Research On the Pricing of Oil Spread Option Based on Geometric Brownian Motion

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Abstract. On Monday, 21 April 2020, for the first time ever in its existence, the price of US crude oil declined. This study constructs a spread option using geometric Brownian motion and simulates the prices of WTI oil and heating oil, both from the Chicago Mercantile Exchange (CME) Group, and processes and simulates them on a Microsoft Excel basis. This paper predicts the pricing of spread option by the heating oil price and compared to the Asian and European options. The results obtained showed that the volatility of the heating oil price affects the premium of the spread option. The significance of this paper is for investors to compare different options and find profit opportunities to make investment decisions.

Keywords: spread option, geometric Brownian motion, Asian option, European option, WTI crude oil & heating oil.

1. Introduction

The option has been an important derivative financial instrument for investors, speculators, or hedgers. An option contract is one that grants the holder the right—rather than the obligation—to purchase or sell the underlying asset at a predetermined price on or before the maturity date. Its underlying assets can be commodities, futures, securities, foreign exchange, etc. Followed by more investment alternatives, unlike traditional options, exotic options have been developed to meet investors' more customizable demands. Among these options, the research focuses on the Spread option, the European option, and the Asian option.

The Spread option studied is a financial derivative that yields a return based on the difference in prices of two assets, taking the difference in price difference in price between two underlying assets. Recently, many scholars or experts have focused on the valuation and hedging of the Spread option. For example, Dempster, Medova, and Tang (2008) investigated long-term spread option valuation and hedging by two prices method [1], while Duana and Pliskab (2003) developed a valuation theory of co-integrated asset prices using the GARCH approach [2]. Moreover, since taking the spread of underlying assets allows traders to discover the profit margin, several kinds of literature investigate the co-movement between them, such as the co-movement between crude oil and refined oil futures prices (Panos Fousekis, Vasilis Grigoriadis, 2017) [3].

An Asian option is a type of financial derivative in which the payoff at maturity is based on the average price of the underlying assets over a given time period. A European option is a version of an options contract that only allows buyers or sellers to execute the option at its expiration date. There is also much research on pricing around them. [4-9].

For investors and speculators, it's meaningful to compare different options helping to make investment decisions. Thus, in this paper, based on the data of WTI crude oil and heating oil price, this paper introduces geometric Brownian motion to price the Spread option, and then compare it with the European option and Asian option constructed from the heating oil price in the aspect of sensitivity analysis.

The paper is organized as follows. Section 2-3 presents the data used and simulation process. Section 4 shows the visualization simulation results in graphs and discusses the comparisons of three options. Section 5 provides an overall conclusion.

2. Data & Method

2.1 Data

This paper collects the data on heating oil prices, and WTI oil prices from the Chicago Mercantile Exchange(CME) Group, and the data sample is between 1st, August 2021 and 1st, August 2022. The data volume is 249.

3. Results

3.1 Stock price Simulation on WTI oil

West Texas Intermediate(WTI) is a mix of crude oil and can be used to refer to the spot price for that oil. WTI used in this paper represents the mix of oil and its price is the spot price since the data of WTI price serves for predicting the future price.

This paper is based on the price-simulation of both WTI oil and heating oil, and by calculating the standard deviation of each oil's return, which is their daily volatility, and then comparing that to annual ones, the annual volatility for each is figured out.

Table. 1. The collected data of WTI oil price

Date	Price	Return	Date	Price	Return
1	71.26		126	88.2	0.000567215
2	70.56	-0.009823183	127	88.26	0.000680272
3	68.15	-0.034155329	128	90.27	0.022773623
4	69.09	0.013793103	129	92.31	0.02259887
5	68.28	-0.011723838	130	91.32	-0.010724732
6	66.48	-0.026362039	131	89.36	-0.021462987
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122	87.35	0.020443925	247	96.42	-0.008636644
123	86.61	-0.008471666	248	98.62	0.022816843
124	86.82	0.002424662	249	93.89	-0.047961874
125	88.15	0.015319051			

3.2 Stock price Simulation on heating oil

Heating oil is a petroleum product refined from crude oil, which means its cost can be roughly divided into three parts: a.cost of crude oil for refiners; b.cost for refiners to produce heating oil; c. costs associated with marketing, distributing, and delivering heating oil to customers.

