The Portfolio Optimization in Different Areas

Yiyi Fu[1, †], Yufei Guo[2, *, †], Jiahui Song[3, †]

[1] Department of Financial Management, Nankai University, Tianjin, China

[2] Department of Business and Management, Beijing Normal University. Hong Kong Baptist University united international college, Zhuhai, China

[3] Energy and power engineering, HuNan University, Changsha, China

[†]These authors contributed equally.

*Corresponding author: q030006295@mail.uic.edu.cn

Abstract: Portfolio optimization is a popular method widely used in the financial industry. This paper analyzes the asset allocation analysis for pension and diversified assets in new energy vehicles, computer software, chip, information industry. There are four assets from different sectors. In this paper, Mean-variance and CAPM model were used for portfolio optimization. Meanwhile, the performance of the portfolio is analyzed in this paper by using weights. In addition we compare the final sharp ratio of accepting the Lump Sum or Defined Benefit. The result shows that in the CAPM model, when accepting Lump Sum, "AAPL" and "NVDA" have the maximum and the minimum weight. When selecting Defined Benefit, "ADBE" and "NVDA" have the maximum and minimum weights in the maximum ratio, respectively. This study may be useful for retirees to use in choosing whether to receive Lump Sum or Defined Benefit.

Keywords: Mean-variance, Lump Sum or Defined Benefit, Sharpe ratio.

1. Introduction

Markowitz first proposed to use variance as the measure of risk and established a mean-variance asset portfolio model [1]. He believed that investors should have two decision-making objectives: "as high as the possible rate of return" and "as low as possible uncertainty (risk)." The ideal combination should theoretically establish an equilibrium between these two opposing conditions [2]. Sharpe proposed the Sharpe ratio based on Markowitz's modern portfolio theory, which assumes that rational investors will invest and hold a portfolio that is most beneficial to them. In short, investors will choose the portfolio with a certain level of risk, but the expected return is maximized, or the risk level is minimized [3].

After the mean-variance model was proposed, the academic circle paid much attention to the portfolio optimization problem. For example, Liao studied the correlation between forestland and timber investment and non-forestry financial assets [4]. The results showed that forestland could be an ideal investment tool mixed with real estate. Abid et al. used mean-variance portfolio optimization and the random domination method to find that American investors who are risk averse tend to prefer domestic investment [5]. If you are a risk seeker, you need to diversify internationally. Corsaro et al. investigated that the application of deep learning in portfolio selection in the mean-variance framework, and the preliminary results show the effectiveness of our method [6]. Contreras-Valdez et al. argue that the correlation between Bitcoin and other assets would show a certain degree of diversification for investment portfolio [7].

It needs to be noted that the portfolio management of different stocks and mutual funds is relatively limited. Moreover, investing in different industries can effectively spread risks and seek incredible benefits for investors.

This study focuses on four representative companies from the information software industry, the high-tech electronic industry, the chip manufacturing industry, and the renewable energy vehicle industry, and they are Adobe (ADBE), Apple (AAPL), NVIDIA(NVDA), and Tesla (TSLA). The specific empirical process of this paper is summarized as follows.

First, this paper chooses these four companies, all from different fields, for the closing prices between September 2017 and December 2021. Also, the data needs to be sorted out in this period. Second, we establish the covariance matrix of different stock returns to verify a positive but less correlated relationship between investment assets. Third, we previously compared the mean variance model and CAPM model and found that the mean variance model seeks the optimal portfolio by calculating the expected return and has a high bias. Fourth, we derive the Sharpe ratio, standard deviation and expected return of each stock and pension bond based on the CAPM model. Fifth, we find the optimal investment weight of four stocks without pension through the programming solver function of Excel. Sixth, we add pension bonds and recalculate the optimal portfolio weights. The results show that we find the best portfolio with and without pension bonds based on the CAPM model. When there is no pension, we choose 45.09% ADBE stock, 45.47% AAPL stock, and 9.4% TSAL stock. When a pension is included, we select 11.15% ADBE stock, 9.87% AAPL stock, 2.61% NVDA stock, 5.37% TSLA stock, and 71% Pension.

The structure of essay is as follows. Part 2 recommends the data. Part 3 describes the method. Parts 4 and 5 show the results and conclusion, respectively.

2. Data

The data we used in the portfolio comes from Yahoofinance (https://finance.yahoo.com/). We choose four companies which are ADBE, AAPL, NVDA and TSLA for their closing prices in about four years from 2017/9/1 to 2021/12/1. The reason why we choose these companies to form the portfolio is because they have a relatively high beta, and they all performed well during the past 5 years. In addition, we picked these companies from different fields to make sure these assets are various and avoid high covariance. Here are some statistics of these assets shown in the Form 1 below.

