Application of Fama-French Three-Factor Model in Firefighter Investment Strategy

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Abstract. Using some models to make the portfolio optimization is a widely used approach nowadays. This paper studies how to choose the right portfolio to diversify for the firefighter who is about to retire. This allows customer to avoid risks while obtaining appropriate benefits. This paper selected five stocks from different sectors that performed better and used two main approaches. It uses the Mean-Variance analysis to analyze how to make these two mutually restricted goals, i.e., minimum variance portfolio and the maximum Sharpe ratio portfolio. At the same time, the FF3F model is used to analyze the investment weight of the selected portfolio, so that the funds can be reasonably invested. The results show that all the stocks are positively correlated. In the FF3F model, the Sharpe ratio of the non-buyout case is higher than that of the buyout case. The results in this paper show that the option with higher Sharpe ratio can obtain more return in per unit risk and is more suitable for firefighters.

Keywords: Mean-variance analysis; FF3F model; portfolio optimization.

1. Introduction

Since 1952 when Markowitz first proposed the Mean-variance model [1], portfolio optimization has been a hot topic in financial research. His paper laid the foundation of portfolio theory and marked the beginning of modern portfolio theory. Most of the traditional portfolio models are based on random environment and use random variables to describe the uncertainty of asset prices. In practice, however, portfolio decisions in financial markets are often subject to many improbability factors [2]. So, this paper should consider the investment allocation of the portfolio from many aspects which is necessary. And in order to make the portfolio perfect, this paper need to use some models for analysis.

This paper finds that there are variety of studies on portfolio optimization. However, there are few studies focus on clients in specific occupations. Currently, most of the studies about portfolio optimization are the studies of the model. For example, Duan, Peng and Li researched the classical portfolio optimization problem with second-order stochastic dominance constraint by establishing a robust second-order stochastic dominance model [3]. Hu and Gao introduced the return weight to calculate the proportion of return in each period to the total return in order to solve the problem that the assumption of return rate in the classical Mean-variance model could not be fully adapted to the actual security market and numerical experiments and simulation results show that the established model is reasonable, and the improved algorithm is effective and feasible [4]. Also, some researchers studied how to solve problems about portfolio optimization by specific mathematical algorithm. Such as Qian, Wei, Zhao and Ni, who made research about using AdaBoost ensemble learning technology to improve the performance of Mean-variance analysis to improve the performance of portfolio strategies [5]. Wang and Yu studied a hybrid algorithm for the design of the mathematical model, which can quickly find the optimal solution by combining greedy strategy, branch and bound strategy and ant colony optimization method [6]. Some researchers made studies about the application of portfolio optimization models such as Wang, who applied the portfolio optimization models into energy futures [7]. Very few people have studied about the application of portfolio optimization models in specific occupations. Therefore, it sets out to make a study about this.

This paper aims to choose a portfolio for the firefighter that suits him best. The empirical process is divided into following aspects. First, this paper chooses five typical stocks from high-tech industry, electronic business industry, internet industry, clothing industry and food service industry. This paper uses the monthly historical data from May 1, 2017, to January 1, 2022. Then it uses the mean returns,

variances, maximum returns and minimum returns of these stocks by applying Mean-variance analysis and make a comparision. Third, it uses covariance and correlation to analyze the relationship between these stocks. Fourth, it comes up with two cases, buyout and non-buyout. And considering the difference that there should be a weight of 0.71 for the government bond in non-buyout case, this paper analyzes the two cases separately. Fifth, this paper applies Fama-French three-factor model into the cases and consider the minimum variance portfolio and maximum Sharpe ratio portfolio for each case. It allows us to choose the one that suits the firefighter better. Finally, this paper gets the conclusion that the maximum Sharpe ratio portfolio in non-buyout option is more suitable for this firefighter.

This paper is structured as follows. Part II shows data of stocks, Part III lists the method this paper uses, Part IV shows the results this paper analyzes, and Part V is the conclusion.

