Pricing of Asian Options and Barrier Options for The Microsoft Corporation Based on Monte-Carlo Simulation

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Abstract. In the current unstable financial market, option is a popular financial tool to hedge the risk of the underlying asset. Compared with the vanilla options, the exotic options can deal with the more complex requirements of investors. This paper focus on evaluating the Asian options and barrier options based on the data simulated from the Microsoft Corporation. This paper has three main research findings: the first is that it prices the Asian options and four types of the barrier options; the second is that it illustrates the reason for the price difference among the European options, the Asian options and barrier options with the help of payoff diagrams; the third is that it is explain the relationship between these two exotic options and the parameters used in the simulation. This paper would help investors better understanding the difference between the Asian options and barrier options from the perspective of the price and the sensitivity to the parameters.

Keywords: Barrier Options, Asian Options, Monte-Carlo Simulation, Sensitivity Analysis.

1. Introduction

Jabbour and Liu (2005) had shown statistically that the options price estimated by using the Monte-Carlo simulation and Black-Scholes model have the similar result when changing the standard error and the price range [1]. Schöne (2015) studied different models for pricing the underlying assets based on the unique characteristics of the stochastic processes [2]. During the study of the Asian options, Ye (2008) indicated the special scenarios that the Asian options had a higher premium than the European options [3]. Fu and Hu (1995) used the gradient estimation method to perform the sensitivity analysis to the European and the America call options [4]. Li (2022) used the Black and Scholes method to price the up-and-in barrier call options and compared with the European options [5]. There would be more deviations when simulating the America options, since the American options can be exercised at any time and it is hard to determine the exercise price accurately. Dion and L'Ecuyer (2010) compared different types of the Monte-Carlo simulation method such as the randomized quasi-Monte Carlo to price the America options [6]. Glasserman and Staum (2001) created an unbiased estimator to deal with the high probability of the barrier options being inactive, which helps to decrease the standard error of the Monte-Carlo simulation [7]. De Weert (2008) comprehensively explained the principle of the exotic options, including lookback options, chooser options, barrier options and so on [8]. On the basis of the Heston model, Leitao (2021) proposed a technique that involving the application of Markov chain, which helps to improve the efficiency of exotic options prcing [9]. An and Suo (2009) demonstrated that a different pricing model would affect the hedge ability of the exotic options, especially for the barrier option, which is higher path-dependent [10].

This paper focuses on discussing the differences between the Asian options and the four barrier options using the Monte-Carlo simulation pricing. In the data section, this paper introduces the source of the simulation data. In the method section, this paper discusses the Monte-carlo simulation method applied on the Asian options and barrier options. For the result part, this paper illustrates the pricing process of these two options and show the sensitivity analysis respectively, then compare the Asian options with the other four barrier options

2. Data

This paper uses the daily stock price ofMicrosoft Corporation (MSFT) from 2022/8/3 to 2022/8/2 to price the barrier options and Asian options. The daily adjusted close price dataset is collected from the Yahoo finance and Fig. 1 shows the one-year movements of the underlying asset. The risk-free rate can be obtained based on the one-year US treasury rate on 2022/8/2 [11].

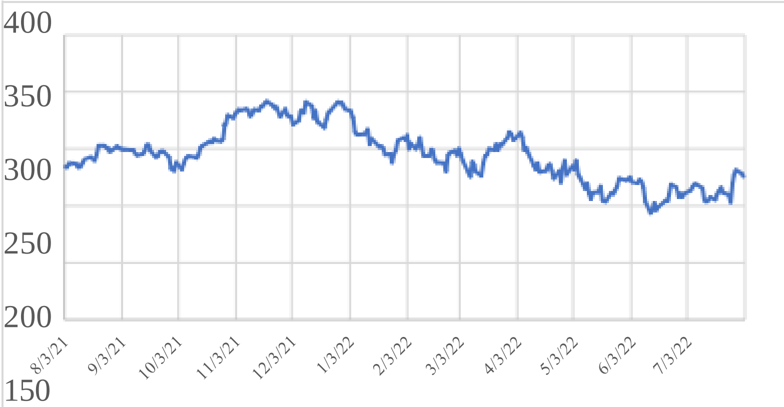

Fig. 1 Microsoft Corporation's Daily Adj Close Price From 2022/8/3 to 2022/8/2.

