Research on the Commodity Pricing Based on Black-Scholes model and Geometric Brownian Motion model

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Abstract. Commodity prices are never easy to forecast in the actual world. To acquire the pricing of commodity options, the price of the underlying commodity should always be anticipated using the commodity future, rather than simulating the price directly. Thus this paper is aimed to directly simulate the commodity price and compare this method to the standard way. The research method are as follows: firstly, collect the original data, then simulate the stock price by using the Black-Scholes model and simulate the commodity price by using the Geometric Brownian Motion model. Last, compare the result. The outcome demonstrates that the stock price may be successfully simulated using the Black-Scholes model using crude oil commodities futures as the underlying asset. The Geometric Brownian Motion model, however, is unable to accurately price commodities directly. This finding implies that it is incorrect to see commodities as a type of financial asset and that models used to value financial assets cannot accurately value commodities.

Keywords: Monte Carlo simulation; Black-Scholes model; Geometric Brownian Motion model; crude oil commodity.

1. Introduction

Before this research, some former researchs have already been conducted. Boyle & Phelim (1977) used Monte Carlo to simulate the European call option, and stated that Monte Carlo is useful in pricing stocks [1]. Glasserman (2004) introdued the practice of Monte Carlo and quasi Monte Carlo in finance and made a comparison [2]. Acworth, Broadie & Glasserman (1996) constract the Monte Carlo and quasi Monte Carlo in the pricing of optioins [3]. MacBeth, Merville, et al. (1979) compared the option price simulated by using the Black-Scholes model with the market price of option [4]. Barles, Guy & Soner (1998) extended the orignial Black-Scholes model on option pricing further [5]. Chesney, Marc & Scott contasted the usefullness between modified Black-Scholes model and random variance model [6]. Postali, AS & Picchetti (2006), Nwafor, Oyedele, et al (2017) and Al-Harthy, Hamood (2007) all priced the oil commodity by using the Geometric Brownian Motion model [7-9]. Ramos, Lubene, et al (2019) used the GBM pricing the iron commodity [10].

In order to assess the possibility of directly using a specific pricing model to value the price of a commodity, this research contrasts the Black-Scholes model and the Geometric Brownian Motion model on the valuation of European up-and-out call options. The valuation of the European up-and-out call option is required for correct comparison, and the option on the crude oil market is chosen. The first model utilized is the Black-Sholes model, while the second model is the Geometric Brownian Motion model. The method employed is Monte Carlo simulation.

The main contribution of this research is to prove feasibility of pricing the option directly using the pricing model in theory and practical these two aspects. In theorical, this research can enrich the analysis of commodity pricing and make preparation for the further commodity pricing. In practical, this research can be a theory basis for day-to-day commodity trading and price prediction.

2. Data and method

2.1 Data

For the data used In both of the simulations, both the crude oil futures prices and the crude oil prices are from the website: www.cn.investing.com, and the crude oil futures chosen is Micro WTI Crude Oil Futures, the crude oil chosen in WTI Crude Oil spot.

2.2 Introduction of simulation method

2.2.1 Monte Carlo simulation

Glasserman (2004) introduced the steps to do the Monte Carlo simulation, and the most important part is to generate random numberof St. And there are several variable to consider, such as the "period of multiplication generator", the "Period of Mixed Congruential Generator", these variables are all assumed to follow central limilt theory[2]. In this research the random number of St is also generated by considering the variables that follow central limit theory, which is the 'Z' value, and for both the simulations, there are multiple steps to simulate the underlyinf asset price according to the Monte Carlo simulaiton.

2.2.2 Black-Scholes model

Black-Scholes model is a useful model to pricing the financial asset such as stock price or options, MacBeth, Merville, et al. (1979) stated that in order to conduct the simulation by using the Black-Scholes model, there are several variables to consider, such as the exercise price, the time to maturity, the variance. It is assumed that there is no cash distributions, there are no transaction costs etc [4]. In this research, the simulation by using the Black-Scholes model also considered the variables such as the exercise, the variance which is the standard deviation in the simulation and the risk-free rate. As for the assumtion, under the basic assumption of Black-Scholes model made, there are several other assumption made in the simulation.