2. Data

The data used in this paper is derived from Yahoofinance (https://finance.yahoo.com/). This paper selected five representative stocks which are AAPL, BABA, TWTR, CHDRY and YUMC for the monthly historical data from May 1, 2017, to January 1, 2022. And the reason why these five stocks are be selected is that the companies they belong to have better prospects which are from different fields, which means that diversified portfolio which have low correlation of assets can spreads risk and reduces some risks effectively. The companies these five stocks belong to are shown in Table 1. Some statistics are acquired by calculating which are shown in Table 2.

Table 1. Selected stocks and their companies

Stock	Company
AAPL	Apple Inc.
BABA	Alibaba Group Holding Limitted
TWTR	Twitter, Inc.
CHDRY	Christian Dior SE
YUMC	Yum China Holding, Inc.

Table 2. Descriptive statistics of the selected assets

	'AAPL'	'BABA'	'TWTR'	'CHDRY'	'YUMC'	'FL GO BOND'
Mean	0.0328	0.0065	0.0255	0.0241	0.0094	0.0085
Variance	0.0074	0.0100	0.0230	0.0078	0.0054	0.0001
Max	0.2144	0.2292	0.5250	0.4598	0.1592	0.0458
Min	-0.1840	-0.2268	-0.2726	-0.1704	-0.1584	-0.0128

Through visualizing the data, it can be discovered that 'AAPL' has the highest average return, while 'BABA' has the lowest one. When it comes to variance, 'TWTR' has the highest variance. And in buyout option, 'YUMC' has the lowest variance. In non-buyout option, 'FL GO BOND' has the lowest one. In addition, 'TWTR' has the highest max return and lowest min return, while 'FL GO BOND' has the highest min return and lowest max return.

3. Method

3.1 Mean-Variance Analysis

The Mean-Variance Analysis, which is also known as Modern Portfolio Theory, is a mathematical framework to choosing portfolio developed by Markowitz in 1952. According to the framework of risk measurement, which allows expression of risk level in terms of variance, portfolio optimization can be formulated as an operation that in order to combine an optimal portfolio with respect to a target for a given target value, minimizing the variance of portfolio expected return. Such an optimal portfolio is often referred to as an efficient portfolio. The reason is that it provides as little risk as possible with its unique level of expected return [8]. The following are the relevant formula.

$$\overline{\mathrm{r}} = \frac{1}{\mathrm{T}}(\mathrm{r}_1 + \mathrm{r}_2 + \ldots + \mathrm{r}_\mathrm{T}) \tag{1}$$

$$\mathrm{r}_\mathrm{G} = [(1+\mathrm{r}_1)(1+\mathrm{r}_2)\ldots(1+\mathrm{r}_\mathrm{t})]^{\frac{1}{\mathrm{T}}} - 1 \tag{2}$$

where $\overline{r}$ is the mean return of the asset and r_G is the geometric mean. The geometric mean must be used when working with percentages, which are derived from values, while the standard arithmetic mean works with the values themselves [9].

$$\sigma^2(\mathrm{r}) = \mathrm{VAR}(\mathrm{r}) = \frac{1}{\mathrm{T}}[(\mathrm{r}_1 - \overline{\mathrm{r}})^2 + (\mathrm{r}_2 - \overline{\mathrm{r}})^2 + \ldots (\mathrm{r}_\mathrm{T} - \overline{\mathrm{r}})^2] \tag{3}$$

where $\sigma(r)$ is standard deviation and $\sigma^2(r)$ is variance which is used to determine volatility and market security. Based on the single asset, this paper can obtain the portfolio.

