Second-Hand House Valuation Based on Binomial Option Pricing Model

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Abstract. More and more families choose to buy the second-hand houses as their first house. Due to a high demand in second-hand housing market, there is a continuously rising trend in the price of second-hand houses. This article mainly focus on the valuation of second-hand housing in Chinese market, based on the binomial option pricing model. Besides valuing the second-hand houses as the American-style options, the time value is another important element in the pricing model. Based on the defined pricing model, the result of empirical analysis is that the estimation of current housing price has a difference from the actual transaction prices to some extent. It can be concluded that the current price of second-hand houses in Chinese market is far from reflecting the time value, and the time value of the second-hand houses is unable to be ignored in the future. It is hoped that there will be a more precise way to measure the value of time in the second-hand housing market specifically.

Keywords: Second-hand housing price; Binomial option pricing model; American-style option.

1. Introduction

In the minds of Chinese people, the house is an extremely important concept in their life. Many people can be considered willing to buy a home. With the rapid growth of China's population and the rapid development of China's economy, coupled with the operation of real estate companies, housing prices are getting higher and higher. In first-tier cities like Beijing and Shanghai, many people may never be able to afford a new home for the rest of their lives. Driven by the pressure of survival and the desire to buy a house, coupled with the current lottery purchase policy, many people turn their attention to the second-hand housing market.

Second-hand homes have several advantages over first-hand homes. The first advantage is that the school district of second-hand housing is relatively stable and will not change easily. The second point is that currently second-hand houses are mainly concentrated in urban areas. At the same price, second-hand houses have better surrounding supporting facilities and are more cost-effective. Third, the first-hand houses are less liquid in terms of delivery time, and cannot be moved in immediately after renovation. However, second-hand houses need to consider many factors compared with first-hand houses, including the age of the house and surrounding facilities (e.g. schools, public relations, and museums), which will affect the price of second-hand housing. Therefore, It is vital to figure out how to quantify the price of second-hand houses and the relevant indicators.

In finance, the two most widely used pricing models are Black-Scholes model and Binomial tree pricing model [1]. In 2012, Gao explained how to adjust the price of second-hand homes based on the amenities surrounding them [2]. Mainly from the area, floor, economic environment and other aspects to consider the general situation of the house price, and then in a more specific correction coefficient to the house price correction to make our second-hand house price is the most accurate [3]. This pricing method fully takes into account the value of the house's own condition and the value of the surrounding environment. In 2018, Xia, Wei and Wang proposed a binary tree pricing model for housing demolition compensation pricing [4]. In this literature, the price of the demolished house is treated as an American option. In the empirical analysis process, the traditional pricing method is used to evaluate the value of the house itself, and then the binary tree model is used to evaluate the time value of the house. The result of this paper is the same as the result of this paper, that is, the present house price is far from reflecting the future time value of the house. The essence of demolition

house is also second-hand house, so second-hand house is a typical American option [5]. Second-hand houses can be bought and sold freely before the expiration of property rights [6]. For American options, the binomial option pricing model is the most applicable pricing method. The binomial option pricing model can simulate the possible change trajectory of the underlying asset price in each time period caused by the rise and fall of the underlying asset price in a given time interval, so the price gradient operation of any previous node can be obtained by reverse operation [7]. The future time value of a house can be calculated by discounting future house prices [8]. This paper selects the second-hand housing in Chengdu as the object of empirical research. In the traditional pricing method, the binary tree is used to carry out the empirical simulation, so that the future time value of the housing can be fully considered in addition to the value of the housing itself. Finally, this paper compares the second-hand house price simulated by the binary tree with the actual transaction second-hand house price to judge whether the current second-hand house price can fully reflect the value of the second-hand house.

