Research on The Lookback Option Pricing of Disney Based on BSM Model

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Abstract. The lookback option is a path-dependent option that enables holders to use the most advantageous pricing to execute the underlying asset. This paper adopts the BMS model to price the lookback option and takes risk factors into consideration to calculate the value of an option contract in the BMS model. The main variables in the BMS model are risk-free rate, current spot price, strike price, volatility, and time to maturity. The results are reliable enough for investors to make future decisions. However, the BMS model still has a few drawbacks which can make the prices derived from real future prices. When predicting future prices, using the BMS model can help investors make better decisions. The lookback option allows holders to minimize their regrets and exercise the underlying asset at the most beneficial price.

Keywords: Lookback option pricing, BSM Model, Disney company, Sensitivity analysis.

1. Introduction

To gain the largest profits and exercise the underlying assets more flexibly, the lookback options are a good choice.A lookback option is a type of path-dependent option that addresses a number of shortcomings in other conventional options and enables investors to use their assets in a way that maximizes their gains. The Black-Scholes-Merton (BSM) model is a fundamental formula of modern finance. It solves the pricing problem by taking the impact of time and other risk factors like volatility, risk-free rate and spot price into account. The lookback options are developed based on the BSM model mathematically. Pricing the option contract based on the BSM model is used extensively in hedging and risk management.

The lookback option is now considered a profitable type of exotic option. The majority of current research on the lookback option is based on mathematical evaluation within different models. For instance, Leung (2013) studied the lookback option pricing method for stochastic volatility [1]. Fallah (2019) analyzed the double Heston model equation's existence and uniqueness and evaluated the Lookback option [2]. Park (2009) used a double-exponential leaps model to assess binomial lookback options. This paper modelled the lookback option using different methods [3]. Some journal papers widely researched the relationship between pricing formulas of lookback options. Cheuk et al. (1997) introduced the lookback option using the binomial approach [4]. Heynen (1995) investigated price models for various full and partial lookback options [5]. Buchen (2005) proposed a new approach to pricing lookback options [6]. This approach simplifies the derivation of exotics in the Black-Scholes model. Another topic related to lookback options is the fractional Brownian motion which is used to simulate some complete markets without arbitrage. Chen (2019) analyzed the stochastic differential equation and established the pricing model of a mixed fractional Brownian motion [7]. Bender (2011) surveyed new progress in the pricing models based on mixed or fractional Brownian motion and collected findings for the transaction cost-based fractional Black & Scholes pricing model [8]. Ma (2021) investigated the behavior of the mixed fractional Brownian exponential population growth system [9]. Azmoodeh and Valkeila (2013) showed that a randomized diagram used to represent the

quadratic variation which derived from a semi-martingale on Brownian motion could also be used for a special type of continuous stochastic process [10].

The paper aims to analyze the pricing method of lookback options. The current pricing method in use is based on the Black-Scholes model. The model predicts the stock's future price based on several variables which are the volatility, the current stock price, the time to expiration, the risk-free rate, and the spot price. The BSM model assumes that there are no dividends for the duration of the option, the market is random; that there aren't any transaction fees; that the risk-free rate and volatility are constant; and that the underlying assets' returns are presented as a trend of normally distributed. Although the BSM model takes several variables into account, the predictions are likely different from reality. The analytical process in this paper also uses this model to estimate future prices.

Based on options theory and the lookback option's distinctive features, this essay discusses it. Using Disney as the underlying asset, this study applies the Black-Scholes Merton approach to model stock price movements and the payouts from fixed and floating lookback call options. In contrast to the floating lookback option, the fixed lookback option and European Option pricing clearly shift when the strike price changes. The lookback option differs significantly from the European Option due to changes in spot price and volatility Additionally, despite having a greater option price, the lookback option solves the market timing issues.

This paper also uses future prices to obtain payoff diagrams and the relationships between payoff and other variables. There are two types of lookback options which are the floating lookback option and the fixed lookback option. The floating lookback option allows investors to change the strike price to the most lucrative asset price at maturity. The fixed option is similar to vanilla options; the strike price is set when investors purchase the underlying asset.