$$\mathrm{COV}(\mathrm{r_i}, \mathrm{r_j}) = \frac{1}{\mathrm{T}}\left[(\mathrm{r}_{1,\mathrm{i}} - \overline{\mathrm{r_i}})(\mathrm{r}_{1,\mathrm{j}} - \overline{\mathrm{r_j}}) + (\mathrm{r}_{2,\mathrm{i}} - \overline{\mathrm{r_i}})(\mathrm{r}_{2,\mathrm{j}} - \overline{\mathrm{r_j}}) + \ldots (\mathrm{r}_{\mathrm{T},\mathrm{i}} - \overline{\mathrm{r_i}})(\mathrm{r}_{\mathrm{T},\mathrm{j}} - \overline{\mathrm{r_j}})\right] \tag{4}$$

where $COV(r_i, r_j)$ is the covariance between asset i and asset j. Covariance measures the directional relationship between the returns on two assets. A positive covariance means that asset returns move together while a negative covariance means they move inversely. Risk and volatility can be reduced in a portfolio by pairing assets that have a negative covariance [9].

$$\sum_\mathrm{i} \mathrm{Weight_i} = 1 \tag{5}$$

$$\mathrm{r_p} = \sum_\mathrm{i} \mathrm{W_i} \cdot \mathrm{r_i} \tag{6}$$

where $Weight_i$ is the weight of asset i in the portfolio. r_p means the portfolio return and r_i means the return of asset i.

$$\mathrm{E}(\mathrm{r_p}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{w_i r_i} \tag{7}$$

where $E(r_p)$ is the expected return of portfolio.

$$\mathrm{VAR}(\mathrm{r_p}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{w_i w_j COV}(\mathrm{r_i}, \mathrm{r_j}) \tag{8}$$

where $VAR(r_p)$ is the variance of portfolio. $COV(r_i, r_j)$ means the covariance between asset i and asset j.

$$\text{Sharpe ratio} = \frac{\mathrm{E}(\mathrm{r_p}) - \mathrm{r_f}}{\sigma_\mathrm{p}} \tag{9}$$

where r_f is the risk-free rate. σ_p standards for the standard deviation of the portfolio. Sharpe ratio can measure the return and risk of investment portfolio at the same time in the monetary and securities market. It is the most commonly used index in fund performance evaluation, and it can provide important guidance in investment portfolio [10].

3.2 Fama-French Three-factor Model

Fama and French established this model in 1993, which is an extension of the Capital Asset Pricing Model (CAPM). It adds two risk factors. The Fama-French three-factor model is commonly applied to evaluate the performance of a portfolio in comparison to market returns. At the same time, it considers the market value factor and the book-to-market ratio factor, which is an effective portfolio evaluation tool.

$$\mathrm{E(r_i)} - \mathrm{r_f} = \beta_\mathrm{i}[\mathrm{E(r_m} - \mathrm{r_f})] + \mathrm{s_i[E(SMB)]} + \mathrm{h_i[E(HML)]} \tag{10}$$

The r_i means the total return of portfolio and r_f means the risk-free return rate. Where $E(r_i)$ is the expected return. r means the market return of portfolio and $E(r_m - r_f)$ means market risk premium. *SMB* refers to Small Minus Big (firm size) portfolio, *HML* refers to High Minus Low (book-to-market ratio) portfolio. They have high correlation. β_i, s_i and h_i are the coefficients of these factors.

4. Results

This paper uses the covariance and correlation function to show the results about these stocks. And Fama-French three-factor model is the core approach.

Table 3. The covariance between the stocks

	AAPL	BABA	TWTR	CHDRY	YUMC	FL GO BOND
AAPL	0.0073					
BABA	0.0019	0.0098				
TWTR	0.0025	0.0011	0.0226			
CHDRY	0.0024	0.0008	0.0008	0.0077		
YUMC	0.0018	0.0032	0.0035	0.0009	0.0053	
FL GO BOND	0.0004	0.0004	0.0001	0.0002	0.0001	0.0001

To make the results more intuitive to compare these stocks, this paper uses correlation which normalizes the results in covariance. The darker the color, the closer the index is to 1.

