Portfolio Optimization for Junior Investors under Different Industries

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Abstract. Portfolio optimization has long been considered as an integral procedure of the financial markets. However, optimizing the portfolio may be difficult for junior investors. Therefore, this paper conducts a portfolio containing five diversified assets covering the e-commerce, entertainment industry, finance and insurance, and energy industries, and explores the optimal portfolio by maximizing the Sharpe ratio and minimizing the variance for each of these five assets through three models which are mean-variance analysis, CAPM and FF3F model. The results show that under both the FF3F and CAPM models, DIS has the maximum investment weight in maximum Sharpe ratio portfolio, while GLD has the maximum investment weight in minimum variance portfolio. We then added a risk-free asset to the existing portfolio, and the results display that adding a risk-free asset can further optimize the portfolio's return and reduce its risk. This research can be useful for potential investors who are into investments in related industries.

Keywords: Portfolio optimization; mean-variance analysis; CAPM; FF3F.

1. Introduction

With the increasing application on securities investment, portfolio analysis has become a heated topic among the investors. Tsaur put forward a fuzzy portfolio model to evaluate the portfolio associated with return and risk [1]. Li also studied the relationship of risk and return in an investment portfolio and found out an optimal strategy to trade off risk and return [2]. To maximize the interest of investment, investment portfolio behaves better than single asset. Therefore, it is significant to consider balancing the return and risk. Newberry also came up with reducing the portfolio risks by diversification [3]. except for being good at the investment, the study of portfolio also helps to moderate the portfolio with real assets, securities, portfolios, the reputation of financial institutions and the financial system from stranded asset risk from climate change [4].

Many papers in recent years have examined the problem of portfolio optimization. Some researchers want to optimize the predictive effect of portfolios through model building. For instance, in response to the great breakthroughs in machine learning for forecasting, Zhang and Chen used the great breakthroughs in machine learning for forecasting to propose the SRI portfolio construction model, which further optimizes the forecasting of portfolios [5]. Meanwhile, Maziar, Andreas and Ralf studied portfolio optimization based on GARCH-EVT-Copula forecasting models [6]. Moreover, there are a number of studies showing that several models can also be utilized to analyze portfolio optimization. Qiao and Guo [7] used the mean-variance model to study the asset growth effect and their results showed that this model was robust to studying the weighting of portfolios. Additionally, Nugraha and Susanti [8] verified that the CAPM has practical values in the process of investment decisions. The FF3F model is also frequently applied to the analysis of stocks return and risk factors. For example, Hui Liu and Huang [9] suggested that the Fama-French three-factor model could better explain stock returns in the Chinese A-share market than the CAPM model.

However, although previous professors used variance techniques to establish optimal portfolio, there is few research concerning the portfolio with stocks in e-commerce, entertainment industry, insurance business as well as gold and oil ETF. Therefore, we use the mean-variance analysis, capital assets pricing model and Fama-French three factor model to establish the optimal investment portfolio by means of maximizing the Sharpe ratio (MVE) and minimizing the portfolio variance (MVP). By

comparison among different methods, we tried to find out the optimal methods to weigh the portfolio. Furthermore, according to Qiguang and Houjie, introduction of risk-free assets will enlarge the investors' interest compared with the portfolio will all risky assets [10]. So, in the last part of paragraph, we discuss how ideal Sharpe ratio we will get from introduction of risk-free assets. With regard in two portfolios above with three methods to evaluate in respect to two directions——MVE and MVP. We could find the FF3F model is superior to other two methods and leading in the risk-free assets performs better in the same condition without it. Thus, the paper is contributed to the find the optimal weight portfolio and comes up with the introduction of risk-free assets will enhance the interests.

This paper is constructed as follows. Part 2 shows the data, Part 3 depicts the methods, Part 4 draws the results and Part 5 discusses the portfolio with risk-free assets. Finally, Part 6 reach the conclusion.