The following section is organized as follows. In section 2, the paper introduced the data from the Disney market and the pricing model. The pricing method is derived from the BSM model, and the paper lists the equations for the pricing. The simulation and sensitivity analysis are in section 3. The comparison between the European option and the Lookback option is shown in section 4. Finally, section 5 concludes the whole paper.

2. Data and Method

2.1 Data

Walt Disney is an American mass media and entertainment company. The data used in this paper is the historical stock price of the Disney company. The data is collected from Yahoo finance. The data used in this study are the stock prices of the previous year. The stock prices the paper used are the close prices which are the standard benchmark prices used by the investors to track the stock performance.

2.2 Simulation methods and pricing model

In this section, the stock prices for evaluating exotic instruments are from the Disney company. The one-year historical stock prices from Disney company show a little volatility which means it's a relatively steady stock. The simulation of the stocks is divided into every 10 days because there are usually 252 market days every year. Thus, there are 25 simulations in total and run 1000 times. The 25th data set of the strike price is the final price at maturity. The maximum and minimum functions are used to get the maximum and minimum prices for each simulation.

There are two types of lookback options that are based on the strike price; one has a floating strike price and the other has a fixed strike price. For the floating contract, the strike price is set at the previous lowest price. The call options and put options are given by formulas (1) and (2).

$$C_{floating} = \mathrm{Max}(S_T - S_{Min}, 0) \quad (1)$$

$$P_{floating} = Max(S_{Max} - S_T, 0) \quad (2)$$

Where $C_{floating}$ represents the floating call option; $P_{floating}$ represents the floating put option; S_T represents the underlying asset's price at the date of maturity T; S_{Max} represents the maximum price of the asset; S_{Min} represents the minimum price of the asset.

The strike price in the fixed contract is fixed, meaning the investor cannot exercise it until maturity. The call options and put options are given by:

$$C_{fixed} = Max(S_T - S_{Max}, 0) \quad (3)$$

$$P_{fixed} = Max(S_T - S_{Min}, 0) \quad (4)$$

Where C_{fixed} represents the fixed call option; P_{fixed} represents the fixed put option; S_T represents the underlying asset's price at the date of maturity T; S_{Max} represents the maximum price of the asset; S_{Min} represents the minimum price of the asset.

The lookback option's pricing approach is based on the BSM model. To simulate the future price, the model takes the time, spot price, volatility, strike price, and spot price into consideration.

The call option's price is calculated based on the Black-Scholes formula (5):

$$C = SN((\ln\frac{s}{k} + (r + t\frac{\delta_v^2}{2}))/\sigma_s\sqrt{t}) - Ke^{-rt}N((\ln\frac{s}{k} + (r + t\frac{\delta_v^2}{2}))/\sigma_s\sqrt{t} - \sigma_s\sqrt{t}) \quad (5)$$

Where C represents call option price; S represents stock price; K represents strike price; r represents risk-free rate; t represents maturity; N represents a normal distribution.

The profits or losses on the call option are given by formula (6):

$$\text{Profits / Losses} = \text{Stock price at expiration} - \text{Breakeven point} \quad (6)$$

2.3 Comparison between Lookback option and European option

A European option is also a path-dependent option which shares similarities with the lookback option. The right to exercise an underlying asset at a certain price prior to the expiration date is provided by these two options to investors. For the European option, the holder can only exercise their underlying assets on the expiration day and the strike prices are fixed.

The call options are given by formula (7):

$$C = Max(S_T - \text{Strike price}, 0) \quad (7)$$

The put options are given by formula (8):

$$P = Max(\text{Strike price} - S_T, 0) \quad (8)$$

The pricing method of a European option is also based on the Black-Scholes option model.

