Application of Mean-Variance Model for Firefighter

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Abstract. A good investment portfolio can make the investment get more returns, which requires investors to adopt appropriate methods to optimize the investment portfolio. A portfolio is not just a simple sum of assets, it requires the use of suitable models to analyze optimal asset allocation. This paper analyzes the asset allocation of firefighters, and uses three methods of mean variance analysis, CAPM model and FF3F model to optimize the pension investment ratio of firefighters. At the same time, this paper analyzes multiple stocks from four industries including retail, technology, manufacturing, and food, and strives to find the investment portfolio with the most potential pension. The results show that in CAPM, the largest weights of both maximum sharp ratio portfolio and the minimum portfolio belong to the possession of 'NSRGY', while in the FF3F model, the greatest weight of the maximum sharp ratio portfolio belongs to 'TM' possession and the greatest weight of the minimum variance portfolio belongs to 'NSRGY'. This research could help retirees on the same pension type better allocate their pension assets and optimize their investment portfolios. At the same time, this paper also analyzes the investment performance of stocks in different industries under the same fixed proportion of pension.

Key words: pension; Mean-variance; FF3F; sharpe ratio.

1. Introduction

Research on portfolio optimization has grown significantly in recent years. Researchers analyze market trends by studying multiple areas. However, there are not many studies focusing on industries such as retail, new energy vehicles, food, and technology. For example, Dai, Zhu, and Zhang analyzed portfolio options for crude oil, gold, and Chinese stock markets [4]. Fichter D P analyzed the oil market using portfolio optimization [5]. According to research by Adi S B, they examined the best portfolio choices for fifteen stocks traded on the Financial Times Stock Exchange 100 Index (FTSE 100) [6]. In addition, Asawa studied portfolio optimization based on clustering, support vector machines, and genetic algorithm [7]. Machine learning and obtained useful results that are close to perfect for researchers. In the real estate market, Gupta P, Mehlawat M K, and Inuiguchi M studied the optimal investment portfolio in the real estate market [8]. Borodin A, Tvaronaviien M, Vygodchikova I researched the structure of investment portfolio of high-tech companies [9]. Furthermore, Hashimoto H, Takayama T studied the investment analysis of the minerals and metals industry [10]. However, it is notices that, as a special group in the society, the firefighter is less been focused, which raises our great interests to optimize portfolio for the group.These studies have made great contributions to portfolio optimization, and also provided ideas for the research methods in this paper.

This paper focuses on a few stocks with investment value and examines how to optimize a portfolio. The stocks selected were Amazon. com, Inc. (AMZN), Toyota Motor Corporation (TM), Nestlé S. A. (NSRGY), Samsung Electronics Co., Ltd. (SMSN. IL). These stocks are all very promising because the companies they represent are heavily weighted in their respective industries. Referring to the stock data of the last five years, we can find that the prices of these stocks are accompanied by market fluctuations, but at the same time they can have very good performances at different stages.The

researching process is summarized as explained below. First, this paper selects 4 stocks coming from different industries for their closing prices during September 2017 to July 2022. Secondly, in this paper, three different portfolio analysis methods, namely analysis of mean variance, capital asset pricing method (CAPM) and Fama-French three-factor model (FF3F), are used to find out the risk and expected return of stocks. Lastly, although CAPM and FF3F has very similar calculating methods, FF3F is still used in this paper as it includes more factors.

The remaining parts are summarized as follows: Part 2 shows and explains data, Part 3 shows and explains the methods, Part 4 shows and explains the results.

2. Data

In the process of data analysis, the data used in this paper comes from Yahoofinance (https://finance. yahoo. com/). This paper selected the following 4 companies which are SMSN, AMZN, TM and NSRGY for their closing prices in five years from September 2017 to July 2022. The raw data comes from the monthly returns of each stock. This paper considers selecting these data because they have a relatively high beta and they have performed well in returns over the past 4 to 5 years. We have a very positive outlook for the future prices of these stocks. At the same time, the stock portfolio selected in this paper also pays attention to the matching of different industries. This paper selects different stocks in the retail, technology, manufacturing, food and other industries for investment portfolios. This is done to diversify asset allocation and avoid excessive risk. Some descriptive statistics of these assets are shown in the Table 1 below.