Table 1. Statistics of the selected assets

	ADBE	AAPL	NVDA	TSLA
GeometricMean	0.0248	0.0295	0.0333	0.0517
Variance	0.0060	0.0077	0.0165	0.0438
Max	0.1742	0.2144	0.2781	0.7415
Min	-0.1535	-0.1840	-0.2516	-0.2243

Table2. Covariance

Covariance	ADBE return	AAPL return	NVDA return	TSLA return	FLGO BOND
ADBE	0.0059	0.0041	0.0067	0.0063	0.0004
AAPL	0.0041	0.0076	0.0070	0.0097	0.0004
NVDA	0.0067	0.0070	0.0162	0.0091	0.0006
TSLA	0.0063	0.0097	0.0091	0.0430	0.0006
FL GO BOND	0.0004	0.0004	0.0006	0.0006	0.0001

Seeing data showed above, we can find that the highest monthly average return is 0.741 in TSLA, while the lowest monthly average return is in NVDA which is -0.252. Comparing their variances,

TSLA has the largest variance while ADBE has the lowest. Also, we can find that ADBE has both lowest max return and highest min return which means that this stock is very stable. On the contrast, TSLA has the highest return and highest risks.

3. Method

3.1 Mean-variance analysis

Markowitz proposed the mean-variance theory to examine the performance of investors' portfolios from two aspects of risk and return, namely, pursuing risk minimization under the same level of return or maximizing return under the constraint of investment risk [1]. The mean-variance model has been widely used as an effective theoretical model for selecting the optimal portfolio [8].

The rate of expected return is generally expressed by the average rate of return. Risk is intuitively expressed as the volatility of the rate of return, that means expressed by the standard deviation of the yield return.

$$\sum_{i=1}^{n} w_i \mathrm{E}(R_i) = \mathrm{E}(R_P) \tag{1}$$

Where $\mathrm{E}(R_P)$ represents the expected return of the portfolio;$w_i$represents the different weight of assets of the portfolio; $\mathrm{E}(R_i)$ represents the expected return of asset i.

$$\sigma_P^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{2}$$

Where $\sigma_P^2$is the variance of the portfolio; $\sigma_i$is the variance of asset i;$\sigma_{ij}$is the covariance of asset i,j. We choose the Sharpe ratio to measure the overall efficiency of return and risk. The higher the sharp ratio, the higher the risk-return.

$$\text{Sharpe Ratio} = \frac{E(R_P) - R_f}{\sigma_P} \tag{3}$$

Where R_f is the annualized risk-free interest rate,$\sigma_P$is the standard deviation of the annualized portfolio return. In a word, we take into account the rate of expected return and standard deviation of each security, choose an appropriate portfolio proportion, and achieve the optimal level of investment utility, so as to obtain an ideal portfolio strategy.

3.2 Capital Asset Pricing Model

Capital Asset Pricing Model (CAPM) is a famous forecasting model in financial industry, basing on the expected return equilibrium of risk assets. CAPM shows the market equilibrium state when investors manage their investment with Markowitz theory. It also uses the simple linear relationship to express the theoretical relationship between the expected return and the risk of stocks [3]. The calculation formula of CAPM model is as follows:

$$\mathrm{E}(R_i) = R_f + \beta_{i,m}(\mathrm{E}(R_m) - R_f) \tag{4}$$

Where $\beta_{i,m}$is the systematic risk of asset i; $\mathrm{E}(R_m)$ is the expected return rate for market m; $\mathrm{E}(R_m) - R_f$ is the market risk premium, that shows the difference between the market expected return and the risk-free return.

The CAPM method can consider the rationality of the prices of different stocks by measuring the quantitative relation of the predicted expected return rate and the standard deviation. Moreover, it

would help entity determine the price of securities that is ready for listing and estimate the micro and macro of market.

Kothari et al. use annual returns instead of monthly returns to carry out tests, and the test results cannot refuse the assumption that the beta coefficient and benefits are relevant [9]. Clare et al. found that the result cannot refuse the hypothesis that the beta coefficient positively relates to the section average British stock market [10]. However, many empirical studies have found that scale, financial leverage, and other factors have significant explanatory power for stock returns, which have raised unavoidable doubts and challenges for the effectiveness of CAPM. Shi believed that opposite of the relationship uncovered by the Capital Asset Pricing Model, though a significant linear negative correlation to show the relation of the expected return and the risk of stocks. At the same time, unsystematic risk plays an essential role in the stock pricing behaviour [11].

4. Results

In order to find the correlation between any two equities, correlation function is an appropriate model to show the correlation between any stocks and mutual funds. From the data below, the correlation of any two equities are considered less relevant which are all less than 0.1. Also, that means different stocks and mutual funds are weakly correlative. It's easy to see the correlation have all positive number, which would imply that the relationship between the two assets have positive relationship. Moreover, all the correlations are non-zero, this meant that all the assets are inter-correlated.