2. Methodology

2.1 Binomial Option Pricing Model

Associated with the paper by Cox, Ross, and Rubinstein in 1979, the binomial option pricing model assumes that the stock price fluctuates only in two directions, up and down [9]. In this assumption, the probability and magnitude of each up or down fluctuation of the stock price remain unchanged during the entire investigation period. The model divides the examined duration into several stages, simulates all possible development paths of the second-hand house during the entire duration according to the historical volatility of the second-hand house price, and calculates the warrant exercise income for each node on each path using the discounted method, specified in Function (1).

$$P = e^{-rT}(pS_u + (1-p)S_d) \tag{1}$$

Where P is the final price of the asset, S_u is the asset price after the price goes up, S_d is the asset price after the price goes down, p is the probability of the price goes up, $1-p$ is the probability of the price goes down, T is one single period, and r is the discount rate, which equals to the risk-free rate according to the Risk-Neutral Pricing Theory.

2.1.1 Single-period binomial tree

For a single period, the asset price and the option price can be specified as Figure 1, where S_0 is the asset price at time $t = 0$.

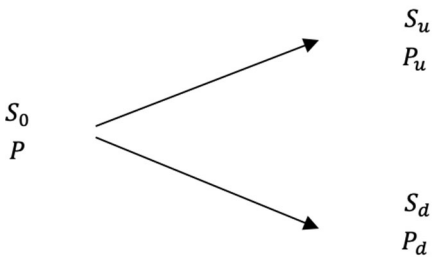

Fig. 1. Single-period binomial tree

In the Figure 1, if the price of the house goes up, the price of the trading portfolio at the option expiration date can be specified as the Function (2).

$$uS_0 \triangle -\mathrm{P}_u \tag{2}$$

Where $\triangle$ is a trading portfolio of the long position of an option or a short position of an option, and u is the ratio of the asset price goes up. If the price of the house goes down, then the price of the trading portfolio at the option expiration date can be specified as the Function (3).

$$dS_0 \triangle - \mathrm{P}_d \tag{3}$$

Where d is the ratio of the asset price goes down. Assuming no probability to make profit without risks, Function (2) and (3) are equal, and $\triangle$ can be calculated by the Function (4).

$$\triangle = \frac{P_u - P_d}{uS_0 - dS_0} \tag{4}$$

Due to the $\triangle$ has been found, which is represented by the $u, S_0, \mathrm{P}_u, d, \mathrm{P}_d$. $\triangle$ can be substituted back into the Function (1) and get the expression of p, as shown in the Function (5).

$$p = \frac{e^{rT} - d}{u - d} \tag{5}$$

In the Function (5), p means the probability that house prices will rise in a risk neutral situation, and $1 - p$ means the probability that house prices will fall in a risk neutral situation. Moreover, $pP_u + (1 - p)P_d$ means the expectation of option payoff in the case of risk neutral. Through the definition of the variance, the variance can be represented as the Function (6).

$$Var(V_0) = E(V_0^2) - [E(V_0)]^2 \tag{6}$$

The variance can be calculated in the Function (7) by substituting p, u, d.

$$\sigma^2 t = pu^2 + (1 - p)d^2 - [pu + (1 - p)d\}^2 \tag{7}$$

Where σ means the volatility of house prices. According to the assumption that $ud = 1$ and the risk-natural condition, the price center of house prices remains unchanged. Then, u and d can be calculated by the Function (8) and (9), respectively.

$$u = e^{\sigma\sqrt{t}} \tag{8}$$

$$d = e^{-\sigma\sqrt{t}} \tag{9}$$

2.1.2 Multi-period binomial tree

Assuming that the single-period binomial tree pricing model is applied at the end of each time node as specified in Function (10), then multiple single-phase binomial tree pricing models can be integrated into a more-phased pricing model, and the probability of a single-period binomial tree fluctuation is the same as the probability of a multi-period binomial tree fluctuation.

$$\Delta t = \frac{T}{n} \tag{10}$$

After calculating the value of the corresponding node of the NTH stage terminal, the option price can be defined as the Function (11), where K is the strike price of the option.

$$P = max(S_T - K, 0) \tag{11}$$

At time $t = T$, which is also the point at Binomial step n, the option price at i_{th} column and j_{th} row, P_{nj}, can be defined as the Function (12) in each trail.

