# Pricing of Asian Options on Tesla Based on BSM model

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**Abstract.** Due to the interaction of the changes in the world political situation and the influence of the global epidemic, the world economy has entered a period of instability and turmoil. This can be an opportunity for people who would like to make an investment. However, without adequate research and good preparation, they may face huge losses. Therefore, this research shows how to choose a reasonable investment approach for a particular market. To be more specific, the research topic of this paper is to show how Asian put options work well in Tesla Market, and the methods of this study include the BSM model, sensitivity analysis, and making a comparison of the Asian options with the European options. Then, the results could be seen that the Asian options will be a more affordable and less risky investment strategy for the Tesla market which is a market with high volatility. Thus, to invest in a market in the current society, it is better for investors to make a clear understanding of their target income and then make multifaceted research. As a result, the investor could reduce their unnecessary losses.

**Keywords:** Tesla; Asian put option; BSM model; Sensitivity Analysis.

## 1. Introduction

Asian options have shown great success in the marketplace. It is useful in avoiding market manipulation and it is a good hedge for stocks and futures. In order to investigate the pros and cons of the Asian options, this study uses European options as a comparison. Asian options' premium is calculated based on the average price of the underlying asset over a certain period. The maturity is pre-set and usually no longer than 5 years. European options' premium is based on the price of the underlying at the maturity date. Compared to European options, the underlying asset's price changes in the last day gives less effect on the premium of Asian options and this is the reason why Asian options could reduce the risk of market manipulation. Fusai and Meucci (2008) analyzed the reason behind Asian options could avoid market manipulations in their research [1]. The using of average price of underlying asset in Asian options leads to higher resistance to risks. To analyzing the price of Asian options, several approaches were used in previous research. Many researchers developed their pricing model based on Black-Scholes model, since it is the most common-used pricing formula. Rogers and Shi[2], Turnbull and Wakeman[3] all introduced using Black-Scholes pricing formula to value Asian options [2,3]. However, the hypothesis of BSM model is hard to be reached in practice, the stochastic volatility model which is called SV models have been created. Wong and Cheung[4], Shi and Yang[5], Kim and Wee[6] used SV models on Asian options in their researches [4-6].

In order to investigate Asian options' merits. This study conducts the evaluation of Asian options and European options using the BSM model. And analyzes the sensitivity of options premium based on three different factors: stock price, strike price and volatility. This study chooses Tesla stock as the underlying asset since it has a relatively high volatility. Using the BSM model, this study simulates the Tesla stock price and then calculates Asian options' premium and European options' premium respectively based on the simulated stock price model. This study also calculated options' premiums based on different stock prices, strike prices and volatilities using what-if function. Through this study, Asian options and European options could be compared under objective data and numbers.

In section 2, this study introduces data and researching methods in detail. In section 3.1, the simulation process was stated. In section 3.2, this article compares the option premium and the last day's stock price. In section 3.3, this study conducts sensitivity analysis from 3 different aspects: stock price, strike price and volatility. Section 4 makes a conclusion of this study.

## 2. Data and method

### 2.1 Data

This study uses the Tesla stock's closing prices from August 2021 to August 2022, the data is collected from Yahoo Finance. Closing prices are used as the underlying asset which is shown in Table 1.

**Table 1**. The Tesla stock's closing prices and returns from August 2021 to August 2022

| Date | Close Price | Return | Date | Close Price | Return |
|---|---|---|---|---|---|
| 2009/1/21 | 775.48 | 0.0540 | 2003/1/22 | 1077.60 | 0.2380 |
| 2010/1/21 | 1114.00 | 0.4365 | 2004/1/22 | 870.76 | -0.1919 |
| 2011/1/21 | 1144.76 | 0.0276 | 2005/1/22 | 758.26 | -0.1292 |
| 2012/1/21 | 1056.78 | -0.0769 | 2006/1/22 | 673.42 | -0.1119 |
| 2001/1/22 | 936.72 | -0.1136 | 2007/1/22 | 824.46 | 0.2243 |
| 2002/1/22 | 870.43 | -0.0708 | 7/28/22 | 842.70 | 0.0221 |
| Average | 0.0257 | StDev | 0.1868 | Annual Volatility | 0.6469 |

Then, the original asset data is used to calculate the return of each month, total average return (0.0257) and return the historical volatility of 0.6469, that is shown in column 2 of Table 1.

