

Application of Mean-Variance Model in the U.S. Capital Market

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Abstract. Portfolio in stock market is an asset that people buy in the hopes of a return or appreciation, or both. Portfolio investment is not active, unlike direct investment, which means managing it by alone. A portfolio can help people spread risk and decrease risk, because it can minimize the negative impact of the economy. This paper analyzes asset allocation for four different companies. In this paper, three methods of mean-variance analysis, CAPM and FF3F model are used to optimize the portfolio. In addition, this paper will calculate different weights to analyze the maximum return of the portfolio. The results show that in the FF3F model. In the Sharpe ratio portfolio, "POAHY" is the security with the most weight. The portfolio with the highest Sharpe ratio weight in CAPM model is "POAHY". The outcome in this paper benefits the related investors in financial markets.

Keywords: Mean-variance; capital market; FF3F; Sharpe ratio.

1. Introduction

In an article titled "The Selection of Asset Portfolios" that was published in the "Journal of Finance" in March 1952, Markowitz addressed various types of movement directions while using probability theory and linear algebra to study securities investment portfolios. He wrote "Portfolio Selection," which was published in 1959 and covered in depth the fundamentals of portfolio management, establishing the groundwork for current Western portfolio investing theory [1].

The portfolio selection theory based on the mean-variance model has been widely concerned, and many scholars have extended research on the model from the perspectives of seeking optimization algorithms for the model, improving the way of measuring risk, and considering constraints [1]. The core of the mean-variance model is to find such a portfolio: the maximum return under given risk or the minimum risk under given return, we usually call the most efficient portfolio or the optimal portfolio [2]. It not only promotes the improvement of enterprise efficiency, but also it enhances the competitive advantage of the enterprise, makes full use of resources, and expands the development scale of the enterprise [3]. Also, for individual investors, a portfolio is a better choice than a single asset because a portfolio can help people reduce risk. It helps people to gain more profit [4]. In 2020, financial fraud case of Luckin Coffee was like a thunderbolt, shattering the wealth dreams of many investors. Looking back at the history of the stock market, there are countless cases of fraud: LeTV, Changsheng Bio, Zhangzidao, Yinguangxia, etc. When we take a heavy investment strategy of one or two stocks, if we are unfortunate enough to buy such stocks, we will suffer huge losses. In the long-term investment practice, stepping on the thunder once or twice is not an accident. Stepping on thunder is not scary, and how to avoid irreparable losses when stepping on thunder is a must-have skill for investors. Assuming that the fraud case will cause 50% of the principal loss, which is the loss of a single stock with heavy positions, if only 10% of the principal is invested in each stock, then the principal loss is only 5%. To reach the level of loss of a single stock with a heavy position, 5 financial fraud cases are required. This probability is obviously very small [5]. Therefore, it is very necessary to diversify investment.

However, investment portfolios are very limited, so this has given rise to research discussions on this issue. This paper selects four stocks as research objects. They are FedEx Corporation (FDX), Porsche Automobil Holding SE (POAHY), Caterpillar Inc. (CAT), NIKE, Inc. (NKE). First, this paper selects these four companies from different fields, and the time period is from September 2017 to May 2022. This paper considers the method of using CAMP and Fama-French three-factor model (FF3F). Using the CAMP method to calculate the expected return only needs to consider one factor, so the error will be large. However, in order to consider practical reasons, this paper will calculate

Fama-French three-factor model through same steps of CAPM to find the maximum Sharpe ratio combination and the minimum variance combination. The FF3F model is more accurate than the CAMP model.

The remainder is summarized in what follows. The data are presented in second part, methods are explained in section 3, and the results and conclusions are presented in part 4 and 5, respectively.