Table 4. The correlation between the stocks

	'AAPL'	'BABA'	'TWTR'	'CHDRY'	'YUMC'	'FL GO BOND'
'AAPL'	1					
'BABA'	0.2199	1				
'TWTR'	0.1935	0.0709	1			
'CHDRY'	0.3150	0.0958	0.0579	1		
'YUMC'	0.2920	0.4440	0.3230	0.1349	1	
'FL GO BOND'	0.3820	0.3466	0.0734	0.1926	0.1379	1

According to Table 4, this paper can observe that stocks are all positively correlated and most of them have low degree of correlation. By contrast, the index between 'BABA' and 'YUMC' and between 'AAPL' and 'FL GO BOND' have higher correlation. 'BABA' and 'YUMC' have the highest correlation. That's because Yum's China business was acquired by Primavera Capital Group and Ant Financial Services Group. And Ant Financial Services Group is Alibaba's financial affiliate. So, the rise and fall of their stocks are highly correlated.

Table 5. The estimations of FF3F parameters

	β_{i}	s_{i}	h_{i}
'AAPL'	1.2372	-0.4646	-0.5168
'BABA'	0.8563	-0.0378	-0.2919
'TWTR'	0.6884	0.7071	0.0581
'CHDRY'	0.8687	-0.0718	-0.0223
'YUMC'	0.7806	-0.0321	0.0723
'FL GO BOND'	0.1119	-0.0537	-0.0450

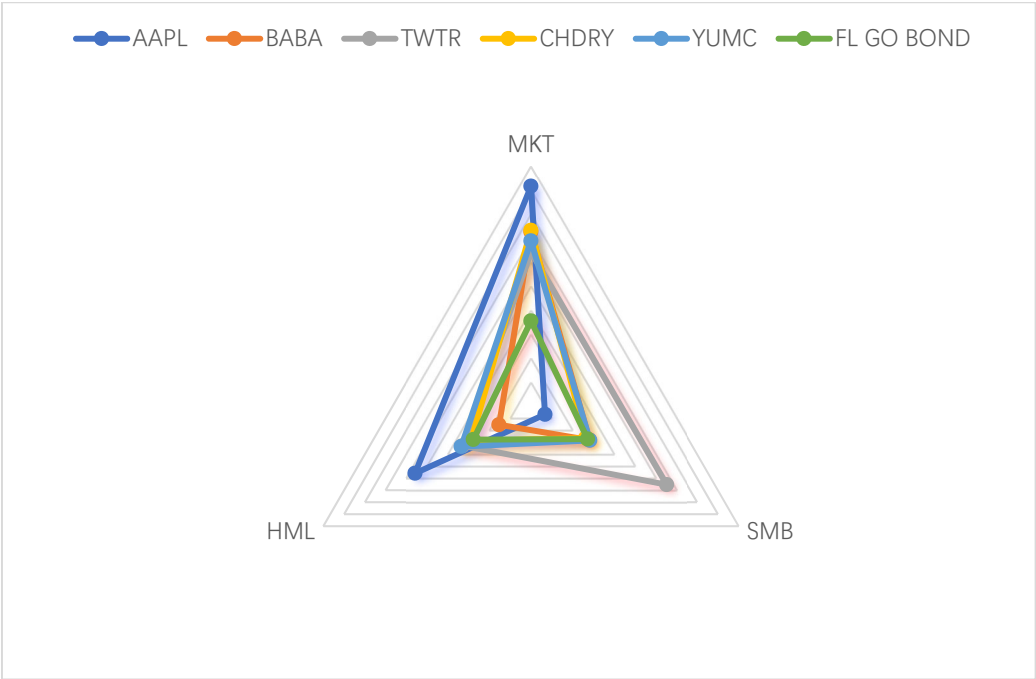

Fig 1. The estimations of FF3F parameters

By analyzing the results Table 5 and Figure 1, this paper discovers that apart from 'TWTR' , all the other stocks have high MKT beta which means that market changes is more affective to these

stocks. The reason why 'TWTR' has high SMB beta is that 'TWTR' is more affected by market value. This paper is going to analyze the results in two cases because the firefighter have two choices: buyout or non-buyout.