3. Method

3.1 Monte-Carlo Simulation

This paper uses Monte-Carlo method to simulate the price of the underlying asset Microsoft Corporation. The risk-neutral Monte-Carlo simulation is one application of the stochastic modeling and it assumes that the price of the underlying asset is lognormally distributed [2]. Based on the input from the date section, this paper applies the stock simulation formula (1) to simulate the daily prices.

$$S_T = S_0 \, e^{(a-\frac{1}{2}G^2)T+zGVT} \quad (1)$$

Where, S_T is the spot price of the Microsoft Corporation at time T, S_0 is the latest price, a equal to the risk-free rate (r) minus the dividend payout rate (d), s is the annual implied volatility, T is the time to maturity, and z represents the standard random numbers. Also, the Monte-Carlo simulation method would share the same result as the Black-Scholes model since the formula can be derived from the Black-Scholes equation through Ito's Lemma. Then, the paper discusses about the simulation process in Excel for pricing the barrier options and Asian options.

3.2 Asian Options Simulation

Asian option is a path-dependent option that requires to track the daily price. The options premium can be calculated by comparing the average price with the strike price. The payoff of the Asian call options is max (0, Savg–K), and the payoff of the put options is max (0, K− Savg), where Savg is the 10 days average price of the underlying asset and K is the strike price. Then, this paper can get the fair value of the Asian options by taking the average of the 500 simulated payoffs.

3.3 Barrier Options Simulation

Barrier option is another type of exotic option that depends on the past movements of the underlying asset. There are primarily two types of barrier options: knock-out options and knock-in options. The knock-out barrier options can further be classified into down-and-out and up-and-out options. A down-and-out call option turns invalid when the underlying asset moves below the down barrier. And the payoff is max (0, ST –K) if the minimum price ofST is greater than the down barrier and 0 otherwise. An up-and-out option turns invalid when the underlying asset moves above the up barrier. And the payoff is max (0, ST –K) if the maximum price ofST is less than the up barrier and 0 otherwise. The Knock-in barrier options can further be classified into down-and-in and up-and-in options. A down-and-in barrier option turns valid only if the underlying asset drops below the down

barrier. And the payoff is max (0, S" –K) if the minimum price ofS" is less than the down barrier and 0 otherwise. An up-and-in barrier option turns valid when the price of the underlying asset exceeds the up barrier. And the payoff would be max (0, S" –K) if the maximum price ofS" is greater than the up barrier and 0 otherwise. The payoff of the barrier put option is the opposite of the barrier call. Under the same condition, the payoffs would become max(0, K-S") since investors benefit from the price of underlying asset going down.

4. Result

This paper focus on pricing the up-and-out call option, up-and-in call option, down-and-out option and the down-and-in option. For each simulation, this paper selects the maximum price and the minimum price from 10 daily prices and compare with the up barrier or down barrier respectively. Similarly, it obtains the fair value of these four options by taking the average of the 500 simulated payoffs. Moreover, this study conducts the sensitivity analysis with different factors such as strike price, volatility and barrier price by What-If Analysis in Excel.

4.1 Data and Simulation Assumptions in the Pricing Model

In order to simulate a 10-day stock price, few assumptions must be made. As listed in Table 1, the risk-free rate is 0.0309 according to the 1-year US treasury rate. Based on the MSFT data, the spot price would be the latest price which is 274.82. The annual implied volatility (sigma) can be calculated from the actual stock prices using the STDEV function in Excel and multiplying by SQRT(252). The dividend payout rate which is delta would be 0.88%. This paper would simulate 10 days stoke price so the time to maturity would be 10/252 (0.0397) year. Assume the strike price is equal to the spot price, then this paper makes an assumption of down barrier 250 and up barrier 300 to further evaluate the price of barrier options.