Moreover, Table 2 shows that the beginning stock price for this study is 842.70, which is the closing price of Tesla stock market on 28th July in 2022. Same as the closing price, Table 2 also includes the risk-free rate, which is the treasury bond's rate for 12 months which is 2.86% from the website of the U.S. department of the treasury that is an official website of the United States Government. Since this study is about a 10-day option, the daily risk-free rate is required as 2.86% divided by square root of 252 which equals 0.18%.

**Table 2**. All the data for this study, including beginning stock price, strike price, volatility rate, risk-free rate, and the time

| Stock | Strike | Volatility (sigma) | Risk-free | Time |
|---|---|---|---|---|
| 842.7 | 842.7 | 0.65 | 0.0286 | 0.0396825 |

### 2.2 Method

This study is about the valuation of a 10-day Asian put option on Tesla stock market. Options are priced based on the BSM model, which is also called "Black-Scholes" model. The BSM model is one of the most important developments in modern financial theory. It is developed in 1973 and the equation and model are named after economists Fischer Black and Myron Scholes who demonstrated in 1968 that a dynamic revision of a portfolio removes the expected return of the security, thus inventing the risk neutral argument. The BSM model is a differential equation widely used to price options contracts. The Black-Scholes model needs five input variables: the current stock price, the strike price of an option, the risk-free rate, the volatility and the maturity time. In this model, future

stock prices were assumed to be lognormally distributed, meaning the future stock price could be represented as a function of "z". "z" is a bunch of normally distributed random numbers. The pricing formula of BSM model is

$$S_T = S_0 e(\alpha - \frac{1}{2}\sigma^2)T + z\sigma\sqrt{T} \quad (1)$$

Where, $S_T$ is the future stock price, $S_0$ is the beginning stock price. $\alpha$ is the growth rate for stocks. $\alpha = r - \delta$, where $r$ is the risk-free rate, and $\delta$ is the rate of dividends of the stock. $\sigma$ is the standard deviation which is also the volatility of the stock. $T$ is the maturity time of the option. The formula shows clearly that the future price is not dependent directly on the expected return of the underlying. This formula is also risk neutrality which means for the purpose of options pricing, the model pretends that there is no risk premium, but the risk premium is already part of the stock price. After building the stock price model, the option price could be calculated by the function

$$C_T = max(S_T - X, 0) \quad (2)$$

Where, $X$ is the strike price. Knowing risk-free rate, random number $z$, beginning stock price, historical volatility, and maturity time, the 10-day price of Tesla stock could be simulated for 1000 times using the BSM model. This study calculates the option premium based on the simulation.

## 3. Result & discussion

### 3.1 Simulation

In addition, Tesla stock does not pay dividends, so when simulating the future prices of Tesla stock using BSM model, α is the risk-free rate. $Z$ is simulated by the function *normsinv(rand ())* in excel. When this function is used, the data of $Z$ will always change, so in Table 4, it is easy to find that this study chooses only one series of $Z$ data by copying and pasting those data.