2. Data

The historical data used in this article are downloaded from Yahoo Finance (https://finance.yahoo.com/), which includes three stocks issued by three companies, namely Disney (DIS), Amazon (AMZN) and Berkshire Hathaway (BRK), as well as two exchange traded funds (ETF), namely SPDR Gold Shares (GLD) and the United States Oil Fund (USO). We selected the monthly closing prices of these five assets from September 2017 to January 2022 and calculated the monthly returns as a preliminary data treatment. We chose these five assets for our investment recommendations due to their relatively high beta and significant performance in the market over the past five years. Furthermore, our investment choice covers diverse industries as e-commerce, entertainment industry, insurance business, gold and oil, which better reflects the diversity of the portfolio. The descriptive statistics and co-variance on these five assets are shown in Tables 1 and 2 below.

Table 1. Descriptive statistics of the selected assets

	AMZN	DIS	GLD	BRK	USO
Mean	0.0248	0.0106	0.0079	0.0118	0.0087
Variance	0.0073	0.0071	0.0015	0.0028	0.0205

Table 2. Co-variance of the selected assets

	AMAZN	DIS	GLD	BRK	USO
AMAZN	0.0073	0.0020	0.0002	-0.0003	0.0020
DIS	0.0020	0.0071	-0.0003	-0.0005	0.0031
GLD	0.0002	-0.0003	0.0015	0.0003	-0.0007
BRK	-0.0003	-0.0005	0.0003	0.0028	0.0032
USO	0.0020	0.0031	-0.0007	0.0032	0.0205

The data shows that over the past five years, 'AMZN' has the highest average return and the 'GLD' has the lowest average return of 0.0248 and 0.0079 respectively. Additionally, 'USO' has the highest variance and 'GLD' has the lowest variance, which reflects the level of risk associated with investing in these five assets. Moreover, by calculating the co-variance of these five assets, it can be seen that the co-variance of each two assets is small, and three groups of assets have negative covariance, thus indicating that these five assets are significantly diversified and prompt the portfolio to be more robust.

3. Methods

3.1Mean-Variance analysis

Mean-Variance analysis helps to predict and assume a fixed return and risk in the following weight solution procedure. Lyle suggested that the fully fundamental portfolios with mean-variance analysis

produce large out-of-sample factor alphas with high Sharpe ratio. There are two indicators in the Mean-Variance analysis, namely portfolio return and variance [11].

The portfolio return is calculated by using weighted average return, as shown in the following:

$$\text{Portfolio return} = \sum_{i=1}^{n} w_i \times r_i \tag{1}$$

wi is the percentage of buying, and ri means the arithmetic mean of different shares.

To calculate the Sharpe ratio, we need to use standard deviation to measure the risk of the share. Portfolio variance is calculated as follows:

$$\text{Portfolio variance} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j \text{Cov}(r_i, r_j) \tag{2}$$

It used the co-variance of these financial assets referred above. Standard deviation is the square root of variance. So the Sharpe ratio can be calculated as following:

$$\text{Sharpe ratio} = \frac{(\text{porfolio return} - \text{Rf rate})}{\text{standard deviation}} \tag{3}$$

Rf rate is the average rate of the risk-free assets in the American market.

This formula excludes the market fluctuation and balance the return and risk of the investment assets. The denominator measures the risk of assets. If the assets fluctuate violently, it is hard to predict the trend, so the risk is high as standard deviation is large. The nominator measure how premium the investor can receive from the portfolio. It is clear that the more return it reaches, the more gain he will receive.

3.2 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM), originally proposed by Sharpe and Lintner based on Markowitz's proposal for mean-variance optimization, has provided a simple and compelling theory for pricing asset markets for over 20 years [12]. The general formula of CAPM is illustrated below.

