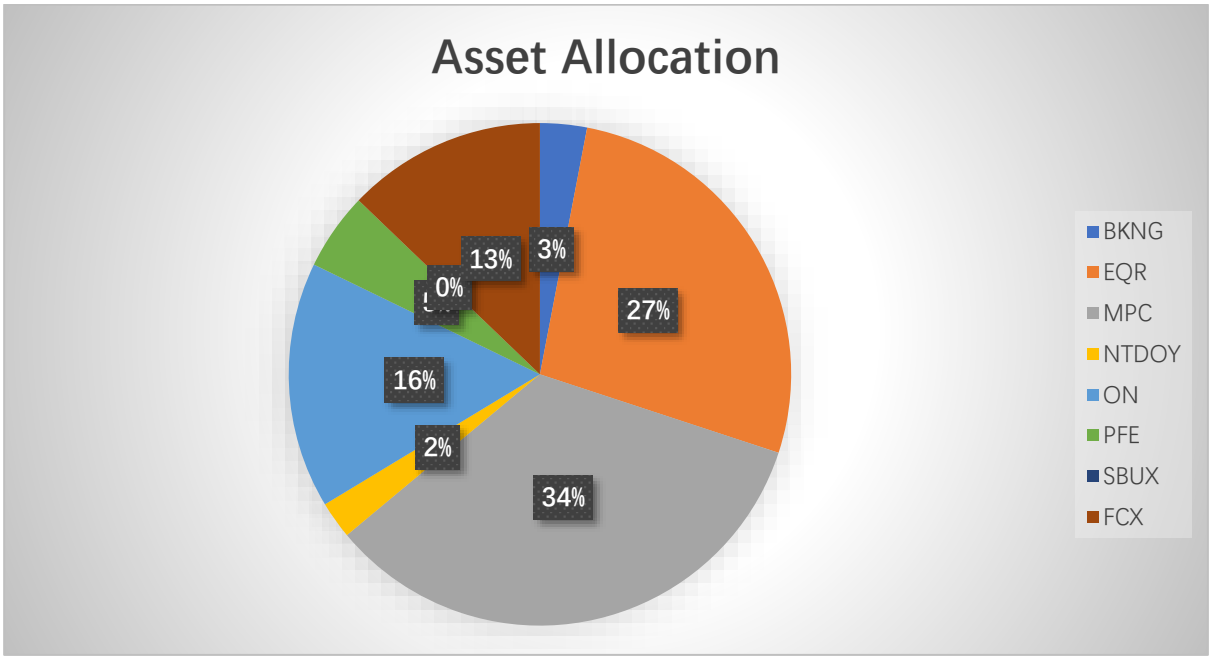

Figure 2. Optimal risky portfolio based on Fama French three-factor model

The figure 2 above provide the asset allocation result of the optimal portfolio under Fama French three-factor model. At this stage, 61% of total assets are allocated in EQR and MPC. Compared with the asset allocation under the CAPM model, the investment weight of MPC has increased by 28%. And investment weight of SBUX decreased from 11% to 0%.

Table 6. Descriptive statistics of portfolio

Monthly Expected return	Variance	Standard Deviation	Sharpe ratio	Annual return
0.0156	0.0079	0.0890	0.1458	18.75%

Under Fama French three-factor model, the Sharpe ratio of the optimal portfolio is 0.1458 and the expected annual return is 18.75% (See Table 6).

5. Conclusion

This paper is based on the US stock market, and combines the CAPM model, Fama-French three-factor model and the mean-variance model to construct optimal risky portfolio. It also provides investment references for investors. The selected stocks for tthis study are from energy, Internet, real estate, and other industries, which effectively diversifying the risk of investors.

Firstly, this study uses the CAPM model to predict the stock returns and brings the obtained returns into the mean variance model to get the portfolio with the largest Sharpe ratio. The weights of each asset in this portfolio are: BKNG (7.97%), EQR (23.72%), MPC (6.02%), NTDOY (11.61%), ON (8.24%), PFE (18.66%), SBUX (10.61%) and FCX (13.16%) respectively. Under the CAPM model, the expected annual return of the optimal portfolio is 11.77%, and the Sharpe ratio is 0.1139. Next, using Fama French three-factor model to predict the returns of each stock, and the expected returns are brought into the mean variance model again. In this case, the asset distribution of portfolio is as follows: BKNG (3.01%), EQR (27.04%), MPC (33.89%), NTDOY (2.40%), ON (15.85%), PFE (4.98%), SBUX (0.00%) and FCX (12.83%). Under this condition, the expected annual return of the optimal portfolio is 18.75%, and the Sharpe ratio is 0.1458.

This paper chooses eight industries in US stock market and select eight equities from these industries. Although selected eight stocks are relatively representative stocks in their industry, the sample size is not too large. Therefore, selection bias issue may exist. Future researches can use the

same model and increase the sample size and select more stocks from a wider range of industries to construct optimal investment portfolio.

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