2.2.3 Geometric Brownian Motion model

Postali, AS & Picchetti (2006), Nwafor, Oyedele, et al (2017) and Al-Harthy, Hamood (2007) all stated that using Geomoetric Brownian Motion model is a better method than the other naive model such as the mean reverting model, and all stated that there are some varaibles needs to consider to simulate the price of the commodity, which are spot oil price, change in time, change in price and drift for mean return and drfit for volatility [7, 9, 10]. In this research, these varaibles are all considered and in order to compare this method with Black-Scholes model, some of the variables are the same with the variables used in the BSM.

3. Results of Monte Carlo simulation

3.1 BSM

When using the Black-Scholes Model, various assumptions are made. The first is that when doing the research, crude oil futures prices are employed. because the prices of crude oil futures are essentially random and fluctuate erratically. And because individuals use commodities for consumption rather than investment, they are not considered financial assets. For this reason, it is incorrect to use the price of crude oil as a direct indicator of the price of a commodity. The Black-Scholes Model also makes the assumption that future stock prices will follow a lognormal distribution. As a result, before starting the simulation, the "Z" value should be simulated in order to get a number with a normally distributed stochastic distribution.

3.1.1 Steps

Before conduct the simulation, there are two assumptions should be made. The first assumption is that the time period is a month, which only has 21 trading days, so the research mainly focuses on simulating the short-term performance of the European up-and-out call option. The second assumption is that the simulation will undertake only 100 times. Because 100 times trading is more appropriate in 21 trading days. In the simulation by using the Black-Scholes Model, the underlying asset is crude oil futures price, so the spot price used is futures price at the last trading day which is 98.6. The maturity date is 21 days, the up-barrier is both \$110, the time T is 1/12 which is 0.0833 show in the data, the risk-free rate used is 3%, the average return used is 0%, the strike price is ‘at the money’. And the standard deviation is 3.64%.

There are mainly five steps to conduct the simulation by using BSM.

The first step is to find the original data of crude oil futures prices, from the data, the close prices are used. Then, the volatility can be calculated by using standard deviation formula, then the daily volatility can be got.

The second step is to collect the variables will be used in the Black-Scholes model. The spot price is the close futures price at the last day, and the strike price is assumed at the money so it is the same with the spot price. The maturity dates is 21 days and the expected return is 0 according to Black-Scholes model. The daily volatility can be calculated from the original crude oil futures price. The barrier price set should be higher than the strike price and the risk-free rate is the latest rate from the official.

The third step is to simulate the Z value, As described above, the stock price is lognormally distributed, so the ‘Z’ value should be the number that can reflect the randomness of the stock price, the simulation here use the formula NORMSINV(RAND()) to get the Z values for 21 days and 100 times each day.

The fourth step, is to simulate the stock price using the Black-Sholes model for 21 days and 100 times each day. Then all of the simulated stock price can be got. However, for the European up-and-out call option, there is a criteria should be achieved, which is to determine whether the stock price is higher than the barrier price or not, so after got the price of the stock for 21 days and 100 times each day, the maximum price among the prices in each time of simulation should be selected to determine whether the criteria could be met. If the barrier price can be achieved, then the option is out, the value is 0, if it could not be achieved, then the value of the option is last price (which is the price of the last day of each time of simulation) minus the strike price, and reiterate that process 100 times, the value of barrier option of each time can be got, then take average, the final result can be got.

Then, the last step, with the value of the barrier option, this value should be discounted by using the risk-free rate to current period and the max error should be calculated along with the present value of the barrier option price.

3.1 2 Pricing results using BSM

After the simulation by using the BSM, a plot graphs can be drawn. The graph of the European up-and-out call option demonstrate that most of the option do not have a value of 0, although some of the plot gathered at the value of 0 at the range of 98.6 to 110, which means those options are failed to exercise, and the rest of the options that have maximum price at the range of 98.6 to 110, are successfully exercised and get a higher return. And there are scattered of plot gather at the value of 0, that means there are some options have a value of 0 out of the region of 98.6 to 110, and these options have a maximum price higher than the barrier price (as shown in the figure 1)