Table 1. Data used for simulation

r	sigma	Spot price	delta	T	Strike price	Down barrier	Up barrier
3.09%	30.08%	274.82	0.88%	0.0397	274.82	250	300

This paper would simulate 10-day price paths for 500 times. The first thing is to generate the 500´ 10 random number matrices using the formula z = normsinv (rand ()) in Excel. Since these random data would change every time when refreshing the sheet, the efficient way is to copy and paste the whole date matrix as values. Then, this paper plugs corresponding random numbers (z) into the simulation formula. For day one price simulation, this paper uses the latest price 274.82 as S% and get the day one simulated price. Then, it uses the day one simulated price as the input "S%" for the day two price simulation and so on. Based on the different payoffs of barrier options and Asian options, this paper would explain the evaluation separately.

4.2 Pricing Results of the Asian Options

4.2.1 Asian options price

After getting the average price of the options, this paper calculates the discounted payoff which is the present value using the formula P = Ae) @", where A is the fair value of options, P is the options, P is the options premium, r is the risk-free rate and T is the time to maturity. Then, this paper calculates the maximum error by the formula Maxerr = 2s/√N, where s is the standard deviation of the simulated options, premiums and the N is the number of simulations. This means that the range of the Asian options price estimation among all simulations is from 3.8939 to 5.0489. Also, table 2 shows that the Asian options has a lower price compared with the European options. One reason could be that averaging the movements of the underlying asset lower the volatility, and lower volatility would lower the, options price generally. However, it does not mean that this relationship is absolute since the Asian options could have a higher price than European options under other conditions related to the dividend payout rate [3].

Table 2. The discounted payoff of Asian options

Asian options		European options
Average price	4.4714	6.7590
Discounted payoff	4.4659	6.7507
Max.error	0.5775	0.8714

4.2.2 Payoff diagram

As shown in Figure 2, the payoff diagram of Asian call options shows the similar trend with the European call options. The y-axis represents the simulated options premium and the x-axis represents the average price of 10 simulated days. The payoff would be 0 when the average price is below the strike price, and the difference between the average price and strike price otherwise. The options premium would be higher as the averaging price increases since investors would take a higher risk while waiting for the price to rise to a greater extent.

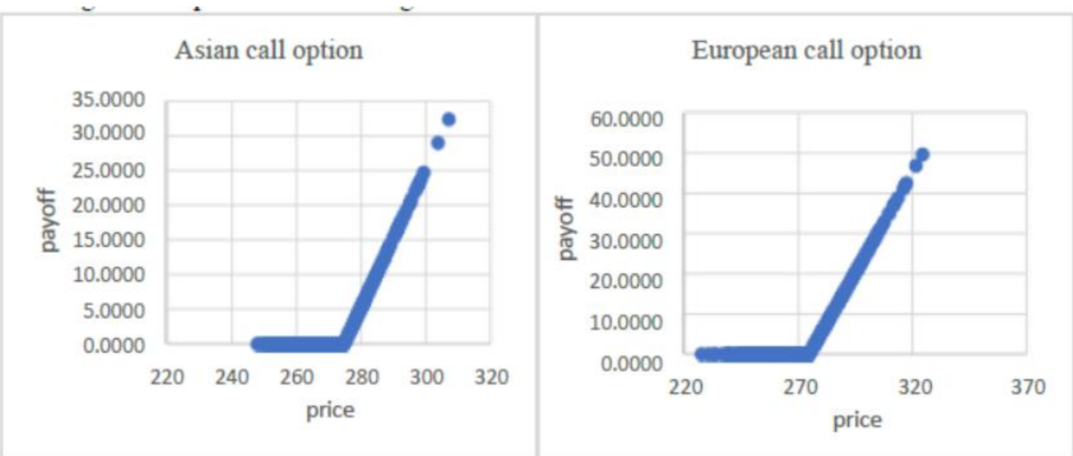

Fig. 2 Payoff of Asian call options and European call options

4.2.3 Sensitivity analysis

Figure 3. shows that the relationship between the payoffs of options and volatility. It indicates both options show the perfect linear relationships. This paper indicates that the European option is more sensitive to volatility than the Asian option because of the higher slope. As the volatility increases by 0. 1, the payoff of Asian option would increase by 1.4623 dollars while the payoff of European option would increase by 2.1975 dollars. One reason could be that the Asian option's averaging operation reduce the sensitivity to volatility.