**Table 3**. Part of the value of Z

| sim | z1 | z2 | z3 | z4 | z5 | z6 | z7 | z8 | z9 | z10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.0078 | -0.6114 | -0.5451 | -0.942 | 0.1189 | 0.9115 | -1.0106 | -0.1161 | 0.0123 | 1.4464 |
| 2 | -0.4036 | 0.9906 | -0.9427 | 2.2637 | 0.5763 | -0.161 | -0.6029 | 0.718 | 0.3703 | 1.2766 |
| 3 | -0.0924 | 1.9152 | -0.4232 | 0.4234 | 0.9261 | -0.463 | -0.0159 | -0.1917 | 0.2128 | 0.4764 |
| 4 | 0.3096 | 0.4645 | 1.1705 | 0.3394 | 0.339 | -2.9957 | -0.1388 | -1.5766 | 0.5449 | -0.6551 |
| 5 | 0.405 | -0.0496 | -0.5266 | -0.1588 | 0.3763 | -0.8917 | 0.4919 | -0.2307 | -0.3136 | -0.3688 |
| 6 | -0.552 | 1.0263 | 0.2369 | -0.3034 | 1.745 | 0.4056 | -1.4009 | -0.1946 | -0.126 | -0.5741 |
| 7 | -0.5137 | 0.0857 | 1.2946 | -0.2608 | -0.3067 | 0.3213 | -0.0813 | 0.1122 | 1.4696 | -2.473 |
| 8 | 0.5581 | 0.9262 | 0.841 | -0.9229 | 0.1784 | 0.2942 | -0.6662 | -0.0994 | -0.1259 | 0.9678 |
| 9 | 2.3386 | -0.7078 | -0.1453 | 0.7802 | -1.6111 | 1.0341 | -1.4298 | -1.3818 | -2.3804 | -1.2594 |
| 10 | -0.2314 | 2.5623 | -0.6086 | 0.0558 | -1.7196 | 1.1319 | 2.4015 | 0.3023 | -0.3641 | 0.2442 |

According to the beginning stock price that is chosen as 842.7, after collecting all these factors mentioned above, a 1000-times simulated 10-day stock price model could be seen in Table 5 by using the formula of beginning *stick price * EXP ((risk-free rate – 0.5 * volatility rate^2) * (1 / 252) + Z * volatility rate * SQRT (1/252))*. The *Z* in this formula means that while calculating the data of Day 1, $Z_1$ should be used.

**Table 4**. Part of 1000-times simulated 10-day stock price model

| Day 1 | Day 2 | Day 3 | Day 4 | Day 5 | Day 6 | Day 7 | Day 8 | Day 9 | Day 10 |
|---|---|---|---|---|---|---|---|---|---|
| 877.4 | 855.2 | 835.81 | 803.76 | 807.08 | 837.03 | 802.68 | 798.32 | 798.14 | 846 |
| 828.36 | 861.86 | 828.79 | 908.23 | 929.15 | 922.41 | 899.37 | 925.41 | 938.81 | 988.24 |
| 838.93 | 906.38 | 890.24 | 905.09 | 939.23 | 921.01 | 919.76 | 911.94 | 919.23 | 936.58 |
| 852.79 | 868.46 | 910.24 | 922.26 | 934.41 | 826.44 | 821.19 | 769.53 | 786.25 | 764.99 |
| 856.11 | 853.77 | 835.04 | 829.06 | 841.27 | 810.67 | 826.49 | 818.17 | 807.2 | 794.59 |
| 823.36 | 857.92 | 865.62 | 854.37 | 916.68 | 931.29 | 878.98 | 871.41 | 866.33 | 845.69 |
| 824.65 | 826.94 | 871.12 | 861.29 | 849.98 | 860.56 | 857.1 | 860.41 | 912.86 | 824.76 |
| 861.47 | 893.96 | 924.47 | 889.71 | 895.56 | 905.71 | 880.82 | 876.63 | 871.52 | 905.93 |
| 926.3 | 899.32 | 893.37 | 921.57 | 862.39 | 898.87 | 847.38 | 800.41 | 725.89 | 689.08 |
| 834.19 | 925.34 | 902.03 | 903.43 | 841.69 | 880.79 | 970.65 | 981.98 | 966.83 | 975.8 |

The strike price was also chosen as 842.7, which is the same as the beginning stock price, since it could enlarge the advantage of choosing Asian put option. According to the data above, this study uses the 10th day's stock price to calculate the average stock price of the 10 days and each value of Asian and European put option respectively by using the MAX formula, and then calculate the premium for European put option and Asian put option respectively.