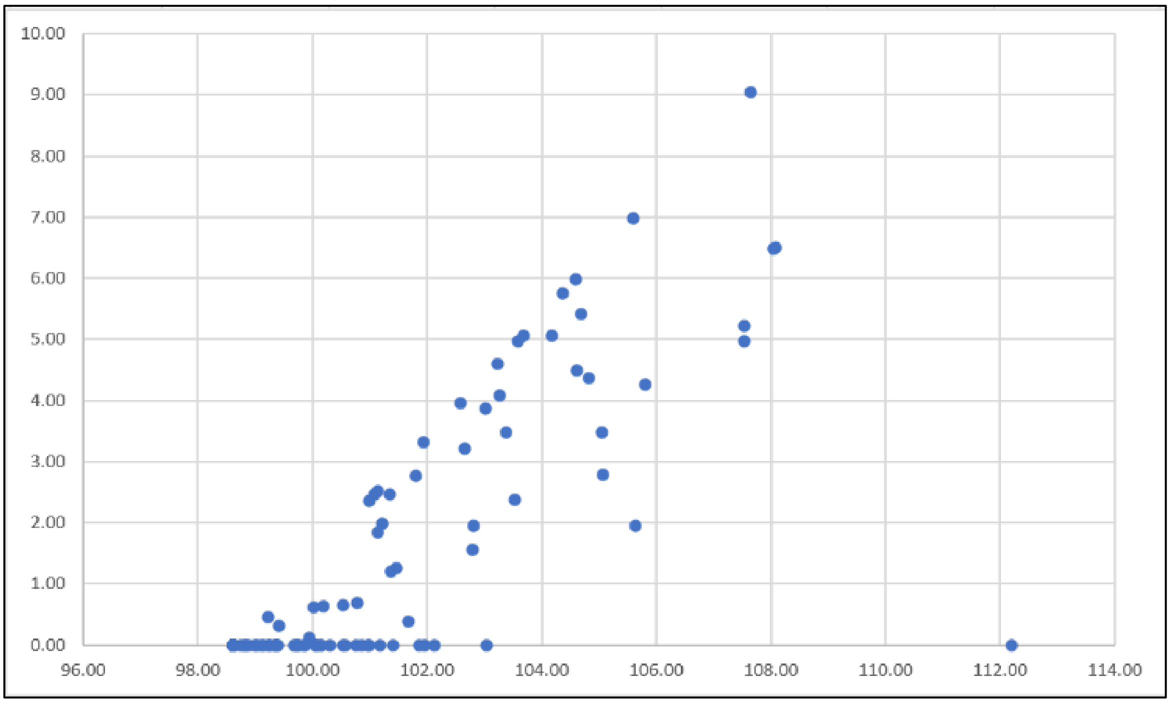

Figure 1. barrier option value & max price

3.2 GBM

There are a few presumptions that must be made while using the Geometric Brownian Motion model. The first is that in order to imitate the commodity price directly, it is assumed that the commodity is a type of financial instrument, hence the price of crude oil is considered to be random walk. The second assumption is that the return of the crude oil commodity is 0%. Because, BSM discovered that the same risk-neutral adjustment for futures could be applied to options, which means the risk-free rate is allowed to be used to simulate future stock prices, in order to increase the comparability, the average return in GBM is also 0%. And, again, the price of the commodity in GBM is lognormally distributed, which is the same with the assumption made in BSM, the 'Z' value should also be simulated first in order to get a stochastically distributed number that follow normally distribution. The last assumption is that the changes of the commodity price is seen as the return of the crude oil commodity.

3.2.1 Steps

Before undertaking the simulation, few assumptions are also should be made and most of the assumption is the same with the assumption made in the simulation by BSM. However, in the simulation by using theGBM, the underlying asset is crude oil price, so the spot price used is oil price at the last trading day which is 100.05. And the standard deviation is 3.5%.

There are mainly seven steps to conduct the simulation by using GBM.

The first step is to find the original data of crude oil prices, from the data, the close price at the last trading day is used for spot price. Then, the volatility can be calculated by using standard deviation formula, and the daily volatility can be got.

The second step is to collect the variables will be used in Geometric Brownian Model. The Drift(daily) is 0% in order to make the contrast with the use of BSM more accurately. With the data of the changes in commodity price, the daily volatility can be calculated, which is about 0.035. Then, the Drift(mean) can be calculated by using the variables in the formula concluded from the GBM, and the result is -0.06147%, the time T is 1/12, which is 0.08333 in the data collected.

The third step is to calculate the 'Z' value. The basis of using GBM to pricing the option is that the price of the underlying asset is stochastically distributed, and to reflect that the price of the underlying asset is stochastically distributed, the price always assumed to follow normal distribution as to find random probability. The simulation here are use the formula NORMSINV(RAND()) to get the Z values for 21 days and 100 times each day.