4.1 Buyout Option

In this case, the firefighter takes the lump-sum payout and invests his nest-egg and the lump-sum to support him throughout his retirement. By using FF3F model, the paper can find minimum variance portfolio whose results are showing below. This type of portfolio has the lowest risk which is suitable for risk averse investors.

Table 6. Results for minimum variance portfolio under FF3F model (buyout option)

	'AAPL'	'BABA'	'TWTR'	'CHDRY'	'YUMC'
Expected return	0.0065	0.0055	0.0070	0.0065	0.0063
Weight	0.1919	0.1287	0.0423	0.2790	0.3582
Portfolio return			0.0063		
Portfolio variance			0.0030		
Portfolio standard deviation			0.0552		
Sharpe ratio			0.0995		

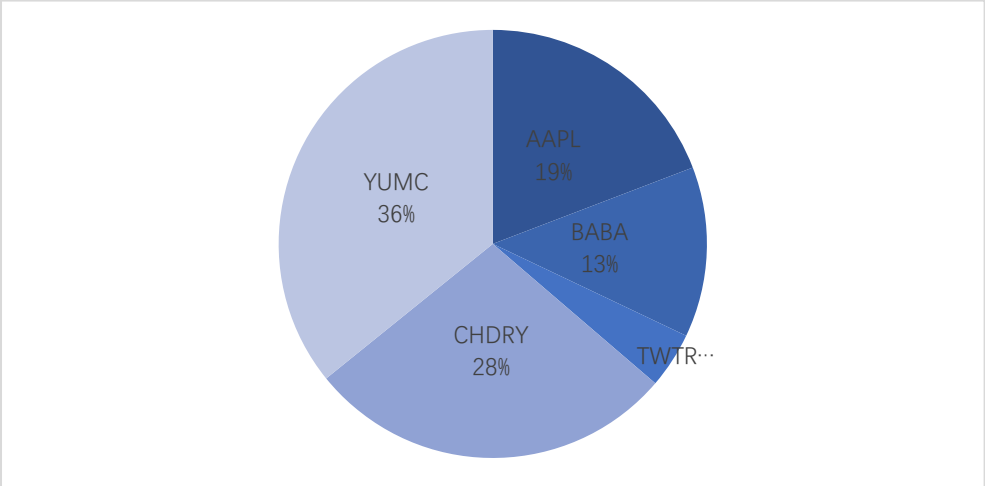

Fig 2. Weights for minimum variance portfolio under FF3F model (buyout option)

According to Table 6 and Figure 2, this paper can find that 'YUMC' and 'CHDRY' have more weight in this portfolio and 'YUMC' has the largest weight. That's because 'YUMC' has a lower risk. Compared to technology industry, e-commerce industry, social communication industry and clothing industry, catering industry has a lower risk which has a narrow competitive advantage and low net interest. The reason why 'TWTR' gets the lowest weight is that Twitter has had a number of incidents in recent years involving data security breaches and personal privacy leaks.

Table 7. Results for maximum Sharpe ratio portfolio under FF3F model (buyout option)

	'AAPL'	'BABA'	'TWTR'	'CHDRY'	'YUMC'
Expected return	0.0065	0.0055	0.0070	0.0065	0.0063

Weight	0.2044	0.0722	0.0581	0.2904	0.3749
Portfolio return			0.0064		
Portfolio variance			0.0031		
Portfolio standard deviation			0.0555		
Sharpe ratio			0.1000		