3. Results

3.1 Simulation using Disney stock prices

Excel is used to run 1000 estimates in one year and every 10-day interval, and the results are shown in Table 1. Then, the 1000 stock prices for floating and fixed options are calculated in Table 2 by Excel. These two Tables are brief versions of all 1000 sets of data

Table 1. Simulated historical option price under one-year simulation and running 1000 times

No. of Times	1	2	3	….	25
1	99.38	96.07	92.17	….	71.37
2	107.67	101.13	93.91	….	149.86
3	101.21	99.57	94.70	….	148.59
….	….	….	….	….	….
1000	105.22	108.02	107.00	….	97.51

Table 2. Simulated stock prices for floating and fixed options

Times	Floating call	Floating put	Fixed call	Fixed put
1	2.89	28.01	0.00	35.02
2	55.95	0.00	46.36	9.60
3	53.90	7.76	52.85	8.81
….	….	….	….	….
1000	19.24	10.51	4.52	25.23

3.2 Sample Valuation

The maximum error test is made as a statistical analysis to evaluate the performance of the simulations. It is calculated as twice the standard deviation of the 1000 observations divided by the square root of the sample size, which is 1000. The results will approach a normal distribution, meaning that the simulation outcomes will be within the maximum error and cover 95% of that random variable. The discounted payoff is calculated through the equation (9)

$$\text{discounted payoff} = \text{average payoff} \times e^{-rt} \quad (9)$$

Statistics shown in Table 3 is an example of using a random variable. The present values of the two types of lookback options can be obtained as discounted payoff +/- maximum error. Since the percentage error is an acceptable range, the error is small to make results.

Table 3. The average payoff, present value, maximum error, and percentage error of lookback options calculated from 1000 random estimates

	Floating call	Floating put	Fixed call	Fixed put
Average payoff	20.78	21.87	25.11	18.51
Discount payoff	20.16	21.21	24.36	17.95
Maximum error	1.53	0.97	1.66	0.90
Percentage error	7.60%	4.55%	6.80%	5.02%

3.3 Comparison between payoff diagrams

The payoff diagrams for call options clearly reveal the profits or losses for call options. It is a graphic way to represent the future potential outcome.

As shown in Figure 1 and Figure 2, the payoff diagrams of floating call and fixed call are similar which are both equal to 0 at first and then a line with a slope greater than 1. The two payoff diagrams of the lookback options are similar to other path-dependent exotic options.

Fig. 1 Floating call option payoff diagram

Fig. 2 Fixed call option payoff diagram

3.4 Sensitivity analysis

Sensitivity analysis is often used to determine the option's ability to bear risks and select the investment option with more significant economic benefit values. Three sensitivity analyses are recommended to compare the effectiveness of a floating call option and a fixed call option, helping illustrate how different independent variables affect the call option price.

3.4.1 Strike price

The first independent variable is the strike price. To visualize the data on how expensive the option is for a different strike, figure 1 is created based on the data table and shows two distinct trends.

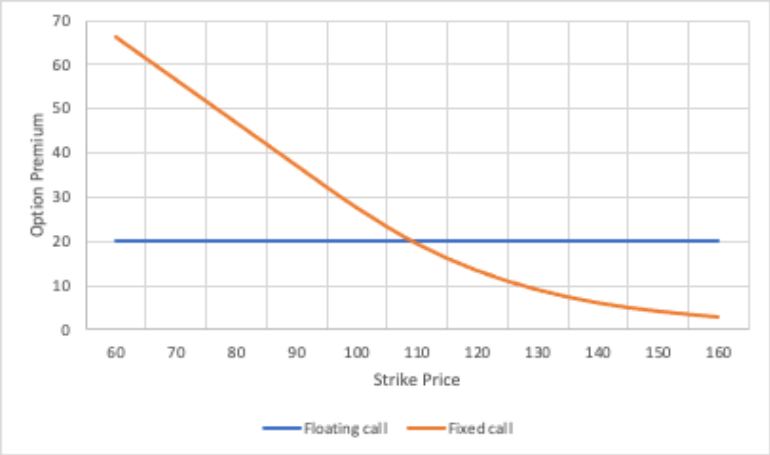

Fig. 3 Scatter of option premium with respect to strike price

For floating call options, the option prices do not change with strike prices. However, for fixed call options, the option prices decrease with the increase of strike prices at a decreasing rate. In this case, a lower strike price gives a more expensive option, which also means a higher probability of profit.