**Table 5**. Part of the 10 day's average stock price and each value of the Asian put option and the European put option

| **Average** | **Asian put option** | **Euro put option** |
|---|---|---|
| 826.1412 | 16.5588 | 0.0000 |
| 903.0632 | 0.0000 | 0.0000 |
| 908.8383 | 0.0000 | 0.0000 |
| 845.6546 | 0.0000 | 77.7125 |
| 827.2376 | 15.4624 | 48.1085 |
| 871.1632 | 0.0000 | 0.0000 |
| 854.9679 | 0.0000 | 17.9431 |
| 890.5789 | 0.0000 | 0.0000 |
| 846.4580 | 0.0000 | 153.6228 |
| 918.2740 | 0.0000 | 0.0000 |
| 792.3705 | 50.3295 | 162.1148 |
| 809.4818 | 33.2182 | 90.5975 |
| 661.1177 | 181.5823 | 286.5983 |
| 814.3128 | 28.3872 | 10.1745 |
| 790.8003 | 51.8997 | 34.1047 |
| 775.1877 | 67.5123 | 67.3194 |
| 886.5202 | 0.0000 | 0.0000 |
| 833.8126 | 8.8874 | 35.1638 |

As for the final value of Asian put option and European put option of the Tesla market, the average value of the Asian and European put option are about 26.2463 and 41.6215 respectively, and the discounted price of the Asian and European put option are about 26.2165 and 41.1879 respectively.

### 3.2 Pricing

The discounted price of the Asian put option is 26.2165 and the discounted price of the European put option is 41.1879, so the Asian put option premium is much lower than the European put option premium. Tesla stock with a volatility of 0.65 is more volatile than the average stocks. In general, the more volatile a stock the more expensive the option. But using Asian option, it has a relatively low price compared to the other options. Although the lower price also means lower profit, Asian option reduces the effect of high volatility.

This study also Analyzed the correlation between the Asian put option premium and the last day's stock price and the correlation between the European put option premium and the last day's stock price. Figure 1 shows that Asian put option premium is not perfectly correlated with the last day's stock price and Figure 2 shows that European put option premium is perfectly correlated with the last day's stock price. Asian put option premium is priced based on the average stock price as mentioned before. As a result, when there are big changes for Tesla's stock price on the last day, Asian option premium will not be affected as much as the European option premium.