2. Data

The data used in this paper comes from Yahoo Finance (<https://finance.yahoo.com/>). This article selects FedEx Corporation (FDX), Porsche Automobil Holding SE (POAHY), Caterpillar Inc. (CAT), NIKE, Inc. (NKE). FedEx's global air and ground network can deliver time-sensitive packages in as little as one to two business days in most cases, with guaranteed on-time delivery and a "On-Time Delivery Guarantee." Porsche is a famous car brand around the world. It is one of the leading brands of both European and American cars, with its headquarters in Stuttgart, Germany. One of the biggest producers of gas engines, industrial gas turbines, and diesel engines worldwide is Caterpillar. It is also a significant producer of mining and construction equipment. NIKE company has a lot of sporting goods, such as clothing, footwear, sports equipment and so on. NIKE is a world-renowned sports brand. Hence, this paper chooses these four brands. These 4 companies serve as closing prices for the nearly five-year period from September 2017 to May 2022. This article considers selecting these data because of their relatively high beta values and their good performance over the past 5 years. At the same time, this paper selects these companies from different fields to achieve asset diversification and avoid high correlation. Table 1 below shows some descriptive statistics for these assets.

Table1. Descriptive statistics of the selected assets

	FDX	POAHY	CAT	Nike
Mean	0.0068	0.0123	0.0123	0.0156
variance	0.0092	0.0118	0.0064	0.0052
Max	0.3054	0.3288	0.1879	0.1519
Min	-0.2974	-0.3372	-0.2063	-0.1238

From this table, the largest average of profit is 0.0156 from "Nike". The smallest average of return is 0.0068 from "FDX". For the variance part, the "POAHY" is highest and is 0.0118, but the "Nike" is lowest. The highest max return and lowest min return are both in "POAHY".

3. Methods

3.1 Mean-Variance Analysis

The expected number of an arbitrary variable can show the average value of random variable but cannot reflect the fluctuation of its value; variance is used to describe the level of fluctuation of all the values of an arbitrary variable around its expected value. One of the variation indicators [6]. Mean-variance analysis can help investors make investment decisions that maximize expected return or minimize risk. A fund portfolio P is composed of funds $i=1, 2, \dots, n$ according to a certain weight ratio, and the weight of each fund i in P is w_i . It means the weight of different assets in portfolio. The relevant formula is as follows [7]

$$\sum_i Weight_i = 1 \quad (1)$$

Portfolio expected return is the total of every product of individual assets' expected return with its weight. The equation is

$$E(R_P) = \sum_{i=1}^n \omega_i r_i \quad (2)$$

A variance-covariance matrix is a square matrix that includes variances. The equation is

$$\text{Portfolio Variance} = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \text{Cov}(r_i r_j) \quad (3)$$

The Sharpe ratio is a ratio that compares the return of an investment to the risk. The equation is

$$\text{Sharpe Ratio} = \frac{E(R_P) - R_F}{\sigma_P} \quad (4)$$

3.2 Capital Asset Pricing Model

The beginning of the CAPM model was provided by Markowitz, William Sharpe, John Lintner, and Jan Mossin. This general equilibrium returns model is linear. According to the CAPM theory, an asset's needed rate of return and its beta value, which represents its systematic or non-diversifiable risk, are linearly connected. The capital asset pricing model (CAPM), which describes how securities are priced in financial markets to estimate projected returns on capital investments, is another idealized representation of this process. The model offers a method for estimating expected return on equity and for quantifying risk [8]. The equation for the CAPM Theory is

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \quad (5)$$

$E(R_i)$ is expected return rate. R_f means risk free return, β_i means a risk measure for the no diversified part of risk. $E(R_m)$ is profit in market portfolio. $E(R_m) - R_f$ is the excess return of other risks [9].

3.3 Fama-French three-factor model

Regression using Three Factor Model has twice advantages over the more straightforward CAPM variant. Firstly, the Three Factor Model, with values of 95% and higher, explains substantially more of the volatility seen in realized returns. Second, the Three Factor Model frequently reveals that, rather than reflecting manager performance, a positive alpha found in a CAPM regression is simply due to exposure to either HML or SMB factors [6]. The equation is