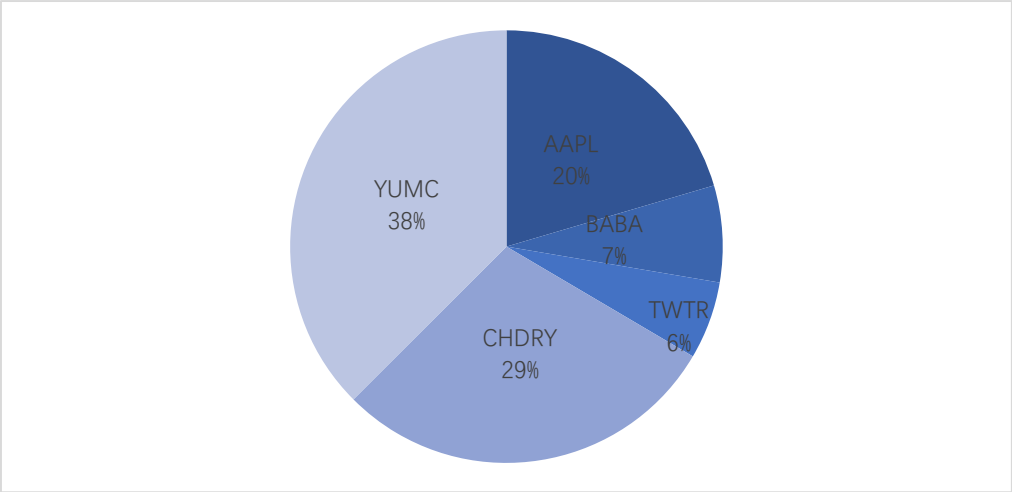

Fig 3. Weights for maximum Sharpe ratio portfolio under FF3F model (buyout option)

Because of the equivalence of risk and return, the minimum variance portfolio yields the lowest returns. Sharpe ratio is a measure of the risk-return ratio, the Sharpe ratio is the highest means that the unit of return bear the least risk. So, it is the optimal venture portfolio. According to Table 7 and Figure 3, it can be discovered that 'YUMC' and 'CHDRY' have more weight in this portfolio and 'YUMC' has the largest weight. Moreover, the portfolio return in the maximum Sharpe ratio portfolio is larger than in the former portfolio. Therefore, the maximum Sharpe ratio portfolio is more optimal in buyout case.

4.2 Non-buyout Option

In this option, the firefighters need to have roughly w=0.71 (1,450,000/2,050,000) of his wealth invested in something with a risk profile similar to that of long-term State of Florida bonds. Because Internal Rate of Return is 7.75%, this paper can get that the expected return of 'FL GO BOND' is about 0.65% (7.75%/12). Then some charts like the four in buyout option are made in this paper. The results are as follows.

Table 8. Results for minimum variance portfolio under FF3F model (non-buyout option)

	'AAPL'	'BABA'	'TWTR'	'CHDRY'	'YUMC'	'FL GO BOND'
Expected return	0.0065	0.0055	0.0070	0.0065	0.0063	0.0065
Weight	0.0351	0.0176	0.0133	0.0884	0.1356	0.71
Portfolio return			0.0064			
Portfolio variance			0.0004			