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \tag{4}$$

$$\beta_i = \text{cov}(r_i, r_m)/\sigma_m^2 \tag{5}$$

$E(R_i)$ means the expected return on individual asset i, β_i means investment's beta value, with its mathematical meaning of $\text{cov}(r_i, r_m)/\sigma_m^2$, which is only the statistical estimate of the true betas. The expected return of the market is $E(R_m)$, and R_f stands for risk-free interest rate. Comprehensively, $(E(R_m) - R_f\beta_i)$ represents the market risk premium.

3.3 Fama-French 3 Factor Model

FF3F extends the CAPM by regressing portfolio returns on several variables in addition to market returns. FF3F extends simple linear regression (one independent variable) of CAPM to multiple linear regression (3 independent variables) in order to calculate the FF3F average return (FF3F Er). According to the Sahai Aniruddh et al., FF3F model is better than the CAPM model in the aspect of capturing assets' return [13].

First, use the historical data of SMB, HML, market excess return and assets return in the 53 months to regress a linear function and solver the coefficients :

$$\text{Assets return} = \beta_0 + \beta_1 \times \text{MKT} + \beta_2 \times \text{SMB} + \beta_3 \times \text{HML} \quad (6)$$

The adjusted return fully considers the size of company, market change and the kind of assets (growth one or value one).

Then, use the coefficients to recalculate the FF3F return.

$$\text{FF3F return} = \text{Rf} + \beta_1 \times \text{MKT RP} + \beta_2 \times \text{SMB premium} + \beta_3 \times \text{HML premium} \quad (7)$$

Rf is the arithmetic average of risk-free rate. MKT RP is the rate of market minus the risk-free rate. SMB premium and HML premium is the SMB rate or HML rate minus risk-free rate.

Finally, use this return to calculate the portfolio return, portfolio variance, standard deviation and Sharpe ratio as above. There are other two factors considered in the FF3F, so it is measured more reasonably, and certainly the weights change.

4. Results

The MVE and MVP method uses the average return and co-variance above to maximize the Sharpe ratio and minimize the portfolio variance respectively to calculate the optimal weights.

4.1 Results of Mean-Variance analysis

4.1.1 MVE of Mean-Variance analysis

MVE is to find the efficient risky portfolio. The objective is maximizing the Sharpe ratio, that is make the standard deviation smallest and average return biggest. In terms of programming solver, through changing the weight of five assets, the Sharpe ratio's maximum is 0.394 and the weight is shown in the Table 3.

Table 3. Results for MVE

	AMAZN	DIS	GLD	BRK	USO
Weight	0.2593	0.0757	0.3274	0.3375	0
portfolio return	0.0138				
portfolio variance	0.0011				
standard deviation	0.0328				
Sharpe ratio	0.3945				

It could find the weight of gold is the second largest, which follows BRK. It can be explained by gold is a relatively stable and safe assets and the average return isn't the smallest. It trades off the risk and return as a ideal assets, so the proportion is biggest. USO stands for ETF of American Oil. Analyzing the recent situation, it is easily affected by politics. Oil fund has a decrease trend due to the Russia-Ukraine conflict [14]. Thus, from the rational analysis, it does not recommend to invest USO.

The overall trend of the size relationship is BRK followed by GLD, AMAZN, and DIS. It is obvious that stable financial company has a better prospect than safe and stable assets and the share in high-tech company is better than the company belonging to certain segregation.

4.1.2 MVP of Mean-Variance analysis

MVP is preferred to the risk-aversion investors. Its aim is to minimize the portfolio variance regardless of the return. Table 4 shows the results with minimized portfolio variance.

Table 4. Results for MVP

	AMAZN	DIS	GLD	BRK	USO
Weight	0.0736	0.1312	0.5234	0.2718	0
portfolio return	0.0106				
portfolio variance	0.0008				
standard deviation	0.0284				
Sharpe ratio	0.3417				

It could find the general trend is BRK and GLD is bigger than AMAZN and DIS. It is also verified that shares with higher return and lower risk are prone to be accepted. Using the minimized variance method, the Sharpe ratio is slightly smaller than using MVE, which is 0.342 and the variance is not different from that to a great extent.