3.4.2 Volatility

The second sensitivity analysis demonstrates how option prices change with respect to the volatility of the asset.

Figure 2 illustrates the trends of the option prices with historical volatility values. The two types of call options show a similar trend: the value of the option price increases with volatility, with the fixed call option presenting a steeper slope. Therefore, at the same option price, the floating call option is more volatile than the fixed call option. For a call option, more volatile means it gets a higher chance of exercising in profit as the stock is more likely to surpass the strike price.

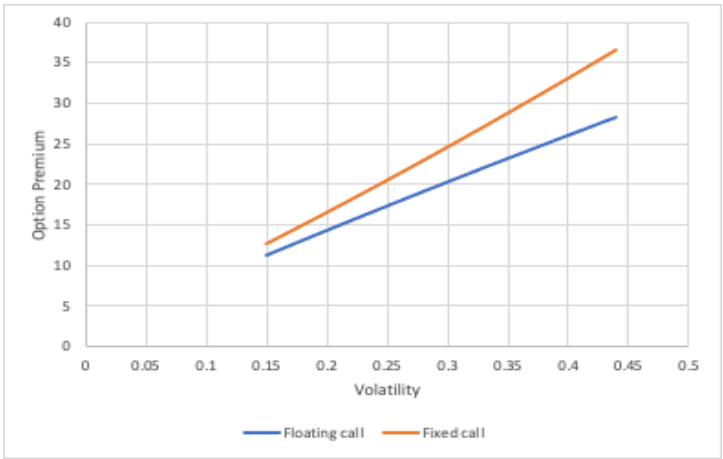

Fig. 4 Scatter of option premium with respect to historical volatility

3.4.3 Spot price

The third independent variable in the sensitivity analysis is the spot price, which shows how the option premium depends on the spot price of the stock. The result is that both types indicate that option price and spot price are positively correlated; the option premium increases with rising in the spot price. The difference is that for the floating call option, a huge change in the spot price leads to a less than proportional change in the option premium.

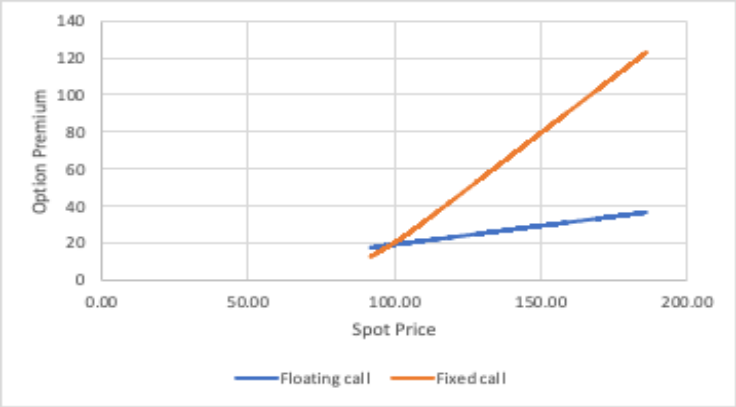

Fig. 5 Scatter of option premium with respect to spot price

To summarize these findings, the fixed call option is always more sensitive to small changes in the independent variable and more likely to give a higher value of option price.

3.5 Comparison between Lookback option and European option

The discounted payoff of European options is around 13.8 which is smaller than the discounted payoff of fixed and floating lookback options which are around 20. The smaller discounted payoff means the investor can repay less under European options. As shown in figure 6, the payoff diagram of European options is very similar to lookback options. The slope is equal to 0 at first, and then it becomes a straight line whose slope is positive and greater than 1.