Portfolio standard deviation	0.0203
Sharpe ratio	0.2765

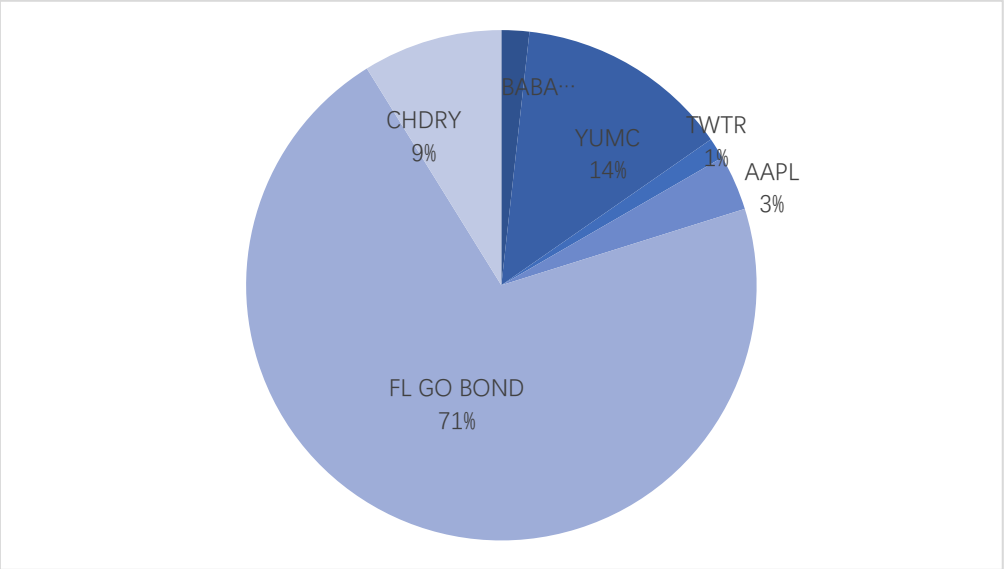

Fig 4. Weights for minimum variance portfolio under FF3F model (non-buyout option)

What is clear is that 'FL GO BOND' has the largest weight in the minimum variance portfolio. There is no doubt that its risk is the lowest and the risk of government bonds is guaranteed by the government. Generally speaking, the default risk of government bonds is very low, and the safety is high. So, it is suitable for 'FL GO BOND' to get such a large weight.

Table 9. Results for maximum Sharpe ratio portfolio under FF3F model (non-buyout option)

	'AAPL'	'BABA'	'TWTR'	'CHDRY'	'YUMC'	'FL GO BOND'
Expected return	0.0065	0.0055	0.0070	0.0065	0.0063	0.0065
Weight	0.0367	0.0101	0.0154	0.0899	0.1378	0.71
Portfolio return	0.0064					
Portfolio variance	0.0004					
Portfolio standard deviation	0.0203					
Sharpe ratio	0.2767					

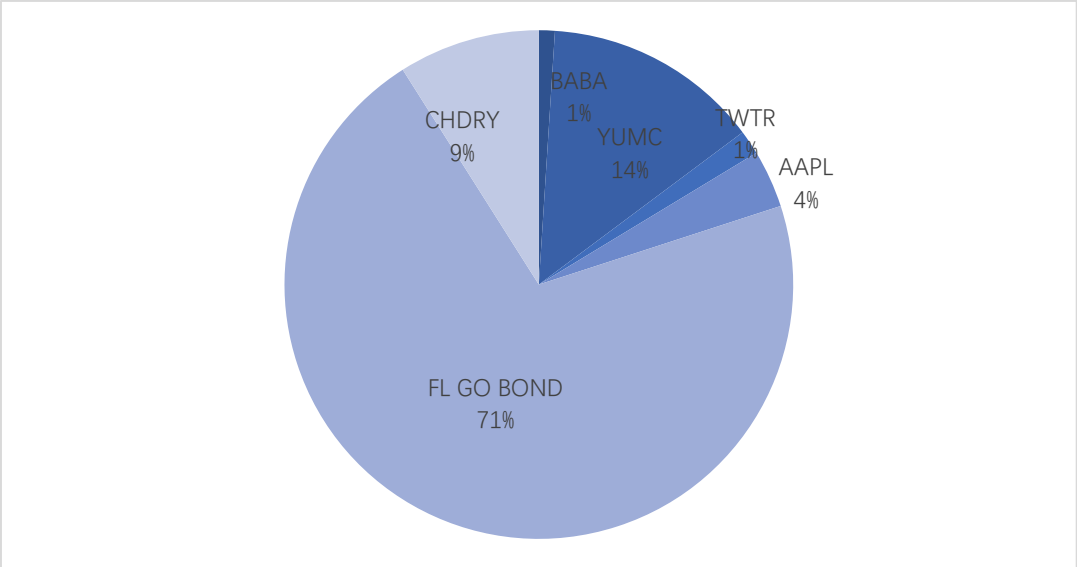

Fig 5. Weights for maximum Sharpe ratio portfolio under FF3F model (non-buyout option)