$$P_{ij} = \frac{\sum_{j=0}^{n}[C_n^j p^j (i-p)^{n-j} max(S_0 d^{n-j} u^j - K, 0)}{e^{rt}} \tag{12}$$

In terms of second-hand housing price, it should not only consider the current fair value of second-hand housing, S_0, the time value also needs to be considered as well, as the house may increase in value or depreciate over time. Therefore, the second-hand housing price can be defined as the Function (13).

$$S = S_0 + P_n \tag{13}$$

2.2 House Price Correction Model

In reality, the price of real estate is formed by the comprehensive effect from several elements, such as the decoration, the floor, the distance from the school, the surrounding development, the supporting conditions of the hospital [10]. Therefore, the current price of second-hand houses needs to be corrected to a more accurate result, as the Function (14).

$$\begin{aligned} Correction\ value\ &= case\ transaction\ price \\ &* transaction\ time\ correction\ factor\ K1 \\ &* transaction\ situation\ correction\ factor\ K2 \\ &* regional\ factor\ correctionfactor\ K3 \\ &* floor\ area\ ratio\ correction\ factor\ K4 \\ &* FRA\ correction\ factor\ K5 \\ &* remaining\ life\ correction\ factor\ K\ 6 \end{aligned} \tag{14}$$

Specifically, K1 and K6 are defined as the Function (15) and (16). K2, K3, K4 and K5 will be viewed as 1 because information about their trading situation is unknown in the same community.

$$\begin{aligned} K1\ &= (1 + month - on - month\ increase\ rate\ of\ land\ prices \\ &* (the\ number\ of\ months\ that\ have\ elapsed\ between\ the\ transaction\ time \\ &\qquad of\ the\ case\ and\ the\ transaction\ time\ of\ the\ parcel\ to\ be\ assessed) \end{aligned} \tag{15}$$

$$K6 = \frac{(1 - (+land\ restoration\ interest\ rate)\text{^}(remaining\ life\ of\ the\ parcel\ to\ be\ assessed))}{(1 - (1 + land\ restoration\ interest\ rate)\text{^}(-remaining\ life\ of\ the\ case\ parcel))\ parcel))} \tag{16}$$

3. Empirical Results

Based on the pricing model defined in the methodology, the data was selected from the second-hand housing market to figure out whether the price of second-hand houses can be valued accurately. Due to the limitation of authority, four typical transactions of second-hand houses in 2020 and 2021 are selected from second-hand housing transaction platform in Chengdu, China were selected for analysis. In terms of the parameters mentioned in the house price correction model, $K1$ can be calculated as follows, through the national land price growth rate shown in the Table 1. Considering the five-year fixed bank deposit rate in the People's Bank of China (2.75%), $K6$ was calculated as 0.908 when the land restoration interest rate viewed as the bank deposit interest rate. In addition, $K2, K3, K4$ and $K5$ are assumed to b1.

After revising the unit price according to the floor, the total present house value, S_0, can be calculated to ¥6,195,187.62. The strike price of the option, K, refers to the cost when investing in a

house. For the buyer of the house when buying the house, the cost price is the price the first owner brought, which is the issuing price of the house. As the unit price is ¥7,498 in 2010, K can be calculated as ¥25,472,220.56. Because the residential area was built in 2010 and has been in use for 11 years until the estimated node, 2021, the remaining validity period T of the unsold house is 59 years. In terms of the rate of return the house buyer can get, the five-year fixed bank deposit rate set by the People's Bank of China (2.75%) can be used as the reference of the risk-free interest rate for valuation. The volatility of house value σ depends on the standard deviation of the fluctuation of local real estate price, which can be replaced by the fluctuation of real estate price in Chengdu from 2016 to 2021, 17.62%.