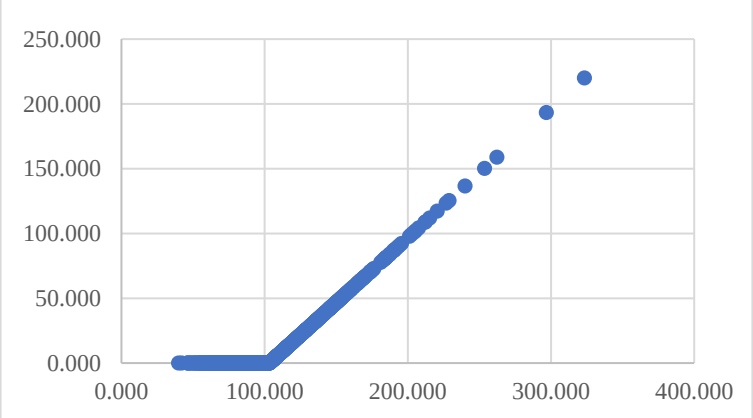

Fig. 6 European call option payoff diagram

The diagram of strike price VS option premium for the European option is the same as the fixed lookback option. That is because the strike prices are both fixed. As shown in figure 7, The difference in strike price VS option premium graph between the European option and floating lookback option is caused by the difference in whether the strike prices are fixed or not.

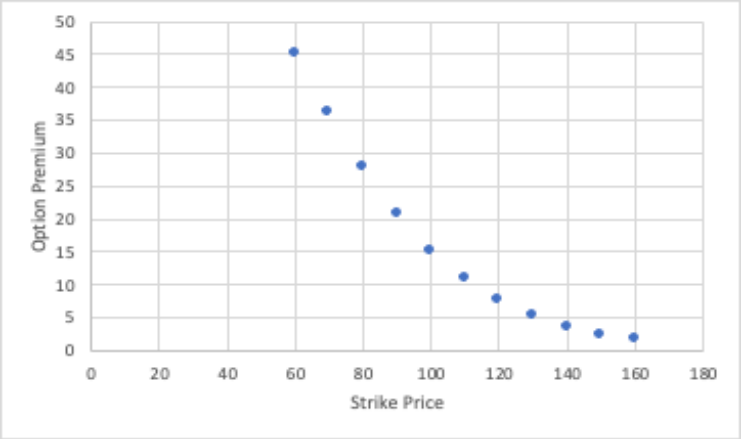

Fig. 7 Strike price VS option premium under European option

The graph of volatility VS option premium under the European option is also the same. As shown in figure 8, when the volatility increases, the option premium increases. Volatility is positively correlated with option premiums. For these two options, when the volatility increases, the likelihood that all options will expire in the money also rises. When the movement of the underlying assets is large, the chance of creating an in-the-money option also grows large.

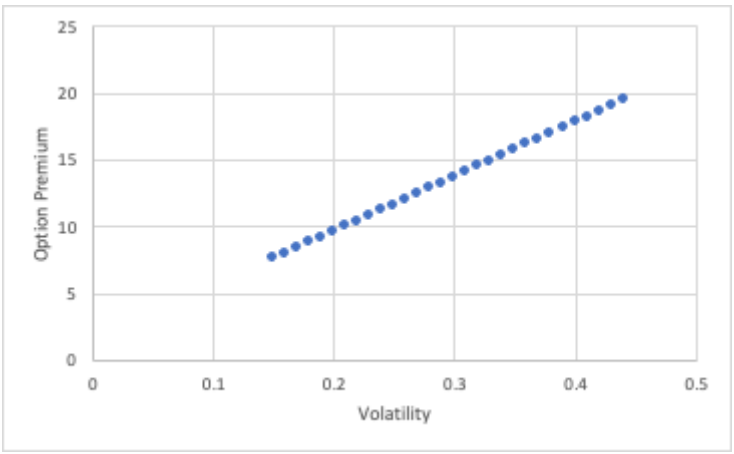

Fig. 8 Volatility VS option premium under European option

4. Difference between Vanilla and Lookback option

4.1 Benefits from using lookback option rather than Vanilla option

4.1.1 Maximize profits

The return of lookback options depends on the underlying asset's maximum or minimum price within the option's validity period. The return of the Vanilla option is equal to the amount by which the cost of the last underlying asset exceeds the strike price. The great thing about lookback options rather than any other option is that investors can review the entire period and get the most out of the difference in profit from the stock.

4.1.2 Minimizes risk

The negative scenario of having a lookback option has been defined, which means that the return is zero and the purchase price of the option is paid. In contrast to other types of financial instruments, lookback options not only increase the degree of leverage but also pose less risk. And lookback options also present an opportunity that provides (at a cost, of course) a way to choose the best market timing. If an investor holds a call option to buy at $X, he can sell the asset at the highest price during this period, thus minimizing risk.