To be more specific, in order to simulate the price of the heating oil, this paper first referred to the Chicago Mercantile Exchange(CME) Group, the data in table 2 are the close price for heating oil between 1st Aug 2021 and 1st Aug 2022.

Since heating oil is similar to zero-coupon bonds and derivatives, the annual volatility can be calculated through the continuously compounded yield formula. And the number of trading days between 1st Aug 2021 to 1st Aug 2022 is 249, thus the annual volatility is 0.4766.

Table. 2. The collected data of heating oil

Date	Price	Return	Date	Price	Return
1	56.39		126	72.37	-0.00652363
2	56.14	-0.004401161	127	73.10	0.010105063
3	54.76	-0.024595561	128	74.96	0.02549749
4	55.60	0.015380165	129	75.90	0.012537419
5	55.03	-0.010208927	130	75.38	-0.006851936
6	53.91	-0.020340609	131	73.72	-0.021993416
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121	70.46	0.015871203	246	98.14	0.037222021
122	72.44	0.028061894	247	97.32	-0.008339386
123	73.77	0.01840379	248	93.69	-0.037246019
124	73.54	-0.003220612	249	90.82	-0.030712877
125	72.84	-0.009441752			

3.3 Results of spread option price based on geometric Brownian Motion

After collecting the data of WTI and heating oil, the correlation of the return of these two commodities can be calculated, which is 0.7084. The correlation reflects a relationship between the return of the two commodities. And the correlation can be used to calculate the parameter z(heating oil) if the z(WTI) is fixed as $norminv(rand(\ \ ))$.

$$z_2 = correlation * z_1 + norminv(rand(\ \ )) * \sqrt{1 - correlation^2} \quad (1)$$

Replacing the z_1 with z(WTI), then z_2 is decided. Using the geometric Brownian motion, replacing S_0 with the current price of each oil, that is the price on 1st August 2022. The α is the risk-free rate, which can be referred to as the rate of a five-year US treasury bond. The σ is the annual volatility of each oil; T is the corresponding time span. This paper only focuses on finding out the price of the spread option one year later, and the T can have multiple numbers to choose from depending on the time span.

$$S_T = S_0 e^{(\alpha - 0.5\sigma^2)T + z\sigma\sqrt{T}} \quad (2)$$

The strike price is set as the spread between the current price of two commodities, and then $S_2 - S_1 - k$ is the price for the spread option.

To make sure that the collection of spreads fits the common situation, this model has simulated 1,000 possible spreads and the results are summarized in table 3. It is reasonable to assume that if the price for the spread option is lower than 0, there is no profit gained from buying this option. Then the graph constructed by the spread of commodities' price and the profit gained can be drawn.

Table. 3. Simulation results of a spread option

	z_1	S_1	z_2	S_2	$S_2 - S_1 - k$
1	0.177454642	94.15211913	-0.399507137	63.23434236	-27.84377677
2	-0.303281179	74.87138846	-0.253519399	68.75878002	-3.03860844
3	0.258073758	97.84047996	1.230764825	161.1243121	66.35783211
4	0.362064942	102.8123202	0.507251783	106.3864573	6.648137059
5	-0.589209282	65.33225391	-1.24792666	38.86474744	-23.39350648
6	0.561272736	113.0529657	1.972666955	246.6140058	136.6350401
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997	0.419540543	105.6678416	0.89649005	133.0059972	30.41215567
998	0.754614587	123.9665417	1.16833475	155.4553362	34.5627945
999	-1.057667894	52.2582497	-0.896596183	47.54396531	-1.640284389
1000	0.639748057	117.3617727	-0.358399718	64.74341009	-49.54436261

3.4 Results of option price of WTI/heating oil

Observing the spread option itself may not provide us with the full picture of what this option can bring investors compared to other options. So, this study would provide the price simulation of the two-oil using the Asian option and European option, thus contrasting the two options and making a more apparent conclusion.

Asian option, to be brief, is a form of option where the payout is based on the underlying asset's average price over a predetermined time period. That's to say, the profit of the Asian option is the price investors bought the option by subtracting the option's average price over a certain period of time. Firstly, this paper uses the information below to simulate the heating oil price for ten days, with each day simulated 1,000 times, thus making the averaged result more reliable.