Figure 4 shows that both Asian and European option have the exponential negative relationship with the strike price. The exponential relationship is because that the movement of option price is greater than the change of strike price [5]. It suggests the payoffs slowly drop to 0 when the strike price is greater than 300 dollars. The reason is that the predetermined strike price is too high to exercise the options under this simulation. The trends tend to be linear when the strike price is below the spot price 274.82. And the Asian option is relatively more sensitive to strike price than the European option since the price of the Asian option is less volatile. Also, the premium difference gets maximized when the strike price is close to the spot price

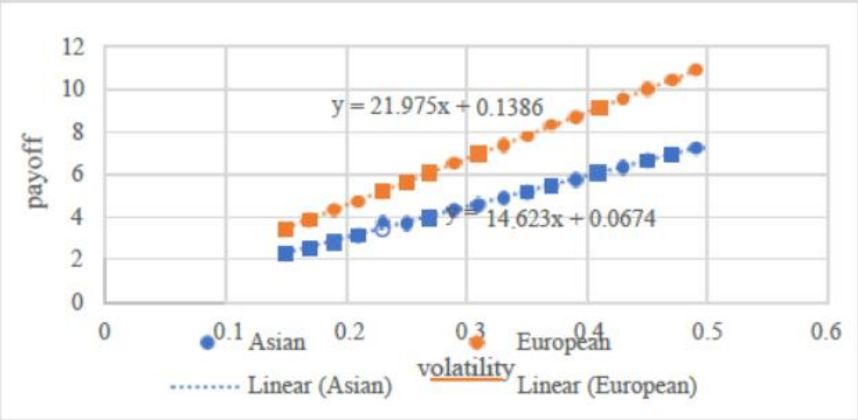

Fig. 3 Asian and European option's sensitivity to volatility

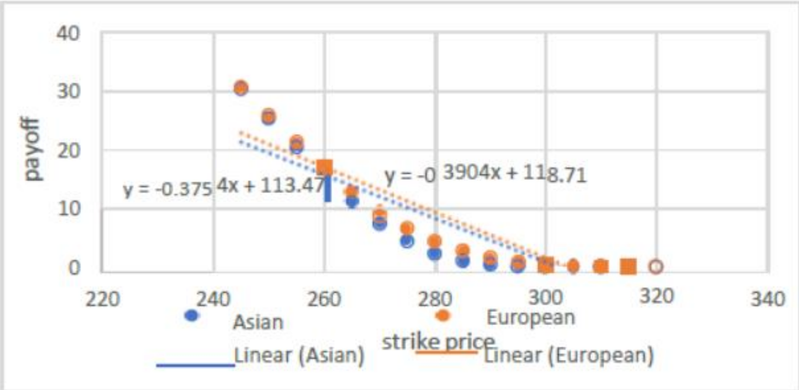

Fig. 4 Asian and European option's sensitivity to strike price

4.3 Pricing Results of the Barrier Options

4.3.1 Barrier options price

The discounted payoff the barrier options can be calculated using the same process for the Asian options. And from the maximal error, this paper indicates that the range of the up-and-out call options price estimation among all simulations is from 3.1114 dollars to 4.195 dollars and the price range of up-and-in call options is from 2.2905 dollars to 3.9019 dollars. Also, the price estimation interval indicates the possible options premiums from the simulation and help investors to make better decisions. Notice that these four barrier options have lower premiums compared with the Asian options and European options. One reason would be that the barrier level limits the opportunity to earn profit.

Table.3 The discounted payoff of barrier options

	Up-and-out call	Up-and-in call	Down-and-out call	Down-and-in call	Asian options	European options
Average price	3.6590	3.1000	3.8420	2.2408	4.4714	6.7590
Discounted payoff	3.6545	3.0962	3.8497	2.2453	4.4659	6.7507
Max.error	0.5405	0.8057	0.5455	0.7269	0.5775	0.8714

4.3.2 Payoff diagram

As shown in Figure 5, there is a clear sudden drop at the up barrier. The payoff would keep rising like the European call options before the underlying asset touch the up barrier. As shown in Figure 6, there is a clear sudden rise at the barrier level since the payoff would be 0 unless the price reaches the up barrier. The other two payoff diagrams of the barrier put is the opposite of the barrier call.