The fourth step is to calculate the log return and simulate the stock price. After collect all the variables, the log return should first be calculated by using the formula concluded from the GBM, in the formula, the log return represents the part in parentheses, by using this formula, the log return can be calculated for 21 days and 100 times for each day. And according to the formula, with the log return, and with the spot price used, the future commodity price can be got.

The fifth step is to collect the variables will be used in the Geometric Brownian Model. The spot price is the close price at the last day which is $100.05, and the strike price is assumed at the money so it is the same with the spot price. The maturity dates is 21 days and the expected return is 0 same with the volatility rate used in the previous part. The daily volatility is also the same with the previous part. And for the rest variables such as the maturity date, the sample size, the up barrier, the risk-free rate and the time T is the same with the data used in BSM in order to make an accurate contrast.

The sixth step is to simulate the value of the barrier option. The value of options in GBM is the same with the simulation in BSM, the criteria should be determined whether is satisfied or not. If the barrier price is lower than the commodity price, then the option should be seen as out and have 0 value, if the commodity price is not surpass the barrier price, then the value of the option is calculated by using the last price minus the strike price.

Then, the last step is that after calculating the average value of the option, this value should be discounted by using the risk-free rate to current period and the max error should be calculated along with the present value of the barrier option price.

3.2.2 Prcing results using GBM

After the simulation by using the GBM, the results are shown in figure 2. The graph of the European up-and-out call option demonstrate that all of the European up-and-out call options have a value of 0, The graph shows that most of the plot gathered in the range of 100.05 to 110, at only one plot situated out of that range. The circumstance shows that most of the options are not exercised and only one of the option, which have a max price that surpass the barrier option, and have a value of 0. By using the GBM, the value of the European up-and-out call option cannot be simulate properly.

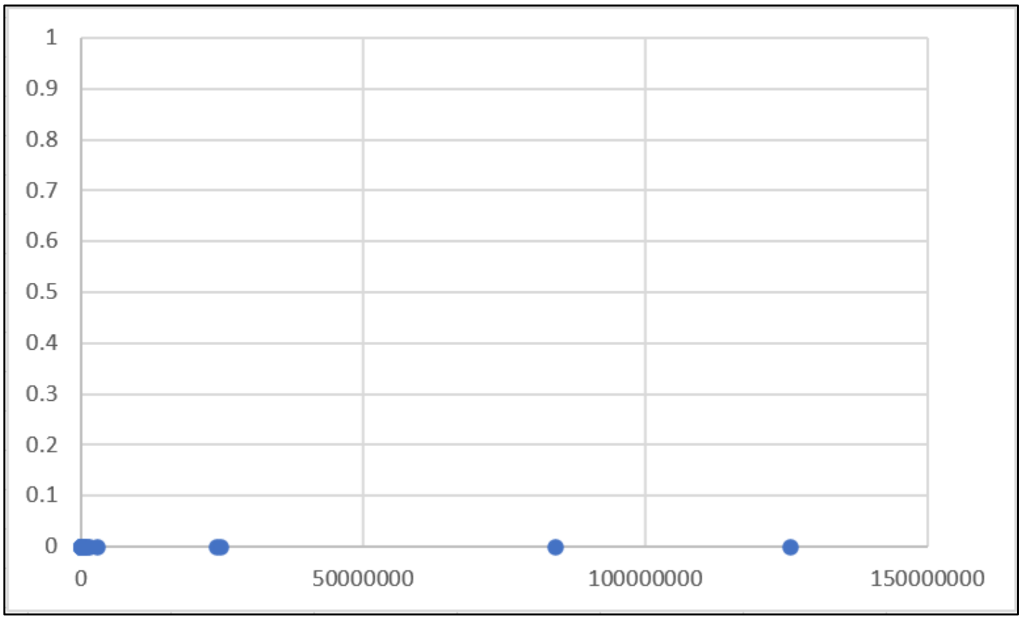

Figure 2. barrier option value & max price

4. Discussion

From previous two plot graph, the graph made after the use of the BSM can explicitly show the distribution of the value of all the options, and basically shows a shape of 'call option', with the some of the option exercised successfully. However, the graph made after the use of GBM also shown the distribution of the value of all the options, which is value of 0 along with the axis, with no options can be exercised successfully and without max price surpass the barrier price, which is uncommon. Hence, the GBM is not suitable for directly simulate the commodity price, the result shown by using the GBM does not correspond with the expected result. There must be several reasons that contribute to the differences.