European option that can only be executed to its expiration date, that's to say the strike point for the call option price is the last day's price rather than the averaged price.

In this sense, the paper provides the simulated prices for Asian and European options.

Table. 4. Data Asian/European Option Required

r	3.18%
Sigma	57.37%
Spot Price	90.82
T	0.0402(10 days out of 249 days)
Delta	0
Strike Price	90.82

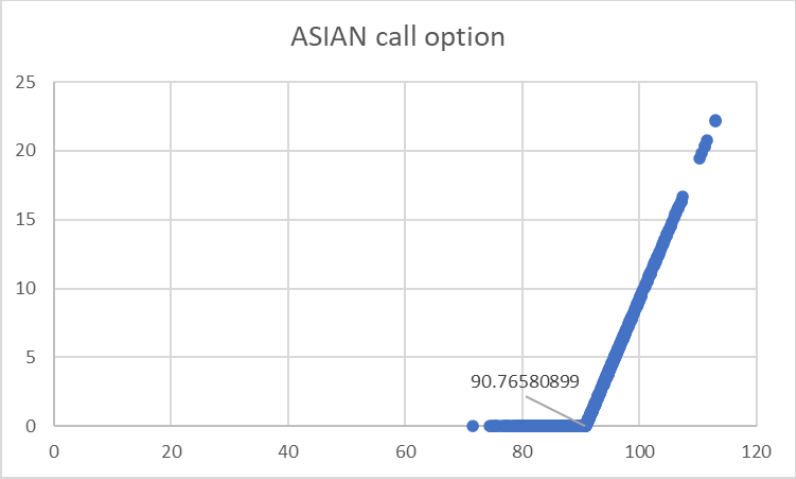

Fig. 1. Simulation of Asian Call Option

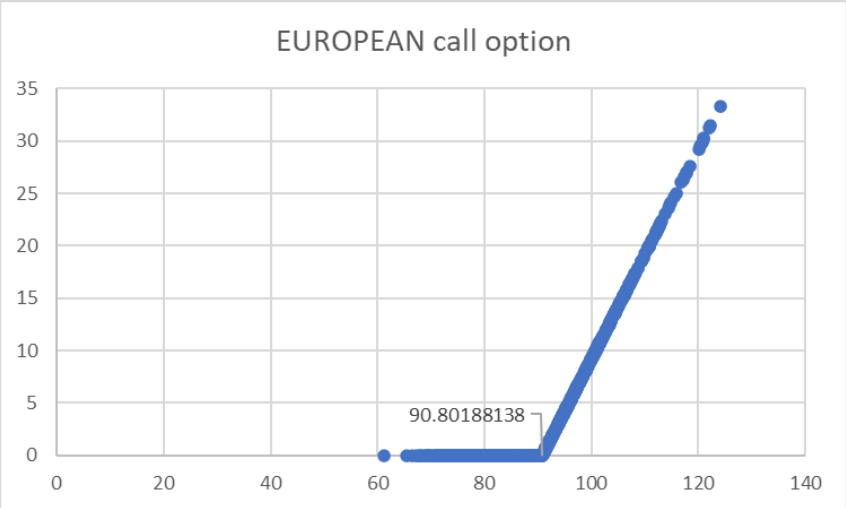

Fig. 2. Simulation of European Call Option

From the Figures1-2, it's easy to find out the turning point for both the Asian option and European option for them to get a positive payoff.

4. Discussion

Then how should investors choose? Should they just buy both and wait for the price to go up and sell them?

In the spread option, investors are given the advice that they can hedge the risk. Since the purpose of buying a spread option is not to gain as much profit as possible. As a matter of fact, the spread option can be a good choice if the future is hard to predict, and the interest rate is not very satisfying. But if the purpose is to make a profit, the spread option may not be a good choice.

Converting data and figures to graphs compares three options more directly. The relationships between three options and strike prices are depicted in Fig.3 and Fig.4, showing how expensive options are for different strike prices. As illustrated in Fig.3, as the strike price of heating oil increases by 1 dollar, the option premium of the European call option declines by 0.7714 dollars, while the Asian call option premium declines by 0.7977 dollars. From Fig.4, it's known that as the spread of heating oil price and WTI price increases by 1 dollar, the Spread call option premium moves down by 0.4091 dollars. Based on the linear relationships contracted from Fig.3 and Fig.4, holding the strike price of WTI constant, when compared to the other two options, the premium for the Spread call option may decline less with each unit increase in the price of heating oil. In this case, investors may prefer to choose the Spread option due to more minor losses.