It concludes that using Mean-Variance analysis which does not consider much external factors has fairly good results.

4.2 Results of CAPM

4.2.1 MVE of CAPM

Under CAPM, this paper can find the optimal investment weights of each asset with maximum Sharpe ratio. See the Table 5 below for specific values.

Table 5. Results for MVE under CAPM

	AMAZN	DIS	GLD	BRK	USO
Weight	0.4226	0.4819	0	0	0.0955
portfolio return	0.0085				
portfolio variance	0.0044				
standard deviation	0.0662				
Sharpe ratio	0.1156				

The results show that Disney has the highest weight of investment, while USO has the lowest, at 0.4819 and 0.0955 respectively, which may be explained by the fact that Disney's profits have been rising over the past five years and that it has already built a full chain of IP-based films, TV, theme parks and derivatives. Moreover, the success of Disney's streaming subscription pay platform——Disney Plus has allowed Disney to weather the epidemic. Additionally, as the largest e-commerce brand in United States and with a surge in demand for e-commerce during the epidemic, Amazon's investment weight was comparable to Disney's at 0.42. Ultimately, the maximum Sharpe ratio for investing in these five assets using the CAPM model is 0.1156.

4.2.2 MVP of CAPM

Moreover, this paper charts the result of minimum variance portfolio under CAPM model.

Table 6. Results for MVP under CAPM

	AMZN	DIS	GLD	BRK	USO
Weight	0.0736	0.1312	0.5234	0.2718	0
Portfolio return	0.0020				
Portfolio variance	0.0008				
Standard deviation	0.0284				
Sharpe ratio	0.0401				

As can be seen from the results in Table 6, GLD have the highest investment weighting of 0.523, which is due to the fact that gold has the lowest variance and also implies that gold is the most stable and has the lowest investment risk. USO, on the other hand, has a weighting of 0, which suggests that

investing in crude oil is the riskiest, possibly due to the impact of new energy sources entering the market in the last five years, leaving traditional energy sources under attack.

4.3 Results of Fama-French 3 Factors model

However, the CAPM has still faced challenges on empirical research as it requires strict assumptions on distribution of returns, investor utility, information, and beliefs [3]. Therefore, this paper introduces a third model, which is Fama-French 3 Factor model.

4.3.1 MVE of FF3F model

Before obtaining the Sharpe ratio, we should recalculate the FF3F return. Table 7 shows the market, SMB, and HML coefficients and the renewed expected return.

Table 7. The market, SMB, and HML coefficients and the FF3F return

	AMAZN	DIS	GLD	BRK	USO
MKT beta	1.2354	1.1649	0.0035	-0.1172	0.7805
SMB beta	-0.4601	0.0044	-0.3618	0.0058	1.2311
HML beta	-0.7565	0.0921	-0.0569	0.2374	1.0096
FF3F E(R)	0.0121	0.0153	0.0018	0.0023	0.0169

When taking the size of company, value of assets and market fluctuation into consideration, the results are much different. Table 8 shows the FF3F weights. We could find the GLD, AMAZN and DIS account for large proportion and BRK and USO account for small proportion. Although the FF3F return of USO is largest, due to the international price of oil, many in the industry believe that not only the supply and demand imbalances, the sharp depreciation of the dollar and geopolitical instability can explain the high international oil price, and that the main driver of the oil price bubble is the flood of speculative funds into the oil futures market[15]. Therefore, the expected weight is small. Furthermore, although AMZN's and DIS's shares aren't the most ideal, the context of the company and its growth are attributed to maximize Sharpe ratio owing to its developing mode.

In terms of the company size, stocks value and market change, the Sharpe ratio becomes more conservative, which is 0.206. The weight of portfolio tends to develope companies' stocks rather than funds.

