Comparisons of Different Arbitrage Strategies in Context of Various Models

Xinyu Hou[1, *]

[1]Shanghai Experimental Foreign Language School, Shanghai, China

*Corresponding author: xingzheng@hanguang-sh.com

Abstract. S Statistical arbitrage is one of the key topics of research today due to its significance statistical features and the ability to control the drawdown. Previous researchers have implemented statistical arbitrage based on various approaches. However, some of these approaches lack depth and coherence. This paper therefore provides an in-depth assessment of the two scenarios. Specifically, the two approaches are distance and time-series models. The former one uses non-parametric distance indicators to identify pair trading opportunities, while the latter seeks to find optimal trading rules by examining and combing through the relevant references. The basic principles as well as procedures of them are presented and clarified. According to the analysis, this paper reveals future research directions and the strengths and weaknesses of the approaches. In addition, the current limitations as well as prospects for the improvements of the models are given. These results shed light on guiding further exploration of pairs trading in statistical arbitrage.

Keywords: Pairs Trading; Relative-value Arbitrage; Distance Approach; Time-series Approach

1. Introduction

Pair trading is a market-neutral investment strategy designed to capture and profit from market imbalances [1]. It requires investors to identify financial instruments whose prices are closely correlated. When price movements are greater than a pre-determined amount, the investor should long undervalued financial instruments while shorting overvalued ones. The simplest form of pair trading is the univariate pair trade [2], which involves only two stocks. The idea of pair trading can be further applied to a quasi-multivariable or multivariable framework.

Gatev et al. tested a simple and convincing algorithm on a large sample of US equities [1]. It obtinaed annualised excess returns of up to 11% with avoidable drawdown (i.e., the risks). Such a profitability cannot be explained by the recorded prior to Jegadeesh [3] and Lehmann [4] or the momentum profits of Jegadeesh and Titman [5]. Scholars have been very interested in the business power of matched trades. The previously mentioned study (GGR) [1] concludes that statistical arbitrage will generate high excess returns for investors with relatively low risk. On the other hand, several studies have shown that this trading strategy is unprofitable or time-varying [4].

In addition to the profitability of pair trading, redistributors are interested in different ways of implementing this arbitrary trading strategy. From the published literature, there are four dominant views: distance methods, cointegration methods, time series methods, stochastic control methods. Depending on these different approaches, researchers use different samples to calculate the excess returns of matched trades.

- Distance approach. During formation, distance indicators are used to identify securities that are moving. Pair trades are identified as profitable across a range of markets, asset classes and time horizons.
- Co-integration methods. During the formation period, co-integration tests are used to identify co-mingled securities.
- Time-series approach. All authors in the field assume that a set of moveable securities has been established through prior analysis and therefore usually ignore the formation period.
- Stochastic control approach: The focus is on determining the optimal portfolio holding in a pair of transactions.

Previous studies on matched trading remains scarce compared to inverse and momentum strategies, and there are few representative in-depth studies and aggregated articles. This study has therefore chosen to analyse and summarise in depth the two most common statistical arbitrage models used by economists and financiers in recent times. These approaches have been widely used in finance over the years and have become indispensable arbitrage tools. The contribution of this survey in this context is threefold: firstly, the paired trading literature on these two approaches is screened and a comprehensive review is conducted. In addition, an in-depth assessment of each approach was conducted from the vast literature of dozens of references, ultimately revealing the advantages and disadvantages. To better illustrate the points in this study, following Sections are presented. Section 2 and 3 focuses on distance approach and time series approach accordingly and respectively. Section 4 compares the differences in performance between the two methods and discusses the advantages and disadvantages of each. Section 5 discusses the current limitations of arbitrage research and an outlook on future research directions. Finally, Section 6 summarises the directions for further research and the research implications of this thesis.