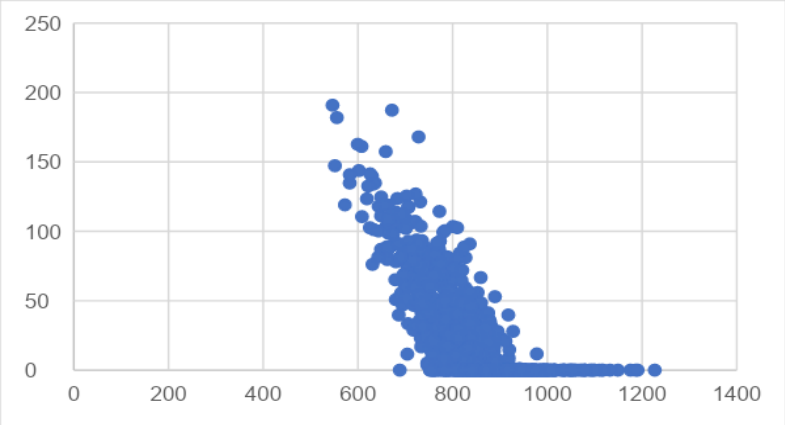


**Fig. 1** Asian put option v.s. 10TH day Tesla stock price

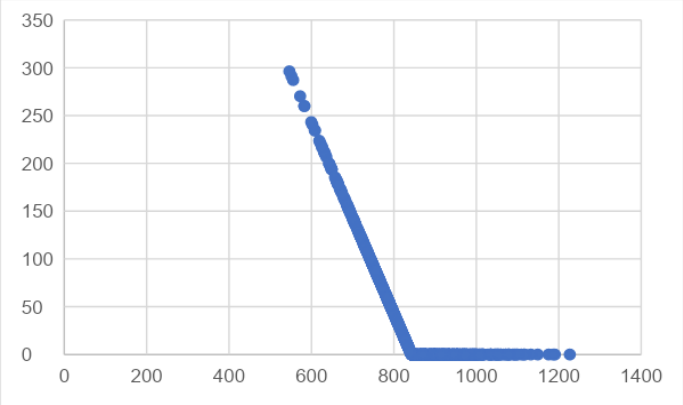


**Fig. 2** Euro put option v.s. 10TH day Tesla stock price

### 3.3 Sensitivity analysis

As for the three Sensitivity Analysis, this study uses the What-if table to find out the relationship between option premium and three essential option factors: stock price, strike price and volatility, so that the investor could know why they should choose Asian put option for the Tesla market more

easily and straightforwardly. For the three Sensitivity Analysis, this study chooses to range the strike price from 700 dollars to 1100 dollars, with the interval of 20 dollars.

### 3.3.1 Stock price

When visualizing the result, a linear relationship between the beginning stock price and the premium of options could be found as Figure 3 shows. In the range of the beginning stock price from 700 to 1100, the premium of both Asian put option and European put option are linearly correlated with the beginning stock price. Figure 3 suggests that the higher the beginning stock price, the higher the premium of both Asian option and European option. The orange line represents the European put option and the blue line represents the Asian put option. The slope of the orange line is higher than the slope of the blue line which means the European option premium is affected more by the beginning stock price than the Asian option premium. This also represents that Asian option is less sensitive to short term risks.

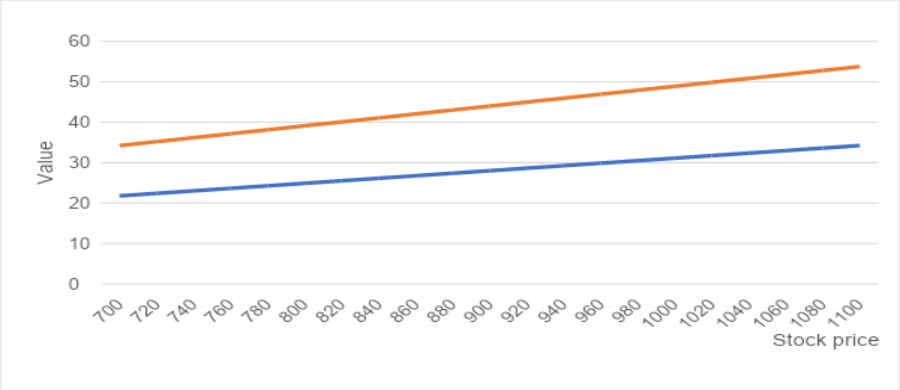


**Fig. 3** Tesla stock price v.s. put option premium

### 3.3.2 Strike price

Strike price, which is also called exercise price, is one of the most essential elements in the options market, especially for valuing the option. It is the transaction price while purchasing or selling an asset. For the call options, strike price means that investors could purchase the asset at this price, and for the put options, strike price means that investors could sell the asset at this price. In this research, the strike price of the Asian put option and the European put option is studied to find out why Asian put option is more suitable for the current Tesla market. To compare the two options, the study includes a sensitivity analysis for strike price.

In this analysis, the strike price and the option premium of the Asian put option and European put option are compared by making an What-if Table under the present value made by the previous value study for the two options.

The relationship between the stock price and the strike price is that the difference of them will determine the value of an option, whatever the call options or the put options. Therefore, the strike price is largely determined by the stock price.

For the put options, while the stock price goes below the strike price, the investors could earn profits. This means that when the strike price is constant, the more the stock goes down, the more the investor could earn, since the difference between them will be growing. However, this does not mean that investors should choose the strike price as high as possible, which is due to the fact that the premium of a put option and the strike price of this option is growing proportionally. This could be found out in the table that when the strike price is increasing, the value of both put options is growing to, whatever the Asian option or the European option.