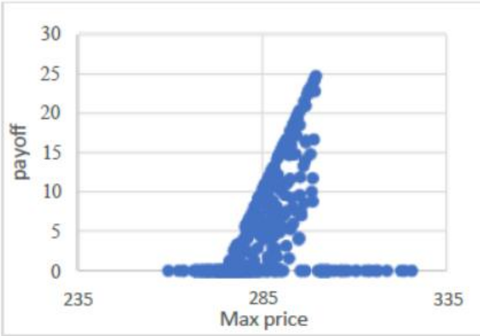

Fig. 5 Payoff diagram ofup-and-out call

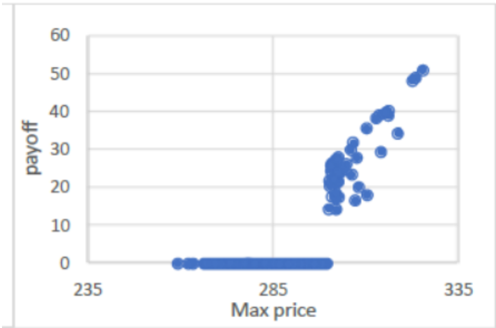

Fig. 6 Payoff diagram ofup-and-in call

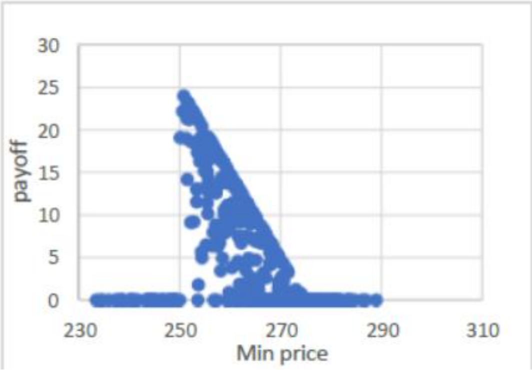

Fig. 7 Payoff diagram of down-and-out put

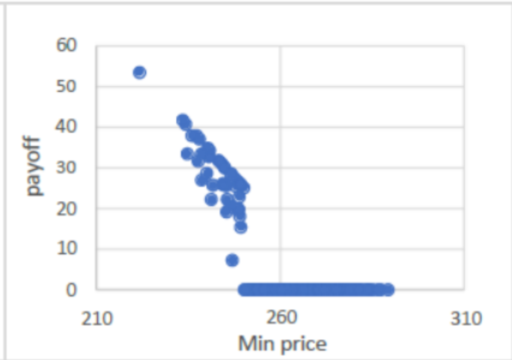

Fig. 8 Payoff diagram of down-and-in put

4.3.3 Sensitivity analysis

(1) volatility

As shown in Figure 9 and Figure 10, the up-and-in call and down-and-in put options have a positive linear relationship with volatility while the up-and-out call and down-and-out put options show the negative linear relationship. When the volatility goes up, the underlying asset is more likely to reach the up barrier and the options premium ofup-and-in call would go up. However, the up-and-out call option is more likely to become ineffective and the payoff would go down as the volatility increasing. Also, the underlying asset is more likely to reach the down barrier and the down-and-in put options premium would increase as the volatility increasing. However, the down-and-out put option payoff would decrease as the options turning invalid