2. Distance approach

The distance method was first proposed by Gatev, Goetzmann and Rouwenhorst (GGR) [1]. In their paper they built a sample of all US CRSP stocks from 1962 to 2002. They set a formation period of 12 months and calculated cumulative returns In addition, they calculated the sum of the Euclidean squared distances (SSDs) for all possible stocks. Finally, they selected the top 20 pairs with the lowest historical SSDs and looked at the performance of these stocks over the next 6 months When the spreads differed by more than 2σ, they went long and bearish on the corresponding undervalued stocks When the spreads differed by more than 2 historical standard deviations, they went long and bearish on the corresponding undervalued and overvalued stocks. They will complete the trade when the share price eventually returns to its long-term average level.

Do et al. point out the advantages of this approach, GGR's approach is free of economic models and therefore this approach is considered to have no risk of missed closes and is not subject to model misspecification and misestimation [6]. Another advantage is that the high risk-adjusted returns generated are statistically significant. GGR's model is also easy to practice or replicate. Unfortunately, the selection pairs using the lowest SSD as the selection criterion are not optimal. The reason for this is that the "perfect pair" has an SSD of zero, which contradicts the investor's goal of maximising returns. Suppose the objective of a rational pair trader is to maximise the excess return on each pair of trades. With a constant initial investment, this is equivalent to maximising the profit of each pair of trades. The latter is the product of the number of trades per pair and the profit per trade. The objective of the pair trader is therefore to cause spreads to deviate frequently and strongly from equilibrium and subsequently converge to equilibrium. Furthermore, when spread volatility shrinks, SSD will fall. These two points explain the fact that the chosen currency pair will have low spread volatility and limited profitability.The empirical spread variance can be mathmatically calculated as:

$$s^2_{P_i-P_j} = \frac{1}{T}\sum_{t=1}^{T}\left(p_{i,t} - p_{j,t}\right)^2 - \left(\frac{1}{T}\sum_{t=1}^{T}(p_{i,t} - p_{j,t})\right)^2 \quad (1)$$

After simplifcation by using the average sum, one derives following solution:

$$\overline{S_{P_i-P_j}} = \frac{1}{T}\sum_{t=1}^{T}\left(p_{i,t} - p_{j,t}\right)^2 = s^2_{P_i-P_j} + \left(\frac{1}{T}\sum_{t=1}^{T}(p_{i,t} - p_{j,t})\right)^2 \quad (2)$$

Later, researchers replicated and extended GGR's model. Do and Faff conducted the same study on a sample of identical stocks with longer maturities from 1962 to 2009 [6]. They found that the distance approach was increasingly less profitable, as many pairs would not converge to their historical average. A typical evaluation results of the stock spreads are presented in Fig. 1 and Fig. 2. The situation could be worse if transaction costs are taken into account. Chen et al. replicate GGR's model on the exact same data sample, but use Pearson correlations in the pair identification step. Trading was done on a monthly basis. These improvements resulted in higher average returns for traders. In addition, they compared the returns of univariate pairs trading with those of quasi-

multivariate pairs trading. The results showed that the latter were usually more profitable [7]. The models can be described as follows:

$$d_{i,j,t} = \beta(r_{i,t} - r_f) - (r_{j,t} - r_f) \quad (3)$$

The distance method is also tested using a sample of inventories outside the US. For example, Perlin selected the 57 most liquid Brazilian stocks from 2000 to 2006 [8] and Andrade et al. chose the Taiwanese stock market to replicate GGR's model [9]. Nath applies data taken from 1994 to 2000 from GovPX to construct pair trading [10]. In this study, the authors use the SSD between standardised dirty prices as a selection indicator over a 40-day formation period. These strategies outperformed the benchmark in terms of Sharpe and P&L ratios. Bowen et al. reviewed GGR's strategies for the FTSE 100 component for the period January to December 2007, which returned 60 minutes [11]. The study used a formation period. Considering a trading cost of 15 basis points and an execution interval of 60 minutes can completely eliminate excess returns.