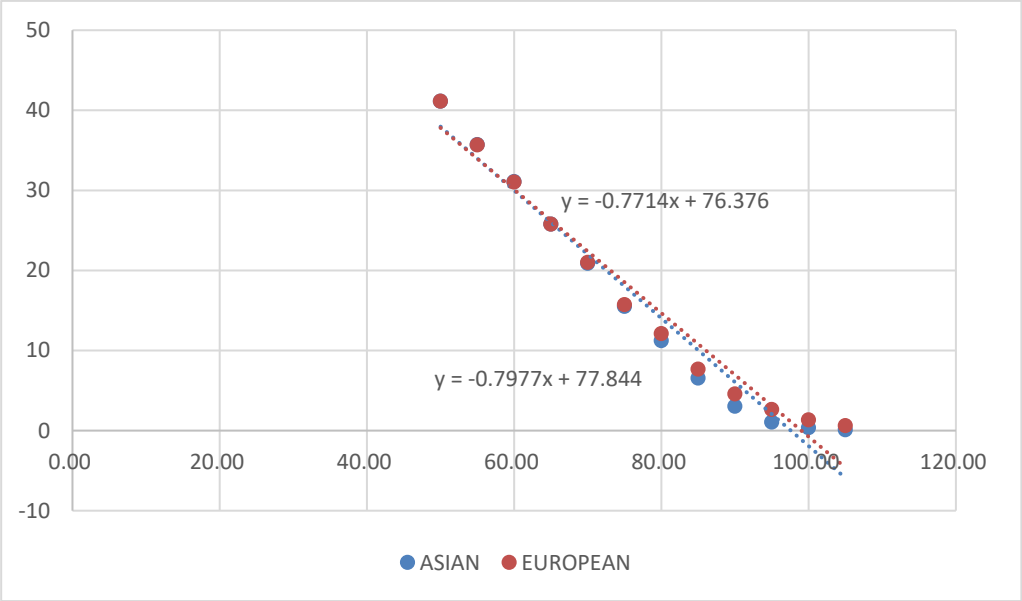

Fig. 3. The relationship between the strike price and Asian, European call option