Table 8. Results for MVE under FF3

	AMAZN	DIS	GLD	BRK	USO
	0.2244	0.3807	0.2208	0.0757	0.0984
portfolio return	0.0108				
portfolio variance	0.0023				
standard deviation	0.0479				
Sharpe ratio	0.2061				

4.3.2 MVP of FF3F model

Same as two methods above, Table 9 indicates the results using MVP. It is opposite to the result in MVE. It implies the portfolio using FF3F in MVE has much risk. If the investor wants to minimize the risk, he should focus on the GLD and BRK which is the same as the Mean-Variance analysis. And in the meantime, the Sharpe ratio is quietly small (0.126).

Table 9. Results for MVP under FF3

	AMAZN	DIS	GLD	BRK	USO
	0.0736	0.1312	0.5234	0.2718	0
portfolio return	0.0045				
portfolio variance	0.0008				
standard deviation	0.0284				
Sharpe ratio	0.1259				

4.3.3 Appraisal of the FF3F model

There are two other factors brought in to the FF3F model. One is SMB. It explains the small companies have large potential to grow in the long-term market when adding it into the regression. The other is HML. It interprets the how much the gain is due to the inclusion of stocks with high book-to-market ratios. Compared with CAPM, FF3F model links the size of company and the company value with the future expect return. For instance, when a company becomes a big company, the management and operation costs gradually increase, the efficiency decreases, and the situation of "dis-economies of scale" occurs. In contrast, small companies have higher efficiency and growth, otherwise they cannot survive.

5. Further portfolios with “risk-free” assets

Belkov, et. Al put forward an evolutionary finance model to describe a portfolio with a risk-free and several risky assets. He found this portfolio make the investors more easily survive in the market [16]. Therefore, after evaluating the portfolio with five risky assets, we introduce a risk-free asset——government bond to revalue the weights and Sharpe ratio.

5.1 Introduction of Florida government bond

Florida government bond isn’t vulnerable to the market. It is issued by government and there is less risk of not being repaid. Referred to the historical data, we calculated the arithmetic mean return, variation, and regress it will market rate, SMB rate and HML rate to obtain the CAPM return and FF3F return as below (Table 10). What’s more, we also test the co-variance among these six assets (Table 11).

Table 10. Four types of expected return and variation of Florida government bond

	FL BOND
average return	0.0082
CAPM Er	0.0104
FF3F E(R)	0.0036
variation	0.0001

Table 11. co-variance among six assets

	FL BOND	AMAZN	DIS	GLD	BRK	USO
FL BOND	0.0001	0.0004	0.0003	0.0000	-0.0001	0.0003
AMAZN	0.0004	0.0073	0.0020	0.0002	-0.0003	0.0020
DIS	0.0003	0.0020	0.0071	-0.0003	-0.0005	0.0031
GLD	0.0000	0.0002	-0.0003	0.0015	0.0003	-0.0007
BRK	-0.0001	-0.0003	-0.0005	0.0003	0.0028	0.0032
USO	0.0003	0.0020	0.0031	-0.0007	0.0032	0.0205

It could find the arithmetic return of the Florida government bond is the smallest among the six assets. However, if taking the market rate, SMB rate and HML rate into consideration, the condition is not bad. The variation is also the smallest, which stands for it nearly has no risk. So we introduce this kind of asset with lower risk but lower return to find out whether investor is likely to choose that.