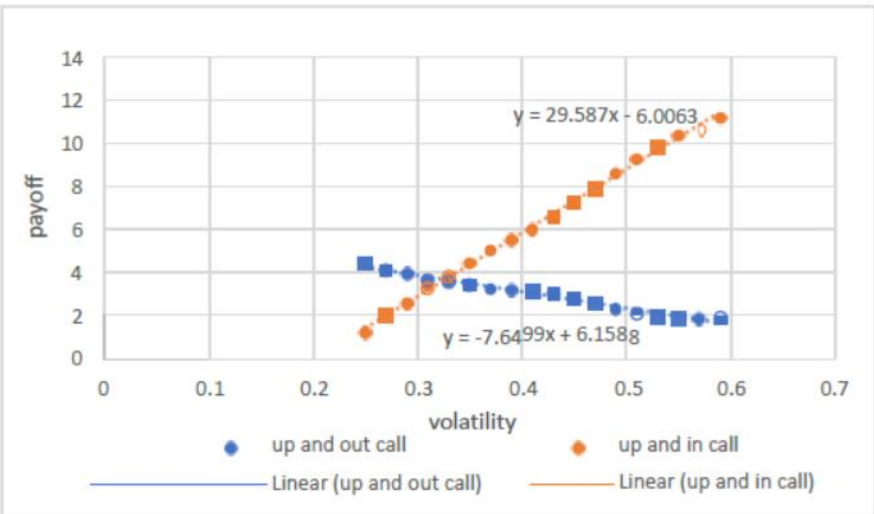

Fig. 9 Barrier call option’s sensitivity to volatility

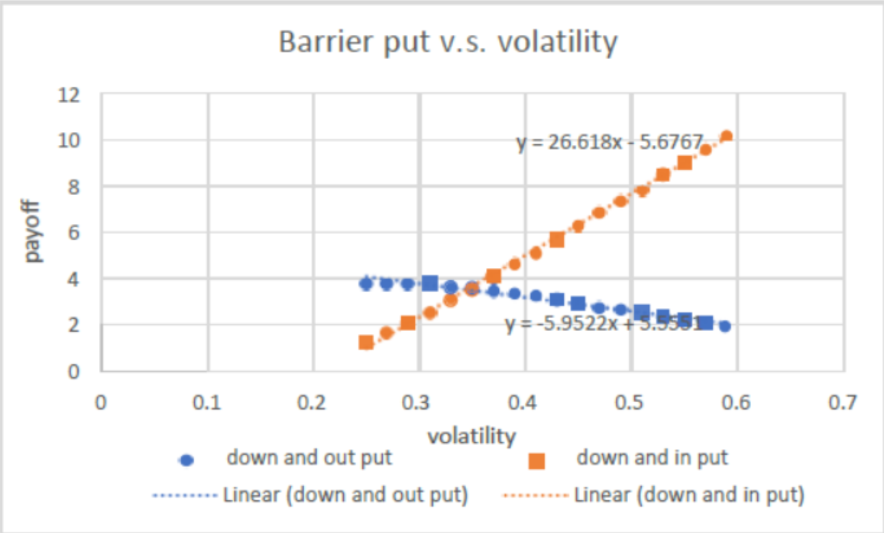

Fig. 10 Barrier put option's sensitivity to volatility

(2) strike price

As shown in Figure 11, the up-and-out call and the up-and-in call have the negative relationship with the strike price. The reason is that the barrier levels of these two call options are set above the strike price, so with the barrier level unchanged, the payoff would be lower as the strike price increases. Also, this paper indicates that the up-and-out call option is more sensitive to the strike price due to the predetermined barrier level. As the strike price decrease by 10 dollars, the option premiums of the up-and-out call and up-and-in call would approximately increase by 2.954 dollars and 0.95 dollars respectively. As shown in Figure 12, the down-and-out put and the down-and-in put have the positive relationship with the strike price. The barrier levels of these two put options are set below the strike price, so with the barrier level unchanged, the payoff would be higher as the strike price increases. As the strike price increase by 10 dollars, the options premiums of the down-and-out put and down-and-in put would approximately increase by 5.35 dollars and 0.745 dollars respectively.