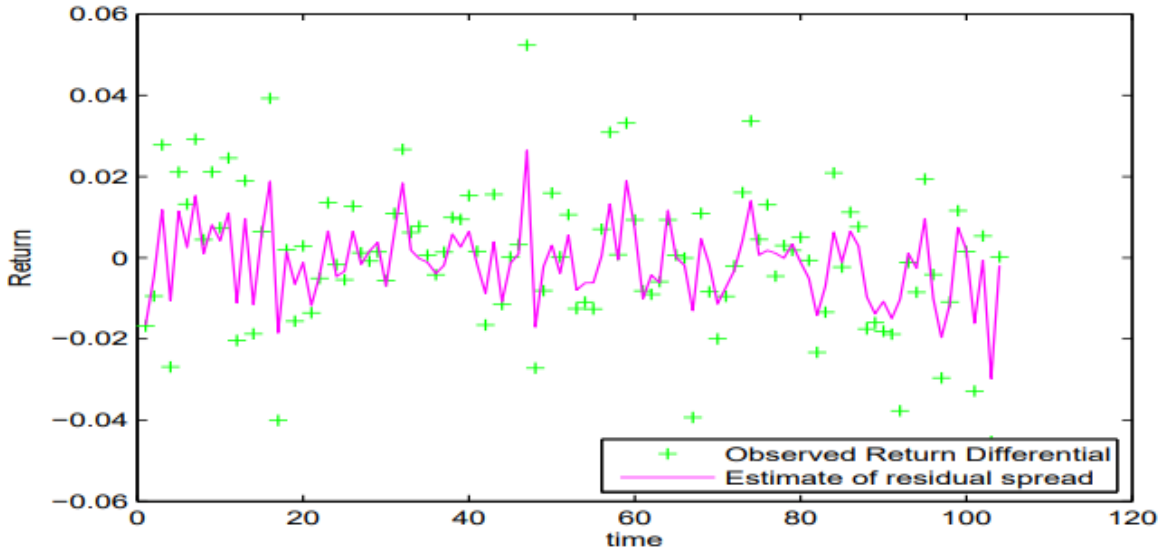

Fig. 1 A typical sketch of the evaluations for BHP-RIO's spread.