5.2 Comparison Portfolio Results Among Three Model in MVE

This paper added Florida government bond into the portfolio and recalculate the weight using Mean-Variance analysis, CAPM model and ff3f model by maximizing the Sharpe ratio. Table 12 shows the results concerning the portfolio with risk-free assets. All the portfolios include a large proportion of the bond. In the Mean-Variance analysis, the risk of AMAZN and DIS is too high because they are listed company and shares' price is vulnerable to the market change. USO is a oil fund, which is also unstable. So if we get rid of the AMAZN, DIS and USO, the Sharpe ratio is rather ideal and the risk is too small. However it is not realized. So we also do CAPM model and FF3F model. CAPM model is willing to maximize the Sharpe ratio by investing high return assets, which is opposite to the Mean-Variance analysis. So the weight of safe assets like GLD and BRK is zero. When we use FF3F model, the condition is more realistic. The weight of bond is the largest. It implies investors are more willing to choose risk-free assets compared with risky assets. Instead of return, people pay more attention to the risk. Except the bond, the second and third largest is BRK and DIS. It is dedicated to the continuously increase in the profit. The Sharpe ratio is not as bad as CAPM model as well as not as ideal as Mean-Variance analysis but in an appropriate range, which is 0.284.

Table 12. Comparison among the three models concerning the portfolio with risk-free assets in MVE

	Mean-Variance analysis	CAPM	FF3Fmodel
FL GO BOND weight	0.8659	0.4598	0.8446
AMAZN weight	0	0.2224	0.0062
DIS weight	0	0.2667	0.0554
GLD weight	0.0512	0	0.0283
BRK weight	0.0829	0	0.0648
USO weight	0	0.0510	0.0007
portfolio return	0.0085	0.0094	0.0041
portfolio variance	0.0001	0.0015	0.0001
standard deviation	0.0097	0.0382	0.0114
Sharpe ratio	0.7782	0.2229	0.2842

5.3 Comparison Portfolio Results Among Three Model in MVP

This article recalculated the weights by minimizing the variance in a portfolio with the addition of Florida bonds, using the Mean-Variance analysis, the CAPM model and the FF3F model. Table 13 shows the associated results. Again, all portfolios include a large proportion of Florida bonds. And surprisingly, the weights of AMZN, DIS and USO are zero in all models, probably because AMAZN and DIS are too risky as they are public companies and their stock prices are vulnerable to market changes, while USO also has an investment weight of zero due to market shocks in recent years. It follows that that when measuring risk, the investment weights obtained under the three models are almost similar as the relative magnitudes of the variances are essentially the same.

Table 13. Comparison among the three models concerning the portfolio with risk-free assets in MVP

	Mean-Variance analysis	CAPM	FF3Fmodel
FL BOND weight	0.8747	0.8747	0.8747
AMAZN weight	0	0	0
DIS weight	0	0	0
GLD weight	0.0578	0.0578	0.0578
BRK weight	0.0675	0.0675	0.0675
USO weight	0	0	0
portfolio return	0.0084	0.0014	0.0034
portfolio variance	0.0001	0.0001	0.0001
standard deviation	0.0097	0.0097	0.0097
Sharpe ratio	0.7753	0.0524	0.2561

6. Conclusion

This paper aims to help the investors find the most optimal investment portfolio in particular industries as e-commerce, entertainment, insurance and energy. After choosing 5 well-performed assets, comparing the results of all the portfolios under 3 models, which are Mean-variance analysis, Capital Assets Pricing Model and Fama-French 3 Factor model. The portfolios using Fama-French 3 Factor model are suggested to be the most optimal investment choice, while the investor can rely on their own level of risk tolerance to choose from the 2 portfolios under FF3F model——maximum Sharpe Ratio and minimum variance. In most circumstances, maximum Sharpe Ratio is more acceptable.

As we found that portfolios with risk-free assets may be more likely to survive in the market, we added a government bond to the original portfolios of five assets under 3 models and analyzed the differences. The results show that the government bond achieve high investment weights under all three models, and portfolios with risk-free assets end up with higher Sharpe ratios than portfolios without risk-free assets, and therefore, for most investors, this paper recommends adding risk-free assets - such as government bonds - to the existing investments in stocks and ETFs.

To sum up, the investigation results in this paper benefit a lot to potential investors who are into related industries in the financial markets.

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