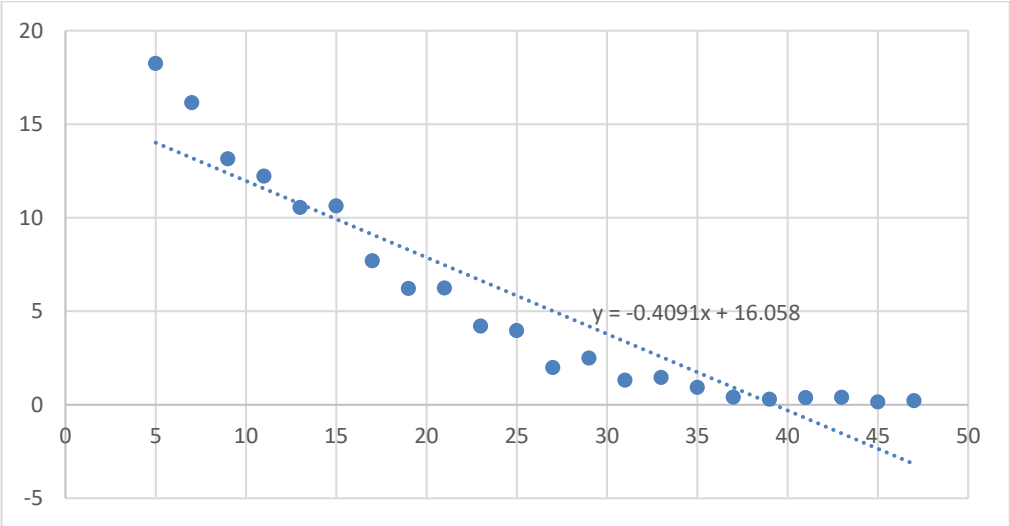

Fig. 4. The relationship between strike price and Spread call option

Subsequently, this paper compares the situations in which three call option premium changes with the volatility of heating oil price as the volatility of WTI oil price is assumed to be unchanged. As illustrated in Fig.5, the line of the European call option reflecting the relationship between volatility of heating oil price and option premium lies above the Asian call option one, and the European option increases faster with volatility. As the volatility of heating oil price rises by 1, the European call option premium can increase by 7.2213, while the increase in Asian call option is 4.4655. The difference in changes in option premium may be due to the nature of the option. For the Asian option, its payoff is determined by the underlying asset's average price over a predetermined time period. In our simulation, the price of an Asian option is averaged by 249 days, which neglects some of the volatility and reduces the effects of volatility on the option premium. The graph of the Spread call option, it's a U-shape curve. This may be because the variance of the spread is a quadratic function. If converting the standard deviation to the dollar and taking the deviation with respect to the volatility of heating oil, a minimum point whose volatility is 0.2526 is obtained. Above that point, the curve is upward-sloping, and one unit increase in the volatility of heating oil leads to a rise of 32.2718 in option premiums. Compared with the European option and Asian option, higher volatility of heating oil price can result in a higher option premium in the Spread option and more chances to get profits.

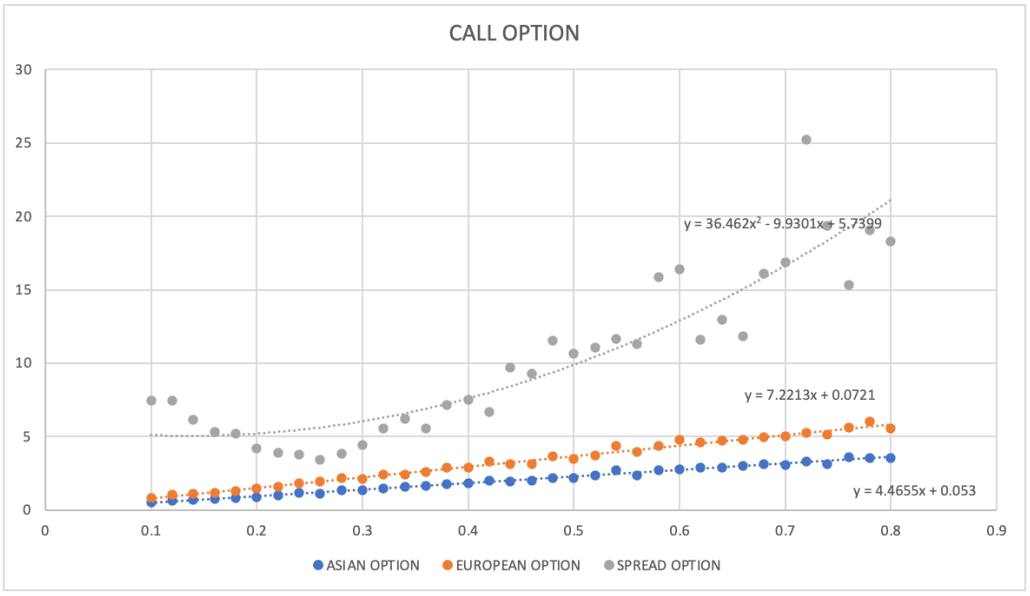

Fig. 5. The relationship between volatility of heating oil and Asian, European and Spread call option

5. Conclusion

This paper applies geometric Brownian motion to construct a spread option using the data of WTI and heating oil. By modelling the price of the two commodities and contrasting the spread option with the Asian and European options made from the predicted heating oil price. This study finds out that: first, holding the strike price of WTI constant, when compared to the other two options, the premium for the Spread call option may decline less with each unit increase in the price of heating oil; second, the difference in changes in option premium may due to the different nature of the option, and holding the volatility of WTI oil price constant, compared with European option and Asian option, higher volatility of heating oil price can result in higher option premium in Spread option and more chances to get profits.

However, this study still has certain limitations. For instant, the volatility of WTI oil price is held constant to examine how the volatility change of heating oil price can influence the option premium of spread option [10-12]. There is still room for further exploration of the practical valuation of the spread option.

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