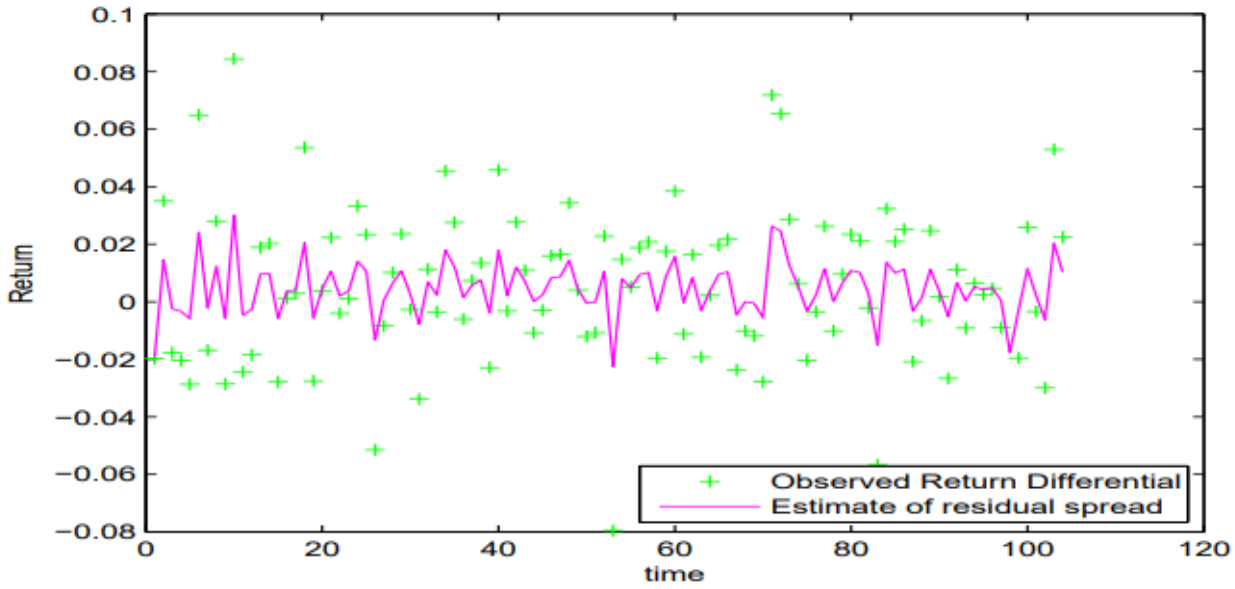

Fig. 2 A typical sketch of the evaluations for Walmart's spread.

3. Time- series approach

The time series approach usually assumes that there is no formation period. The reason for this is that highly correlated stock pairs are identified a priori. Consequently, researchers focus their studies on the trading period. The most widely cited time series approach is provided by Elliott et al. [12]. They capture the propagation in a state space that includes states and measurement formulas. They make two assumptions. One was that mean regression characteristics existed in the state variables. The other is that the observed propagation consists of state variables and Gaussian noise. This time series model has advantages in the following respects. Firstly, it can be used to predict future performance as it is a continuous model. Furthermore, the parameters in the model are considered to be the best due to expectation maximization (EM) and minimization of mean squared error. It is assumed that the latent state variable x(k) follows a mean-reverting process:

$$x_{k+1} - x_k = (a - bx_k)\tau + \sigma\sqrt{\tau}\epsilon_{k+1} \tag{4}$$

which can be simplified as:

$$x_{k+1} = A + Bx_k + C\epsilon_{k+1} \tag{5}$$

Here, $A = a\tau, B = 1 - b\tau$ and $C = \sigma\sqrt{\tau}$, where all the variables are coefficients (i.e., a, b, τ, σ). Afterwards, the variations of y can be described as follows:

$$y_k = x_k + \Gamma U_k + D\omega_k \tag{6}$$

Here, the U_k and ω_k are two noises, while Γ and D are coeficients.

Do et al. developed and improved on the state space model of previous studies who focused more on pricing errors [6]. They argued that the observed spread should be the difference between the returns of two assets. In addition, arbitrage pricing theory is used to support their analysis of trades. Similar to other literature, it is suggested comparing spread values and thresholds before taking long and short positions. However, this paper does not explain the time frame and the determination of thresholds. Therefore, this paper is of more theoretical value, but it is practically difficult to replicate this approach in a large empirical analysis. The evolution equation for different steps can be described as Eqs. (5) and (6). The scenario allows for greater flexibility in the parameters and use Bayesian estimation in their study. Based on the mean regression characteristics of spreads, other researchers have developed a number of time series models for paired trades [13], where a typical scale following the descriptions is given in Fig. 3.