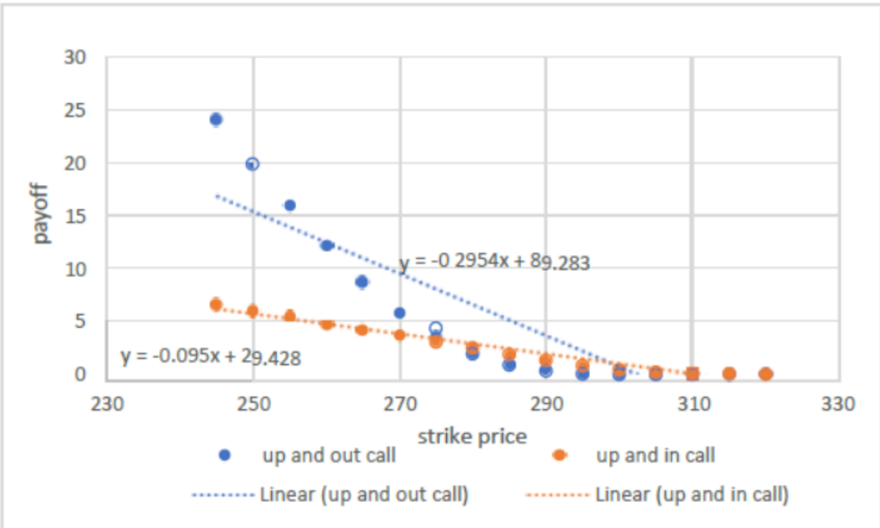

Fig. 11 Barrier call option's sensitivity to strike price

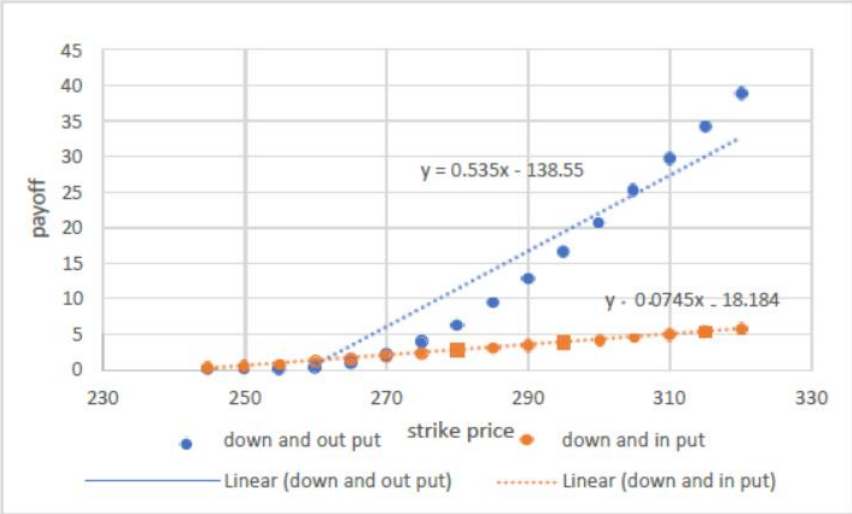

Fig. 12 Barrier put option's sensitivity to strike price

(3) up/down barrier price

As shown in Figure 13, the payoff of the up-and-out call option is 0 when the up barrier is below 280 dollars. Because it is easy to touch such a lower up barrier to terminate the options. Also, the option premium stays at 6.75 when the up barrier is above 330 dollars. The reason is that it is hard to reach such a higher up barrier to terminate the options. Then, the up-and-out call option would perform the same payoff as the European call options. Within a certain range of up barrier price, the payoff of the up-and-out call is positively linear correlated with the up barrier, while the payoff of the up-and-in call is negatively linear correlated with it. As shown in Figure 14, the payoff of the down-and-out put option is 0 when the down barrier is above 275 dollars. Because it is easy for the underlying asset to move below such a higher down barrier, and this put option would be terminated. Also, the option premium stays at 6.095 dollars when the down barrier is below 225 dollars. Because it is hard for the underlying asset to move below such a lower down barrier, and this put option would not be terminated. Then, the down-and-out put option would perform the same payoff as the European put option. Within a certain range of down barrier price, the payoff of the down-and-out put is negative linear correlated with the down barrier, while the payoff of the down-and-in put is positively linear correlated with it