Thus, making a good choice for the strike price will help the investors to balance their investment cost and the profits they can make. For the Tesla market, in order to compare the two options, several characteristics need to be concerned: the trend of the stock price of the market, the profits made under the same conditions and the option premium under different strike prices. The previous research of

the value of the two options has proven that although the Asian option may not make as many profits as that of the European option, it will still be suitable and profitable for the investors who have lower expected payoff. To prove more about this point, the analysis of strike price could show the advantages of choosing the Asian option, since it is more affordable while the strike price is close to the stock price. Excluding the consideration of profits, the needed strike price will be the one with less cost under the same circumstances, which requires to consider the option premium algorithm of two options. Therefore, the graph that is made by the data from the what-if table of the research could show that when the strike price is very low or very high, the option premium of the Asian put option and the European put option is almost the same. The only difference between those prices, which is not close to spot price, is that when strike price is very low, both put options are not profitable and that is why they are cheap. Moreover, for put options, it is almost impossible for the price (average) of the Asian option to go below the strike price, but for the European option, it will be more likely to earn more because fundamentally it is necessary that the stock just dump in value before expiration. Although this is also a rare condition, it would be more likely than average price being under the strike. Also, when the strike price is very high, both put options are very expensive, which means there will also be no cost advantage for the Asian option. Only when the strike price is equal or close to the current spot price, the option premium of the Asian option is much lower than that of European options, which means investors could take advantage of Asian option with a cheaper price.

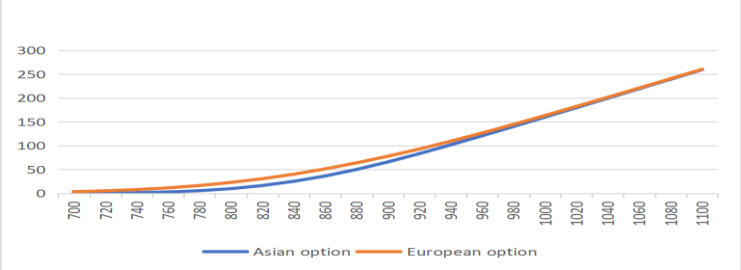


**Fig. 4** Option premium at the different strike price for Asian option and European option

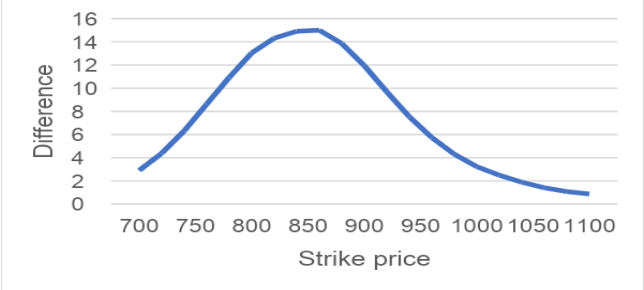


**Fig. 5** The notional difference of the option premium between Asian and European option

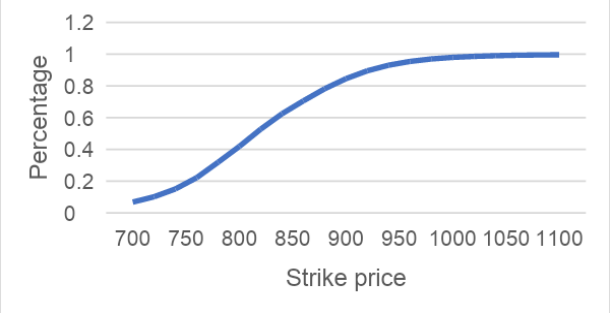


**Fig. 6** The percentage difference of the option premium between Asian and European option

### 3.3.3 Volatility

This section continues to analyze the sensitivity analysis of volatility. As shown in figure 7, the slope of the Asian option is 40.99 and the slope of the European option is 64.68.