The first reason is that the there are some flaws about assumptions used in the GBM. The first flaw is that the crude oil commodity itself is assumed to be a kind of financial asset, The crude oil commodity should not be seen as a kind of financial asset because the commodity does not have the property as the financial asset do, such as stocks, futures etc, And the root of this error is that there is no return of crude oil commodity, the commodity itself does not generate return as the financial asset do. The other flaw is that the changes in price is seen as the return of the financial asset. And both the return in the simulation by using the BSM and GBM is 0%. However, the actual changes in price of commodity is -0.31%, and the commodity do not contain dividend itself like the futures, so there is no additional return to balance the return of the commodity. The difference of 0.31% can be a reason that attribute to the abnormal valuation of option in GBM.

The second reason is that there are some flaws about data used in the simulation. When undertaking the simulation by using the GBM, the 'Z' value is the first variable that should first be calculated, so the first flaw may from the abnormal distribution of the 'Z' value. After calculating the 'Z' value, the results are shown in figure 3. From this line chart, it demonstrates that most of the 'Z' value fluctuate at the range from -2% to 2%, and there are also some 'Z' values reached -3% and 3% or in the range of -3% to 3%. There are some extreme 'Z' values reached almost -4% and 4%. And this extreme value may be the root reason, in order to go deep further, the volatility of the log return should also be visualized. After calculating the log return, the results are shown in figure 4, and it depicts that the general form is quite similar to the line chart of 'Z' value. And other information can be got from the line chart. Most of the log return is in the range of -5% to 5%, and there are some returns reached -10% and 10% or in the range of -10% to 10%, the extreme values are also exist, which are higher than 10% or lower than -10%. In contrast, figure 5 of 'Z' value by using the BSM shows that most of the 'Z' value fluctuate also at the range from -2% to 2%, and there are also some 'Z' values reached -3% and 3% or in the range of -3% to 3%. There are some extreme 'Z' values reached almost -4% and 4%. The flunctuation is almost the same, But the difference is still exists. Hence, finding the flaw from the aspect of price is necessary. From figure 6, the chart demonstrates that most of the price is close to 0 and with few exteme value surpass $2milllion, $4million, $8million, $10million or even $16million respectively. While figure 7 shows the prices flunctuate at the range of $95 to $101, with most of the prices ranging from $97 to $100. For both methods used, the original formula the price is at the logarithm form, but in order to calculate the price, an exponential function with e as the basis should be used. And according to the property of exponential function, the higher the negative number, the larger the result can be get, so the extreme number will appear in the simulation. The line chart explicitly shows that there are few extreme value in GBM but not in BSM. Hence, the root of this flaw cannot determined.