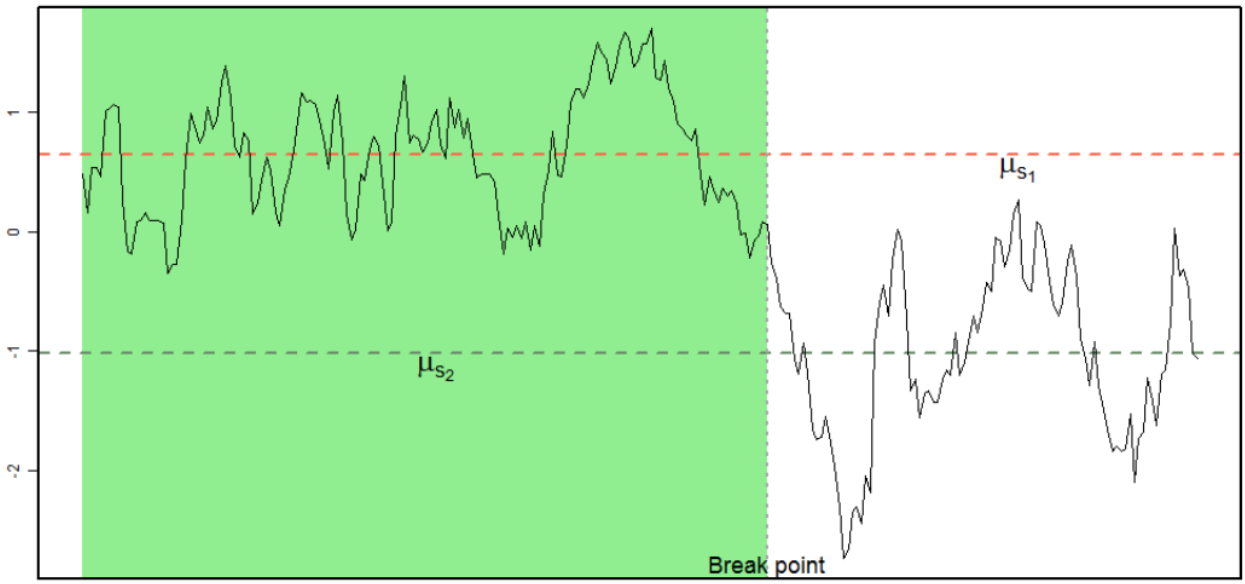

Fig. 3 A typical scale of the model.

4. Comparison

According to the analysis, SSD has several drawbacks, resulting in low variance spreads, limited profit potential and high variance risk. Pearson performs slightly better in terms of correlation at the return level. Pair trading profitability exposure to systematic sources of risk is low and declines over time, partly due to information dissemination, pair visibility and market frictions. There are applications in other asset classes (bonds, commodities) or time frames (daily data, high frequency data) where GGR's initial findings are usually confirmed. It has two main advantages. In addition, matched trading has proven to be profitable in a variety of markets, asset classes and time frames.

The time series approach has three advantages. Firstly, it uses statistics to predict the future based on past trends, usually in line with how things develop; secondly, it is more accurate by focusing on the impact of seasonal and cyclical changes on specific points in time while considering trends; and finally, it recognises the impact that random variables may have on the final outcome. However, this method also has disadvantages in that it uses only time as a factor of analysis and does not take into account the influence of other factors; and it forecasts only on the basis of historical data and does not take into account the possibility of changes in the market..

5. Limitations & Prospects

In theory, it is possible to fully hedge the arbitrage risk arising from the lack of similar substitutes; therefore, any investment signal associated with excess returns can generate real trading profits. In practice, however, arbitrageurs cannot fully hedge all the risks associated with perfect arbitrage. Mispricing can arise as investors demand greater compensation for taking on increased arbitrage risk, a situation that exists primarily in stocks with high heterogeneous volatility. As a result, investors seeking to profit from accrual and asset growth anomalies must assume a greater degree of uncertainty in outcomes than previously understood. This uncertainty comes in the form of high heterogeneous risk, which can increase costs and discourage profitable arbitrage. The arbitrage risk arising from the lack of similar substitutes may impose significant limitations on arbitrage for investors seeking to profit from asset misprizing. Even if seemingly serious mispricing is identified and persists over time, investors may not be able to outperform the market on a post-cost basis.

6. Conclusion

In summary, plenty of research and empirical studies have been done by scholars under the term pair trading. To be specific, five major approaches are discussed. Each method has its own strengths and weaknesses and there is room for model improvement. In this paper, the two most used methods are selected for analysis and comparison. Contemporarily, arbitrage models are subject to risks arising from the lack of similar alternatives, and the restrictions imposed by several major stock markets make the situation unpromising. Future research should address such short selling restrictions and pricing risks and carefully examine the profitability of each method. Overall, these results offer a guideline for implementation of arbitrage.

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