By performing the programming operation in five-hundred periods, the time value of the second-house price P_n can be calculated as ¥3,647,500. Therefore, the estimated second-hand house price S can be obtained by adding $S_0$and P_n, ¥9,842,687.67. However, the actual transaction price is ¥6,470,000, which means the error rate is 52.12%. Judging from the current situation of second-hand housing transactions in Chengdu, the transaction price is far from reflecting the future time value of the house. Different from the traditional second-hand housing pricing, this paper mainly discusses the time value of second-hand housing before the property rights expire on the basis of conventional second-hand housing pricing, and this part is precisely the neglected part of traditional second-hand housing pricing. Through this huge error, it can be clearly seen that the time value of second-hand houses occupies an important part in second-hand houses, although the proportion may not be as large as the estimated price. Therefore, in the process of pricing second-hand houses in the future, the time value of houses cannot ignored.

Moreover, second-hand housing pricing not only needs to consider the objective conditions of the house in the community, but also needs to include the subjective opinions of buyers, for example, low floor lighting is not good, high floor escape is not convenient, there may be water leakage, the degree of perfection of the decoration, the buyers of the existing deco ration of love and so on. Therefore, how to set a satisfactory price for both buyers and sellers has become a more important issue in the secondary housing market transactions. In the second-hand housing appraisal, because there is no good use of scientific quantitative methods, only based on the experience of the evaluator to judge the score, so that the credibility of the appraisal results, science and objectivity have been greatly affected. Therefore, in order to be able to evaluate the second-hand house scientifically, the influence factors and comparative factors must be considered, and reduce the randomness of the correction. It is hoped that there will be a new pricing method in the future to evaluate the time value of second-hand houses more accurately.

4. Discussion

At present, this binary tree determination model based on the traditional second-hand housing pricing model can only show that the current second-hand housing prices are far from reflecting their proper value, because the traditional second-hand housing pricing does not incorporate the future time value of the house. A reference range for housing pricing. Thus, this model still has some defects. The price simulated by the binary tree includes the time value of the house until the property right expires, which will lead to the house price under the binary tree model for most people possibly affordable price. Therefore, it needs a more accurate model to find a suitable model or indicators to measure the time value of house prices, so that the future time value of houses can be fully reflected in the range of most people's burden. In addition, with the increase of the age of the house and the continuous appreciation of the house, these second-hand houses may fall into an embarrassing situation, that is, no one is willing to buy these old and expensive houses. There are many other aspects that need to be considered in the measurement of the time value of second-hand housing.

5. Conclusion

In terms of limitation, this paper only considers the above objects. The transaction time and the remaining years are two factors, so the changes made to the price of the house during the correction process are not very large, and the factors that affect the unit price of the house in actual life are still not there. The degree of interior decoration of the house, forward, whether it is a school district room, and the desire and urgency of the second-hand housing holder to sell. The consideration factors of this project are still not perfect, so the error is so small according to this, it is very likely that these factors have not been fully considered and perfect, of course, it is not ruled out that the model itself fits well. For example, the problem of house decoration is a problem in the trading market where buyers and sellers of second-hand houses frequently dispute.

Because the house has been renovated or the decoration is relatively high end, the seller hopes that the decoration cost can be added to the seller as the house cost when selling the house. However, the seller believes that the decoration style of the house does not meet their own needs, and therefore does not agree to include the house price in the house decoration cost. In this case, we cannot use the binomial tree pricing model to evaluate the house, but should be used in actual operation. The general substitution principle of real estate price appraisal is handled flexibly based on the principle of highest and best use. Moreover, for different houses in the same city, the water and electricity levels in urban and rural areas are not exactly the same, so the unit price of houses will also vary. In this project, we only considered the Jinjiang District where the property is located. This area will also have many highly end properties and low-end residences. We only took the average value.

Due to the continuous exposure of defects in traditional methods, accurate price evaluations of houses for sale have not been possible, and many disputes have arisen in the trading market, which have attracted widespread attention from the academic community. Therefore, more scholars choose to use the theory of real options to compare traditional methods Be improved. Moreover, for first tier cities such as Beijing, Shanghai, Guangzhou, and Shenzhen, the pace of urbanization is accelerating, which intensifies the uncertainty of housing location factors. Therefore, we need to consider the impact of uncertain factors on the basis of the method of transmission, and the scope of the price evaluation of the uncertain factors to meet the demands of both parties.

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