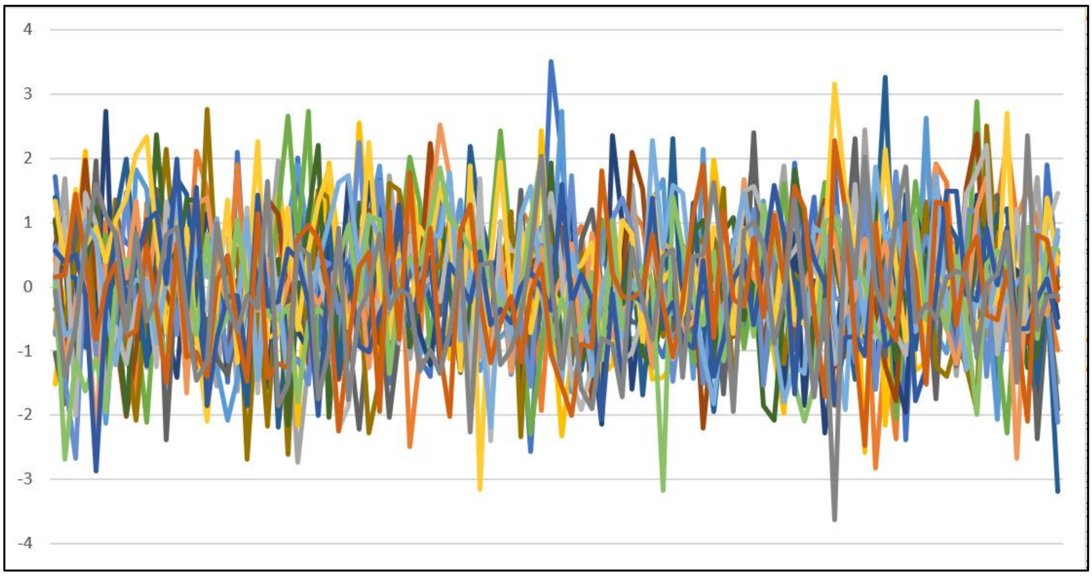

Figure 3. Volatility of Z value

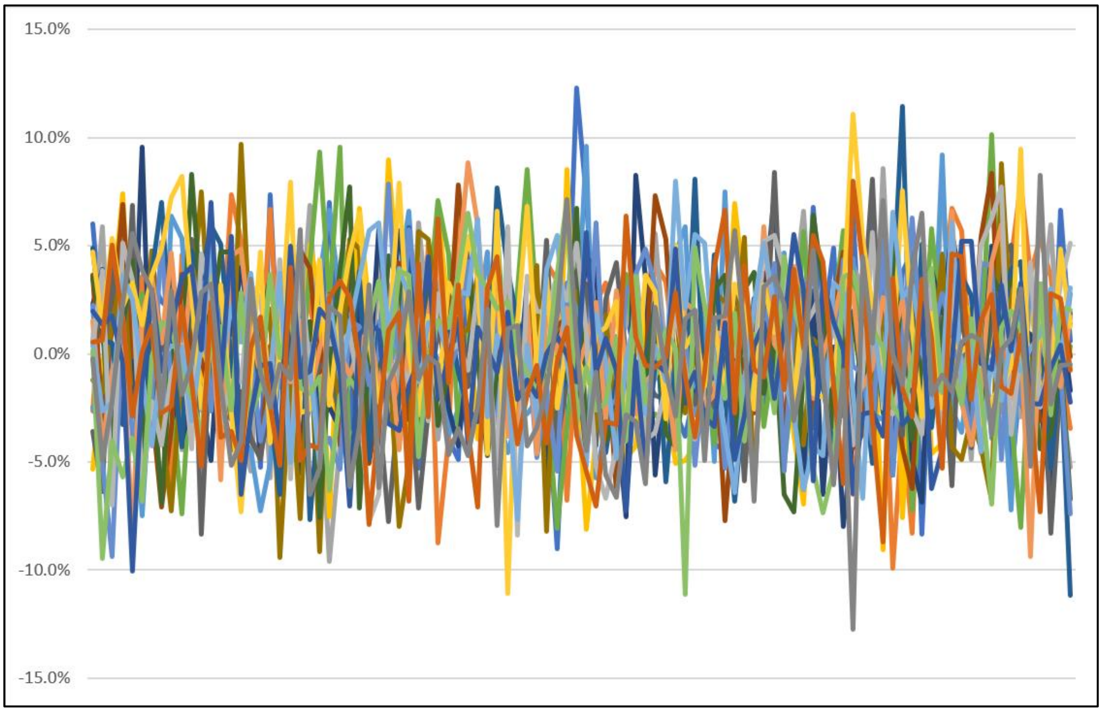

Figure 4. Volatility of log return

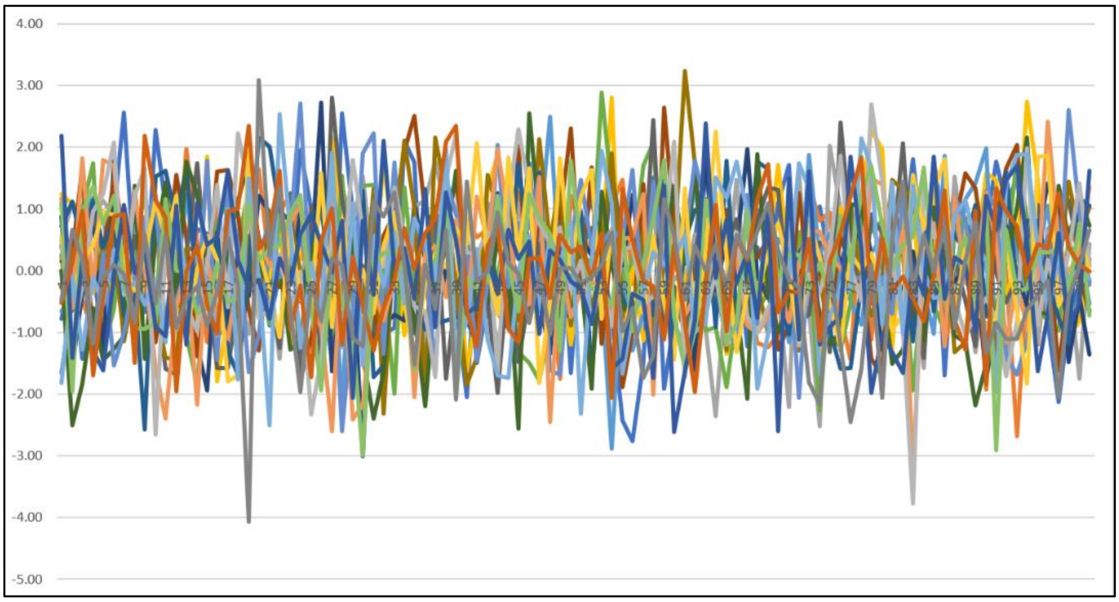

Figure 5. Volatility of Z value

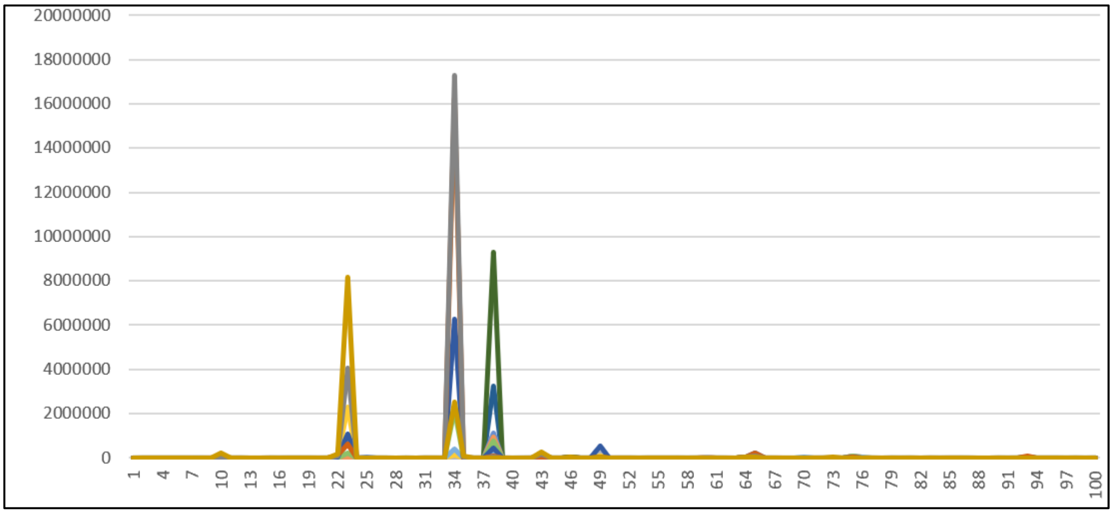

Figure 6. Volatility of price

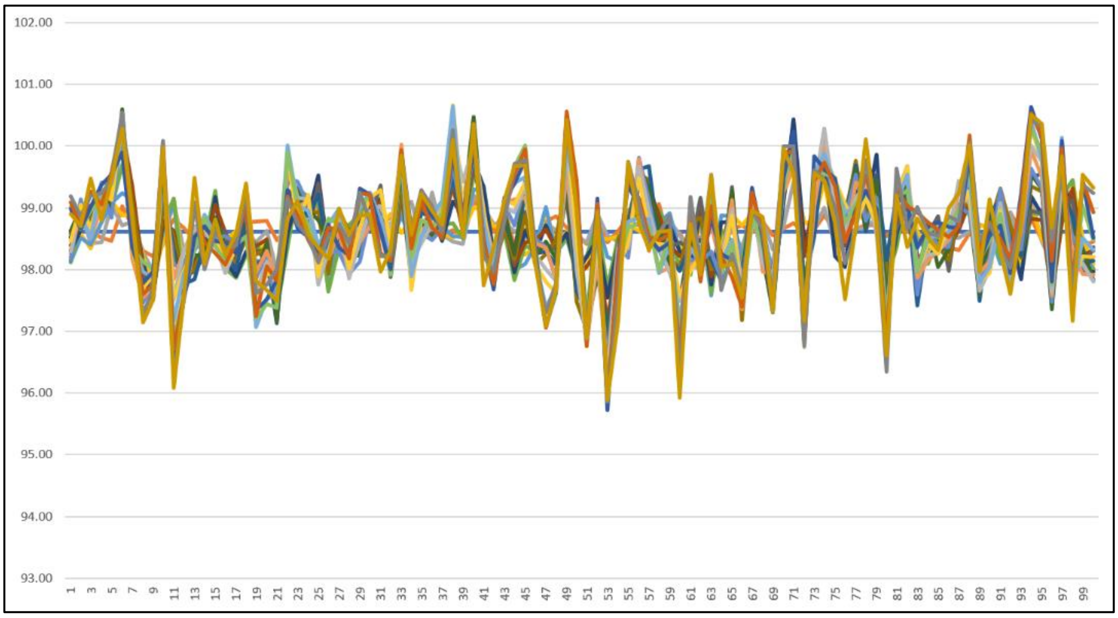

Figure 7. Volatility of price

Nwafor, Oyedele, et al (2017) used two international oil benchmarks and an oil futures to conduct the simulation [9]. The difference between this simulation and the simulation undertaken by in this paper is that all of the parameters used is based on the relationship of these three ‘products’, and then take an average of each parameter calculated. Hence, the flunctuation is decreased by this operation, such as the volatility, the expected return and the growth rate, and it is more possible for the parameters to be positive, rather than 0 or a negative number. And for other steps such as the simulation process is quite the same with simulation in this paper. The result the arthour showed that GBM outperformed in pricing oil than other model. From this paper, the reason for why the crude oil commodity cannot be got still cannot be found.

However, Al-Harthy, Hamood (2007) mainly focus on the comparision among Geometric Bownian Motion mode, Mean Reversion model and Mean Reversion with jumps model. What’s more, the underlying asset the arthour used is the output of oil in the oil-filed [10], which is quite difference with the underlying asset used in simulation in this paper. The assumption