4.1.3 Volatility means better

The change range of the stock price of the lookback option within a month. The larger the rise and fall range of the lookback option, the more intense the price trend is, the greater the volatility, and the smaller the reverse. Volatility can be used to measure the risk of the option. The price trend of the highly volatile option is highly uncertain. They may face a sharp rise or fall. For example, many small-cap and junk stocks tend to be highly volatile. However, the price trend is very stable for the lookback option with small fluctuations as the beta calculated during sensitivity analysis, thus volatile vs premium is less fluctuate than other options. After buying, investors will only face a small rise and a small fall, and the profit and loss will not be large.

4.1.4 Solve market timing and get maximum possible gains

Risk and return are inseparable in any case. The longer the holding period of risk assets, the chance of getting higher maximum assets, and the lower the probability of serious losses.

4.1.5 No regret of holding for too long or selling too early

Understand that the overall market realization of a diversified portfolio is very different from the performance of individual assets in the portfolio

4.2 Disadvantage of Lookback option: expensive and sometimes cannot cover premium

When buying an option, the minimum decline value is 0, but the premium can be a pain because with a net gain of 0, the stock market still forces investors to pay a premium for the buy, so it will create a negative profit situation. However, due to lookback option's unique pricing characteristics, the odds of generating a gain are much greater than with European-style options. However, there is no guarantee that the difference between the buy and sell prices of the asset will be enough to cover the cost of the option itself.

5. Discussion

The research in this paper is similar to the previous studies and the conclusions obtained are basically the same. The difference is that this paper shows the process of proving the results by simulating and running a total of 1000 simulation attempts in EXCEL. The conclusions are justified by looking at the option payoff graph. In addition, in previous studies, sensitivity analysis was not used to analyze the impact of volatility on lookback option pricing. This paper also goes a step further by examining the main causes of pricing volatility through sensitivity analysis.

The main direction of inquiry in this paper is to compare lookback options with standard options, which in general tend to have higher returns, but this comes with a correspondingly high "price". However, this does not mean the floating lookback option is profitable. The ability to understand the past and the future is undoubtedly very expensive. Therefore, if the minimum price is not low enough or the maximum price is not high enough, and the profit does not cover the premium of the "lookback" option, the option buyer will lose money. Investors embracing the floating call option pay more premiums than the fixed call option because the latter is less volatile than the former. High volatility means that the strike price will most likely be surpassed for profits. The more volatile the stock price is expected to be. Closely looking at the strategies around the call option, it can be seen that an investor tries to minimize exposure to stock price fluctuations by choosing one option over another.

Consequently, the loss or the profit awaiting the lookback option will be large. Many investors study the market and predict the behavior of stocks before investing. Some investors mitigate the risk by buying the floating lookback call option and sellers a fixed lookback call option to enjoy the full space when the stock price increases. The seller and the buyer both anticipate future price changes, but we can see that the benefit derived from the trade is higher for the investor who predicts the market Correctly.

When discounted European Options are compared to the discounted payoff of fixed and floating lookback options, an investor will repay less under European options. However, the strike prices for

both options are fixed, and volatility increases the premiums to be paid for each option. At expiration, the option's value is the amount of money the holder of the option can realize if they exercise the option. Without the intrinsic value, the option is worthless at expiry. As a result, the premium paid for the option initially is forfeited.

Volatility in the lookback option is taken as the risk of the option for any investor who decides to invest in it, but it is defined as a good risk due to the nature of the lookback. And the greater the volatility, the benefit tends to be the greater the option premium paid. From sensitivity analysis, the higher the volatility in the stock market, the higher the premium. Since volatility has a guaranteed downside, but the maximum profit range is infinite, so volatility means higher returns, which increases the price, where higher risk attracts higher premium payments, and vice versa. If the stock is expected to rise sharply, buy the appropriate standard call option; If investors are worried that there is still room for decline before the rise, investors can buy a floating lookback call option. No matter how the market changes before the expiration, the price of the option buyer's "buy" stock will always be the lowest price before the expiry; If a trader pays more attention to selling at the highest price than buying at a low cost, the seller can buy a fixed lookback call option to grasp the maximum space for the market to rise. Although the lookback option can not bring the buyer back to the past, it can make up for our regret in investment to a certain extent. Of course, the price of "taking" this regretful medicine is very high. No matter how complex the option strategy is, it is essentially the packaging of our market expectations. Therefore, the essence of options trading is still a contest of who is more discerning.