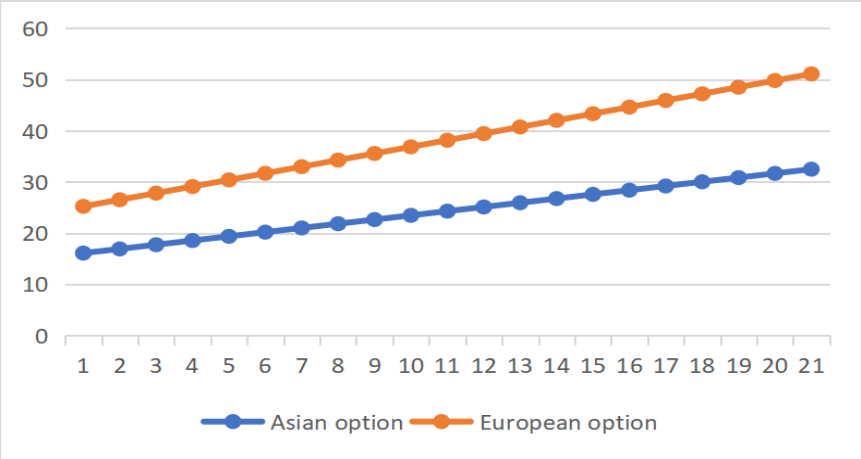


**Fig. 7** The sensitivity analysis about volatility of Asian option and European option

First of all, the data shows how the option price changes when the volatility increases. The slope shows the degree to which volatility affects prices. The European option is more expensive and it increases faster with the volatility. The line shows that the volatility affects the European option more than the Asian option. The slope is lower and the value is lower, so the Asian option is lower risk and lower sensitivity to volatility changes.

Secondly, because the Asian option chooses the averaging price, it may be less affected by the uncertainty of the market. As the investor is in the stock market, it may be more suitable for them to choose the Asian option when they can't predict the future price, and it also helps them to avoid the risk.

Finally, the reason why Asian options are more popular than European options is that the Asian option choosing the average price can be better at reducing fluctuations and reducing the risk. That's why Asian option's price is usually cheaper than the other option's price. Anything that reduces the outset price of the options contract can make the Asian option more popular.

Above all, to meet the needs of kinds of investors, Asian option is better for the investors who prefer buying the option at a lower price and meet less risk at the investment because the use of averaging price reduces the volatility.

Scholars and practical financial workers show that, Volatility can be described as the speed and magnitude of the price movement of the underlying asset. In the valuation of options, it can also be said to be a calculation of the uncertainty about the future price movements of the underlying asset [7]. High volatility represents large fluctuation, and higher volatility would cause more significant difference between the Asian option and the European option. That's why the price of Asian options are cheaper than their European counterparts, in the case when the stock price fluctuates a lot [8]. Hansj¨org Albrecher (2022) showed that statically hedging an Asian option in terms of a portfolio of European options is a good alternative to other methods which is easy and quick to be carried out. [9]. Moreover, compare to the most of the existing techniques, this approach is applicable in general market models whenever the risk-neutral density of the asset price distribution or an approximation of it is available [10].

## 4. Conclusion

In conclusion, this study shows the reason for choosing the Asian put option for the Tesla market according to the BSM model and three sensitivity analyses: stock price, strike price, and volatility. According to the results, it could be known that Tesla's stock market is on a downward trend, so it is better to choose the put option. Furthermore, Tesla is an investment with relatively high returns, but its original cost is also quite large, so choosing the Asian option could help people to reduce costs compared with the cost of the European option. Moreover, Tesla is a market with high risk and high volatility. Asian options can neutralize certain risks by using the average price method. Therefore, in

general, under the decreasing trend of Tesla stock price, the Asian put option will be a cheaper and safer investment strategy due to the high volatility and high cost of Tesla investment. Today's financial situation is very turbulent, which leads to high investment risk and an unstable economic situation, Thus, it is better for the investors to make more comprehensive preparations, by understanding various characteristics of the market and investing strategies, especially for small and medium-sized investors.

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