Table 3. The correlation between different equities

	ADBE return	AAPL return	NVDA return	TSLA return	FLGO BOND
ADBE	0.0059	0.0041	0.0067	0.0063	0.0004
AAPL	0.0041	0.0076	0.0070	0.0097	0.0004
NVDA	0.0067	0.0070	0.0162	0.0091	0.0006
TSLA	0.0063	0.0097	0.0091	0.0430	0.0006
FLGO BOND	0.0004	0.0004	0.0006	0.0006	0.0001

Based on the CAMP model, the report can find the MVE weight in different stocks and come to different sharp ratio, expected return, and standard deviation. Moreover, the paper forms two different Sharpe ratio portfolio, one is with bond, another is without bond. And these two portfolios will come different expected return and standard deviation. Table 4 shows the maximum Sharp ratio portfolio.

Table 4. The maximum sharp ratio portfolio

MVE weight	ADBE	AAPL	NVDA	TSLA	PENSION
	0.4509	0.4547	0	0.0944	0
Expected return		Standard deviation		Sharp ratio	
0.029494121		0.07861		0.3408	

As the table above, the 'APPLE' has the largest weight, which is 0.455, but the smallest weight appears in 'NVIDIA' - 0. It is not hard to see a more prominent feature is 'Adobe' weight is close to 'APPLE' weight, which is 0.451. The 'APPLE' gets this weight probably because iphone has a reliable quality, a smooth and stable system, and a good reputation which maintains a high development trend in the global market. 'Adobe' is a second big weight probably because it mainly

research and development of multimedia production software. Also, it has started to develop marketing applications, Internet applications, financial analysis applications and other software development in recent years, this meant that it adapts to the current development trend. And the reason for 'NVIDIA' weight probably relate to this current industry prospects, which has received a great influence by the economic depression. From the table, we known that the sharpe ratio equal to 0.34, which is aimed to measure the relationship between return and risk, expressing the profits is less than the risks. The standard deviation reflects the volatility of fund return, that means the higher standard deviation, the greater risks. In the table above, the standard deviation equals 0.79, that is a relatively higher number. Thus, this portfolio has the high risks accompanied by relatively high earning.

Table 5. The minimum sharp ratio portfolio.

MVE weight	ADBE	AAPL	NVDA	TSLA	PENSION
	0.1115	0.0987	0.0261	0.0537	0.7100
Expected return		Standard deviation		Sharp ratio	
Standard deviation					
0.0146		0.0300		0.39819	

Also, this paper finds the result of minimum variance portfolio. The largest weight, 0.710, appears in 'FL GO BOND' which shows that this bond has the smallest variance. It has this weight maybe because credit rating of bond is generally considered to be the highest and is guaranteed safer by the government. Compared with other four equities, FL GO BOND has high security and stable earning. And 'NVIDIA' has the smallest weight 0.026, this can infer that it has a higher volatility. There are many possible factors, such as the global economic downturn, monetary inflation, and the impact of the pandemic on the business environment. But the main thing is that chips are in short supply and its prices have risen sharply, then cause the unmarketable products surplus. As shown from the table, the sharpe ratio equals 0.40, which means that this portfolio has a high return compared with the minimum sharpe ratio portfolio. The standard deviation drop to 0.030 which is a significant figure, showing that the return is relatively stable and the lower risks. Therefore, the maximum sharp ratio portfolio has the lower return.

5. Conclusion

According to the Yahoo finance, the paper calculate the portfolio of stocks in different field, in order to help investors make the best investment decision. We chose four companies, Adobe, Apple, NVIDIA, and Tesla, and collecting their closing price about four years, from 2017 to 2021. These four stocks come from different fields, such as energy and technology company, Software company, high-tech company, and the information technology industry. Firstly, this paper computes the correlation between any stocks by using Mean-variance analysis. Then it selects the Capital Assets Pricing Model method to calculate the optimal portfolio. Based on the Capital Assets Pricing Model, it delivers the relation with the assets expected return and the risk and finds the maximum sharp ratio portfolio with 0.40 and the minimum sharp ratio portfolio with 0.34. It's a intuitive way to think about that the high sharp ratio portfolio would provide a high return accompanied with a low risk, which is called optimal portfolio. However, looking at such a situation from another perspective, maybe the weight of funds has a big proportion, leads to the risk doesn't fluctuate as much and the payoff is not very well. Actually, the CAPM analysis model still have shortcoming, because this quantitative data analysis is calculated in a very ideal state, there are more factors affecting the portfolios that are probably not considered.

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