As an extension of the paper, If the maximum lookback option is viewed as a ladder option, investors can quickly spot the gamma risk characteristics of the lookback option. As long as the asset price rises, the lookback option will have gamma risk. As the asset price decreases, the gamma value also decreases rapidly. When an option priced below the barrier price is knocked off, the gamma will be 0. Thus, the gamma of the maximum lookback option reaches its maximum when the asset price goes from the opportunity expiration date to the current high and continues to increase.

6. Conclusion

This paper used the BMS model to investigate the pricing of the lookback options. The option contract's value was calculated by considering the various risk factors. The main findings are that the lookback option compared to European options is always more sensitive to small changes in the independent variable giving an option price with a higher value. In addition, through the sensitivity analysis, spot price, volatility and strike price corresponding to the option price display chart reveal that the fixed call option is always more volatile than the floating call option, which indicates that it has high sensitivity characteristics.

The research implications for this study are that the pricing of the lookback option affects the value of the contract, and the buyer may have losses or profits. It depends on how an investor anticipates future changes. The second implication is that due to each option has different properties, and it is important to discover when to use the right properties. Through sensitivity analysis, the reader will be able to better determine the specificity of the option, allowing the reader to take the right approach in svolatile market.

This research was limited to access to data to be used in the simulation. Most of the data is private, and less is disclosed by the companies making access to valid data a problem. To better analyze the data, the best solution is to see the company's dividends through the company's annual financial statements, which effectively confirms the delta value. The Monte Carlo simulation method could evaluate the average price movement for the fixed exercise price lookback option and the floating exercise price lookback option. The value of the options impacted by risk in numerical computation would be used as a forecasting model. For further research, lookback options may be compared to other options with different companies to obtain a broader understanding of stock options complexities.

References

[1] Leung, K.S. An analytic pricing formula for lookback options under stochastic volatility. Applied Mathematics Letters, 2013, 26(1): 145-149.

[2] Fallah, S., Mehrdoust, F. On the existence and uniqueness of the solution to the double Heston model equation and valuing Lookback option. Journal of Computational and Applied Mathematics, 2019, 350: 412-422.

[3] Park, H.S. Analytical binomial lookback options with double-exponential jumps. Journal of the Korean Statistical Society, 2009, 38(4): 397-404.

[4] Cheuk, T.H.F., Vorst, T.C.F. Currency lookback options and observation frequency: A binomial approach. Journal of International Money and Finance, 1997, 16(2): 173-187.

[5] Heynen, R. C., Kat, H. M. Lookback options with discrete and partial monitoring of the underlying price. Applied Mathematical Finance, 1995, 2(4): 273-284.

[6] Buchen, P., Konstandatos, O. A new method of pricing lookback options. Mathematical Finance, 2005, 15: 245-259.

[7] Chen Qisheng, Zhang Qian, Liu Chuan. The pricing and numerical analysis of lookback options for mixed fractional Brownian motion. Chaos, Solitons & Fractals, 2019, 128: 123-128.

[8] Bender, C., Sottinen, T., Valkeila, E. Fractional Processes as Models in Stochastic Finance. Advanced Mathematical Methods for Finance, 2011, 75-103.

[9] Ma Weijun, Liu Wei, Zhu Quanxin Zhu, Shi Kaibo. Dynamics of the Exponential Population Growth System with Mixed Fractional Brownian Motion. Complexity, 2021.

[10] Azmoodeh, E., Valkeila, E. Spectral characterization of the quadratic variation of mixed Brownian–fractional Brownian motion. Stat Inference Stoch Process, 2013, 16: 97–112.