$$R_a = R_f + \beta_a(R_m - R_f) + s_a \text{SMB} + h_a \text{HML} \quad (6)$$

R_a is the expected return rate on portfolio, R_f shows rate of risk free in finance market, R_m is the return rate on the market portfolio and the co-efficient beta to be estimated of the 3 factors is the market portfolio premium of risk, premium of size, and PB ratio. The impact on three reasons of premium, in which beta conception on the portfolio risk premium is similar to beta coefficient. In CAPM model, the differences between the return of the portfolio of companies and the price-to-book ratio premium. HML is the difference between the return on the portfolio of higher-carrying companies and the return on the portfolio of lower-yielding companies. α is the excess returns, and the excess returns on a portfolio can be showed by three factors, so that alpha should be statistically same as 0. Depending on the price-to-pound ratio and company size, the data of the three factors divide each brand in the finance market into 10 portfolios with equal market capitalization in the regression analysis and use their historical data to determine the three factors' respective premium levels [10]. β_a is a measurement of the exposure of an asset to market risk. Despite the fact that due to the additional components, this beta is not same as beta in a model of CAMP. s_a shows exposure degree to size risk and h_a points the exposure level to risk [11].

4. Results

By using correlation function label, it shows all correlations are small because they are less than 0.5. In addition, all correlation numbers are positive so that the relationship between the two assets have positive situation. Due to small correlation, people will be more desired about these, since these stocks are not affected each other dramatically. Finally, all statistics aren't zero which means they are inter-correlated.

Table 2. The correlation between the equities.

	FDX return	POAHY return	CAT return	Nike return
FDX return	0.0092	0.0051	0.0046	0.0027
POAHY return	0.0051	0.0118	0.0048	0.0031
CAT return	0.0046	0.0048	0.0064	0.0025
Nike return	0.0027	0.0031	0.0025	0.0052

Table 3. Results for Sharpe ratio portfolio optimization under CAPM model

	FDX	POAHY	CAT	Nike
CAPM portfolio weight	0.4954	0.5046	0	0
CAPM return	0.0101			
CAPM variance	0.0078			
CAPM standard deviation	0.0885			
CAPM sharp ratio	0.1048			

The largest weight is 0.5046 which show in "POAHY", but the "CAT" and "Nike" have the smallest weight which is zero. The "POAHY" has the biggest weight because "Porsche" has a growing profitable business with high profit margins, a strong global brand awareness, and an excellent development policy [12]. "POAHY" company belongs to consumer cyclical sector in stock market. "CAT" has the smallest weight probably because recession and its impact on the company's business. "CAT" company belongs to industrial sector in stock market.

Table 4. Results for Sharpe ratio portfolio under FF3F model

	FDX	POAHY	CAT	Nike
ff3f ER	0.0099	0.0144	0.0083	0.0057
ff3f portfolio weight	0.2206	0.5624	0.1309	0.0861
ff3f portfolio return	0.0118			
ff3f portfolio variance	0.007			
ff3f portfolio standard deviation	0.0838			
ff3f portfolio sharp ratio	0.131			
ff3f portfolio return	0.0118			

From the table above, "POAHY" has the biggest weight, which is 0.5624, but the "Nike" has the smallest weight, because many people may fight with China where has worsening situation and where ongoing lockdowns to stop the spread of Covid-19 infections [13]. "Nike" company belongs to consumer cyclical sector in stock market, too.

5. Conclusion

According to the current information data, a lot of research on the portfolios only based on analysis of specific calculation and calculation of specific companies. In this paper, it help investor make decisions before they want to invest. This paper uses three kinds of analysis ways to find the optimal

portfolio, which are Mean-variance analysis, Capital Assets Pricing Model and Fama-French 3 Factor model. Different models have their different purposes. Both Capital Assets Pricing Model and Fama-French 3 Factor model suggests that “POAHY” company is more profitable, and it has the biggest weight among these four companies. Although “POAHY” company is classified by consumer cyclical sector in stock market, whether consumer cyclical sector in stock market is profitable or not depending on specific company, since the “Nike” company has smallest weight as mentioned before.

Finally, these models still have some disadvantages. There is no denying that there are other factors affecting stock and portfolios, such as epidemic. In 2020-2022, due to the influence of the epidemic, the situation in market is more special and deserve further discussion.

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