According to the Table 9 and Figure 5, this paper discovers that the weight of each stock is similar to the data in the minimum variance portfolio because the expected return and weight of 'FL GO BOND' are fixed.

Based on the analysis and results comparision, the minimum variance portfolio in buyout case is considered as a portfolio which has the lowest risk because of its smallest variance which is about 0.0030. Generally speaking, the larger the variance, the larger the fluctuation range of this set of data will be, which means that its stability will be less. However, investors must pay attention to whether the risk of the portfolio is within his acceptable range. If it is, then the firefighter will invest. If it is more than the firefighter can afford, then the firefighter probably won't take a big risk. That's why this paper considers the minimum variance portfolio in buyout case as the lowest risk portfolio. In addition, the maximum Sharpe ratio portfolio in non-buyout case is considered as a portfolio which has the highest return. Its Sharpe ratio is about 0.2767 which is the largest one. Sharpe ratio is an index which can take into account both returns and risks. It also reflects the degree to which the net growth rate of unit risk fund exceeds the risk-free rate of return. The higher the Sharpe ratio, the higher the excess return the portfolio can earn per unit of risk.

In conclusion, the firefighter is more recommended to choose the minimum variance portfolio in buyout case if the firefighter is a risk averse investor. And if the firefighter is an investor who chases higher yields, the firefighter is recommended to choose the maximum Sharpe ratio portfolio in non-buyout case. As far as this paper know, the firefighter is approaching his retirement at the age of 55 and the firefighter has 25 years of service as a firefighter and paramedic for a number of fire departments. It means that the firefighter is in good physical condition. According to the government life expectancy tables for men, this paper knows that the man in age of 55 is expected to live about 25.69 more years. The firefighter is also in good health and expects to live a long time, so the firefighter is expected to live more than 25.69 years. This firefighter is unmarried and has no children, so the firefighter only need to consider himself. A sum of money will be saved, and his economy will be ample. The firefighter rents a 2-bedroom lakeside cottage and plans to fish and travel when the firefighter retires. It can be seen that the firefighter wants to enjoy life and do what the firefighter wants to do instead of intending to rest in his old age, which shows that his mentality is not old. The firefighter has a certain pursuit of life, so the firefighter is not suitable for low risk and low return option. Besides, fishing and traveling require certain financial support. Moreover, the variance of maximum Sharpe ratio portfolio in non-buyout case is not too different from the others. In other words, this portfolio is not much riskier than other portfolio. So, the portfolio with a high Sharpe ratio is more suitable for him, that is, the non-buyout case.

5. Conclusion

According to the current studies, most of them are about the portfolio models such as changing some algorithms. Making this research is aiming to study portfolio optimization for the client in specific occupation. Not only consider how to choose the portfolio which suits the client better but combine his job with actual situation. People in different jobs will be influenced by their manners of working, which means that it may influence their decisions. This paper chooses five typical stocks from different industries. By using Mean-variance analysis, it acquires the variance, covariance and correlation. The paper compares these data, and it discovers that they all have positive correlation with each other. This paper also use Fama-French three-factor model to create the minimum variance portfolio and the maximum Sharpe ratio portfolio in buyout and non-buyout cases. It turns out that the maximum Sharpe ratio portfolio with non-buyout option suits the firefighter better.

However, there are some shortcomings in this paper. This paper does not use volatility to analyze the data. It cannot predict precisely whether this firefighter will live to the expected age. It still need to be considered more in the further discussion.

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