Table1. Descriptive statistics of the selected assets

	'SMSN'	'AMZN'	'TM'	'NSRGY'
Mean	0.0076	0.0191	0.0069	0.0081
Variance	0.0061	0.0082	0.0024	0.0017
Max	0.2054	0.0281	0.1496	0.0605
Min	-0.1867	-0.1165	-0.0733	-0.0457

From the data showing above, this paper finds that the average return of 'AMZN' is 0.0191, which is also the highest average return, while the lowest average return is 0.0069 from 'TM'. Comparing their variances, 'AMZN' has the largest variance while 'NSRGY' has the lowest. In addition, the lowest max return is shown in 'AMZN', and highest min return are in 'NSRGY'. But the highest max return and the lowest min return are both in 'SMSN'.

3. Methods

3.1 Mean-variance analysis

Investors often use mean-variance analysis as a tool to measure investment decisions. Its advantage is that it can obtain maximum returns within a fixed risk range.

The Markowitz mean-variance model is used to find the ratio of optimal asset allocation, and it is also the first time that mathematical statistics methods are introduced into portfolio theory. This research has important implications for the development of portfolios, as well as providing us with new approaches in computing. The calculating equation and method is as below:

$$\sum_{i} Weight_i = 1 \tag{1}$$

The $Weight_i$ stands for the weight of different assets in the portfolio.

$$\mathrm{E(RP)} = \sum_{i=1}^{n} w_i r_i \tag{2}$$

E(RP) stands for the portfolio's expected return.
w_i represents the weight of the total capital invested in asset i.
r_i is the expected retrun of asset i.

$$Variance\ of\ Portfolio = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) \tag{3}$$

$Cov(r_i, r_j)$ represents the variance-covariance between the expected returns of asset i and asset j.

$$\text{Sharpe ratio} = \frac{\mathrm{E}(R_P) - R_f}{\sigma_P} \tag{4}$$

What the Sharpe ratio does is measure the risk of a financial portfolio and show a measure of risk-adjusted returns. Sharpe ratio helps investors to find the comparisons between returns and risks. i. e. portfolios with higher Sharpe ratios are considered to outperform similar portfolios.

3.2 Capital Asset Pricing Model (CAPM)

WIlliam Sharpe, Jack Treynor, John Lintner and Jan Mossin developed CAPM in the 1960s. (https://aeaweb. org) It shows the relationship between expected returns and the risks. As it is simple to use and could show comparisons at ease, it is widely used in the financial industry, to help investors evaluate the fair value of a stock when the expected return and the risk are compared. This paper also fully incorporates the CAPM model when optimizing the portfolio of case assets. In this paper, the four target stocks are added into the CAPM model for calculation and analysis. The calculating equation and method is as below:

$$\mathrm{E}(R_i) = R_f + \beta_i(\mathrm{E}(R_m) - R_f) \tag{5}$$

Where:
$\mathrm{E}(R_i)$ stands for the expected return of investment.
R_f stands for the risk-free rate.
β_i stands for the beta of the investment.
$(\mathrm{E}(R_m) - R_f)$ stands for the market risk premium.
If β_i is more than one, then it indicates that the asset has a higher risk relatively to the market. Oppositely, if β_i is less than one, then it indicates that the asset has a lower risk relatively to the market.

3.3 Fama-French three-factor model

Eugene Fama and Kenneth French co-developed the Fama-French three-factor model in the 1990s, and they are also well-known Nobel laureates.The model introduces scale risk into market risk analysis, while also taking into account the impact of value risk factors. So, this model is actually an extension of the original CAPM model. There are three factors being used by Fama French Model:

The first is SMB (small minus large), HML (high minus low) plus the return of the portfolio, then minus the risk-free rate. Market value factors and book-to-market ratio factors also need to be considered, so it aims to provide an effective tool for calculating investment portfolios. The calculating equation and method is as below:

$$r = rf + \beta 1\,(rm - rf) + \beta 2\,(SMB) + \beta 3\,(HML) + \varepsilon \quad (6)$$

Where:

r stands for the expected rate of return.

rf stands for the risk-free rate.

β stands for the factor's coefficient.

(rm - rf) stands for the market risk premium.

SMB (Small Minus Big) stands for the excess returns of small-cap companies over large-cap companies.

HML (High Minus Low) stands for the excess returns of value stocks overgrowth stocks.

ε stands for risk.

According to Nobel Lauren Eugene Fama and researcher Kenneth French, based on the observations that value shares tend to outperform growth shares and small-cap stocks tend to outperform large-cap stocks. The same with the CAPM model, it is best for the Sharpe Ratio of the FF3F model to also stay between 1 and 2.