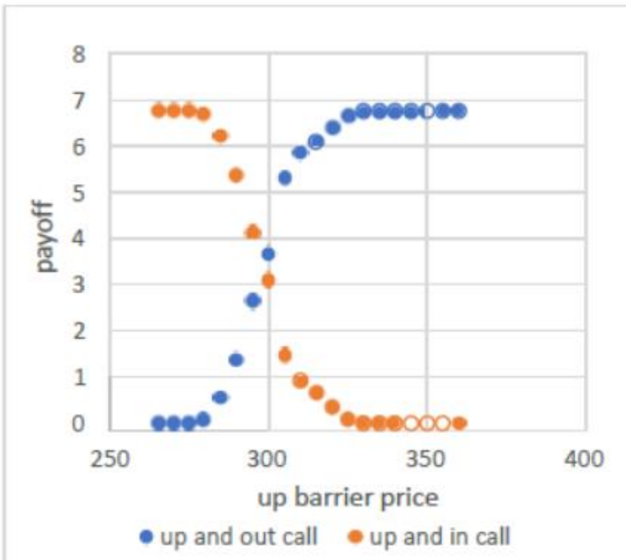

Fig. 13 Barrier call option's sensitivity to up barrier price

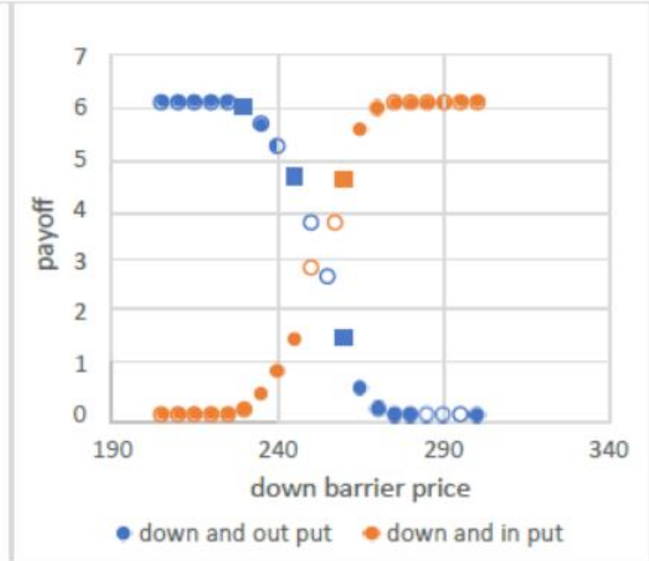

Fig. 14 Barrier put option's sensitivity to down barrier price

4.4 Comparison between the Asian Options and Barrier Options

The result part of this paper shows that four types ofbarrier options have a lower discounted payoff than the Asian options. This paper can reveal the reason through the payoff diagrams. For the up-and-out call option, the payoff would be the same as the Asian options unless the underlying asset hits the up barrier level. For the up-and-in call option, the payoff is 0 when the underlying asset moves between the strike price and the barrier price. Similar with the barrier put option, this paper indicates that the up or down barrier level lower the option price. In addition, both the up-and-in call and the down-and-in put options are more influenced by the volatility than the Asian options, while the other two barrier options are less influenced. However, the down-and-out put option is more affected by the strike price than the Asian options, and the up-and-out call option is less affected while the strike price only has a little influence on the other two options.

5. Conclusion

This paper use Monte-Carlo simulation to evaluate the Asian options and barrier options for the underlying asset Microsoft Corporation. There are three main conclusions: the first is that four barrier options discussed in this paper have lower premiums than the Asian options and European options; the second is that the up-and-in call and the down-and-in put options are more sensitive to volatility and less sensitive to the strike price compared with the Asian options; the third is that the up-and-in call and the up-and-out call options have the opposite relationship with the up barrier, and the down- and-in call and the down-and-out call options have the opposite relationship with the down barrier.

The Monte-Carlo simulation is widely used in the finance industry. By simulating the option price, investors can compare with the actual option price to decide whether it is profitable to buy an option. Also, exotic options like the Asian options and barrier options can satisfy investors' more complex requirements. This method can be applied to price other exotic options such as chooser options, basket options and so on.

There are two limitations need to be discussed. The first is that this paper simulate the price based on the parameters of Microsoft Corporation. It cannot cover other specific industries' characteristics such as volatility. Different underlying assets move randomly with different volatility. The second is that this paper only make a 10-day simulation. The time to maturity of the options usually is much longer than 10 days. The simulation could be completer and more accurate if extending the time to maturity

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