and the parameter used is not stated explicitly. And from the result of the simulation conducted by the arthour, the GBM is useful in long-term change of oil price [10], which can indirectly prove the result of simulation in this paper.

The biggest difference between these research and research of this paper is about the underlying asset, the Underlying asst used in this paper is WTI Crude Oil spot, whereas, the underlying assets used in the essay "Simulation and hedging oil price with geometric Brownian motion and single-step binomial price model."[9] and "Stochastic oil price models: Comparison and impact." [10] are not crude oil spot but index or futures or output of oil, which can be seen as the proxy of the crude oil commodity. The use of these proxy, makes the research on data easier.

5. Conclusion

The commodity value was priced in this paper using the Geometric Brownian Motion model and Black-Scholes models, and the value of the options was then determined based on this value. The research has yielded relatively few conclusions. The first finding is that the options value can be properly simulated using the Black-Scholes model and the price of a commodity future. The second conclusion is that utilizing the Geometric Brownian Motion model to directly mimic the price of a commodity cannot produce an appropriate value for the options. However, following data analysis, it is unclear why using the GBM directly cannot produce an accurate value for the options.

The research described in this study goes a step further and uses a pricing model to simulate commodity prices directly rather than attempting to discover proxies for those prices. This study can serve as a theoretical foundation for the short-term commodities option transaction, which will increase the transaction's effectiveness. Additionally, this research can offer an analogy approach that views a commodity as a financial asset, albeit it is necessary to utilize a parameter to make the analogy more precise.

However, there are some limitations for the operation of variables in this paper. Firstly, In this research, only the European up-and-out call option is considered to be contrast with the European call option, however, there are other type of barrier call options such as the up-and-in call option, down-and-out call option, down-and-in call option. Hence, the objects of comparison are not comprehensive. Secondly, the strike price set is the same with the exercise price, which is at the money, there are other way to set the strike price for the call option which is in the money, hence, if consider the in the money situation, the barrier price level could be changed. In the simulation, the barrier price is set higher than the strike price. At in the money situation, the barrier price can be another situation. Thirdly, this research focus on the short-term performance between the European up-and-out call option and the European call option, which is only in 21 trading days in a month. Normally, the options are performed in a much longer term, especially the commodity option.

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