4. Results

We first report the covariance matrix in the following Table 2. By using MMULT function in excel, we could calculate the MVE, and MVP returns with the covariance of these four stocks. MMULT function is a Math & Trigonometry function in excel which stands for 'Matrix Multiplication'. It returns the product of the two companies.

Table 2. Covariance matrix.

	SMSN return	*AMZN return*	*TM return*	*NSRGY return*
SMSN return	0.0061	0.0028	0.0017	0.0008
AMZN return	0.0028	0.0082	0.0011	0.0012
TM return	0.0017	0.0011	0.0024	0.0005
NSRGY return	0.0008	0.0012	0.0005	0.0017

Table 3. MVE and MVP sharpe ratio.

	SMSN	AMZN	TM	NSRGY
MVE portfolio weight	-0.0807	0.2884	0.2779	0.5144
MVP portfolio weight	0.0100	0.0125	0.3878	0.5898
MVE Portfolio return	0.0110			
MVE Portfolio variance	0.0017			
MVE Standard deviation	0.0418			
MVE Sharpe ratio	0.2630			
MVP return	0.0078			
MVP variance	0.0012			
MVP Standard deviation	0.0352			
MVP Sharpe ratio	0.2212			

According to the Table 3, The MVE sharpe ratio is 0.2630. However, on the other side, instead of maximum the sharpe ratio of portfolio, investors could also choose to minimum the risk for the investment which is called Minimum Variance portfolio (MVP). In the Table 4 above, the sharpe ratio for the MVP is 0.2212which a little bit lower than MVE sharpe ratio. In order to analysis which is the best portfolio to get the maximum return, we construct 2 models which are CAPM and FF3F. We can see the data in the following Table 4. These data clearly reflect the performance of each stock

under different models. This means that the proportion of the target stock is not fixed or balanced. The proportion of these stocks in the portfolio varies with the model.

Table4. CAPM and FF3F sharpe ratio.

	SMSN	AMZN	TM	NSRGY
CAPM portfolio weight	0.1539	0.2872	0.2669	0.2920
CAPM port return	0.0075			
CAPM port variance	0.0020			
CAPM port Standard deviation	0.0451			
CAPM port Sharpe ratio	0.1065			
FF3F portfolio weight	0.3905	0.1014	0.5048	0.0034
FF3F portfolio return	0.0080			
FF3F portfolio variance	0.0026			
FF3F portfolio standard deviation	0.0512			
FF3F portfolio sharpe ratio	0.1039			

Based on the CAPM model, it takes the insights from mean variance analysis that although everyone has the same portfolio, all the assets will be determined by the risk and return on the tangency portfolio. The portfolio weight for SMSN is 0.1539, AMZN is 0.2872, TM is 0.2669, and NSRGY is 0.2920. In addition, the CAPM return is 0.0075, and sharpe ratio is 0.1065. Furthermore, we also do the FF3F model as it includes more factors such as expected returns and market size. Momentum and liquidity will be added as a factor of FF3F model. The FF3F model portfolio weight for SMSN is 0.3905, AMZN is 0.1014, TM is 0.5048, and NSRGY is 0.0034. The FF3F model has a portfolio return 0.0080 and sharpe ratio 0.1039. Compare the sharpe ratio of CAPM and FF3F model, by considering all the factors, FF3F has a more comprehensive benefit than CAPM which will be chosen as for the investors. Because FF3F considers more risk factors in addition to the traditional risk model, such as scale risk and value risk factors.

5. Summary

Based on the current information data, most of the calculations on the portfolio is the weight of how allocating money could get the most return. In this paper, all the research are aim to help the investors to decide to choose which stocks to buy. The firefighter could have the opportunity to get the most return by using his money according to these different models. This paper uses three methods

of mean variance analysis, CAPM model and FF3F model to optimize the pension investment ratio of firefighters. In the CAPM model and FF3F model, this paper find the highest sharpe ratio portfolio for the four stocks. CAPM is most used in the financial industry which help investors to determine the return of a combination of stocks with their risk. Furthermore, FF3F model is an extend version of CAPM that it includes more factors related to the risk of market. Therefore, by considering the market factors, we choose to recommend FFF3F model for the firefighter. This paper believes that the FF3F model can provide a well-suited portfolio for the assets described in the case, from which investors can obtain a balance of risk and return.

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