How to Optimize a Financial Portfolio for a Firefighter

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Abstract. How to increase the amount of pension is a major concern for many retirees. In this paper, the question of whether retirees need to buy out their pensions is examined. Two portfolios are set up and their Sharpe rates are compared under two assumptions. The purpose of the study is to effectively assist retirees in their asset allocation. In this paper, diversified assets from four different sectors were selected to construct the portfolios. Three methods, the mean-variance model, CAPM model, and FF3F model, are used in this paper for portfolio construction and comparison. Meanwhile, the performance of the portfolios was analyzed using the weighting analysis method. The results show that "Disney" has the largest share when it chooses to buy out its pension and reinvest it using the CAPM model, while "LVMH" has the largest share in all other cases. This study may be useful for retirees who are faced with the decision of whether to buy out their pensions.

Keywords: Pension, Mean-variance, CAPM, FF3F.

1. Introduction

Since the 1970s, U.S. financial theory and practice have put forward and continuously improved the basic theory of securities investment for general public investors: first, insisting on diversified long-term investment; second, insisting on the rate of return of diversified long-term investment can beat inflation and achieve the preservation and appreciation of pension funds; third, general public investors can realize the effective amateurization of personal pension funds through the above investment methods Third, the general public investors can realize the effective management of their pension funds through the above investment methods, and then achieve the financial goal of personal pension protection [1]. Therefore, Most U.S. state governments offer their employees defined benefit pension plans, which provide a new possibility for retirees to get more pensions, i.e., retirees can choose to buy out their pensions and reinvest them to increase their income. In a defined benefit pension plan, the employer promises the employee an annual payment that begins when the employee retires, where the annual payment depends on the employee's age, tenure, and late-career salary [2]. As for the individual retiree, each individual needs to have at least one plan to ensure that his or her pension is at the level he or she needs [3].

In recent years, there has been significant growth in research on pension investment optimization. Several researchers have studied pension investment patterns under different models. For example, Tianlu Li studied pension investments under the 401K model [4]. Jin Gao [5], on the other hand, studied what kind of pension plans private sector employers should offer to retirees in the context of an aging population, comparing the defined benefit plans - DB plans, the defined contribution plans - DC plans, and hybrid plans. Yingli Feng, on the other hand, specifically studied the three-pillar system in the US pension finance [6]. Wei, on the other hand, studied the U.S. college and university teachers' pension system through a case study approach using the California Higher Education System Pension Plan and the Yale University Faculty Pension Plan as an entry point [7].

However, the studies on occupational and pension systems are quite limited. Therefore, depreciation has aroused interest to investigate this issue in depth. This paper chooses to study the pension investment options of retired firefighters. The following four stocks were chosen to construct the portfolio: The Walt Disney Company (DIS), Apple Inc (AAPL). , LVMH Moët Hennessy - Louis Vuitton (MC.PA), Société Européenne and Starbucks Corporation (SBUX). The research process can be summarized as follows. First, this paper chooses these five companies which are all from different fields for the closing prices during the period between MAY 2017 and January 2022. And the data needs to be sorted out in the specified period. Secondly, the paper considers using the mean-variance

model to value the two portfolios, but the underlying data is "expected return" which leads to a large bias in using this method. Therefore, this paper next considers the CAPM model for valuation. However, since this model is a one-factor model and the assumptions are more stringent and less consistent with reality, the CAPM-based Fama-French Three Factors Model (FF3F) is further chosen. The results obtained using this multi-factor model are more accurate and closer to reality than the CAPM model.

The following is a summary of the remaining parts. Part 2 shows the data, Part 3 depicts the methods, and Parts 4 and 5 show the results and conclusion, respectively.

2. Data and methods

2.1 Data

The data in this article is derived from Yahoo finance (https://finance.yahoo.com/). This paper select representative companies. i.e., DIS, MC.PA, SBUX, AAPL for closing prices, from May 1, 2017, to January 1, 2022. Then, to implement further investigations, the job of data cleaning to match the time has been finished. Finally, 285 data are collected. These closing prices have been transferred to returns and the pension was replaced with a long-term Florida bond. And some basic information is shown in Table 1.

Table 1. Descriptive statistics of the selected assets

	'DIS'	'MC.PA'	'AAPL'	'SBUX'	'FL GO BOND'
Mean return	0.007774	0.023877	0.032763	0.012807	0.008545
Geometric mean	0.004497	0.022174	0.029165	0.010318	0.008478
Variance	0.006925	0.003561	0.007405	0.005041	0.000137

Through visualizing the data, besides FL GO BOND, it would be discovered that the 'AAPL' has the highest average return, while the 'DIS' has the lowest average return. When it comes to variance, 'MC.PA' is the lowest, while 'AAPL' is the highest. What's more, the 'AAPL' has the highest geometric mean return, while the 'DIS' has the lowest geometric mean return.

2.2 Methods

2.2.1 Mean-Variance Analysis

The Markowitz model, also known as the mean-variance model, is a portfolio optimization model proposed by Markowitz in 1952, which measured the utility of a portfolio from both the risk and return perspectives of investment and introduced the first quantitative measure of financial risk [8]. This means that the model is looking for a portfolio with the minimum variance at a certain rate of return. The use of the model is based on these five assumptions.

(1) there are no frictions in the market; (2) all investment assets are infinitely divisible; (3) investment decisions are made based only on the risk and return of the assets; and (4) the rational investor assumption. That is, the asset with the least risk is chosen for a given return or the asset with the greatest return for a given level of risk; (5) short selling is allowed, i.e., the weight $x < 0$ is allowed for an asset [9]. The following are the relevant formulas.

$$\sum_i w_i = 1. \tag{1}$$

w_i means that the proportion of each asset in the portfolio and the sum of the proportions of all assets equals 1.

$$E(r) = \sum_{i=1}^{n} w_i r_i. \quad (2)$$

r indicates the return of the portfolio. Therefore $E(r)$ indicates the expected return of the portfolio. implies that the proportion of asset i in the portfolio, r_i then denotes the expected return on investment asset i.

$$Var(p) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \, w_j \, Cov(r_i, r_j). \quad (3)$$

The variance of the portfolio will be expressed using $Var(p)$. $Cov(r_i, r_j)$ in the formula reveals the variance-covariance relationship between the expected return of asset i and the expected return of asset j.

$$Sharpe\ ratio = \frac{E(r) - R_f}{\sigma_P}. \quad (4)$$

R_f represents the risk-free rate, while σ_P is the standard deviation of the portfolio, which is the standard square root of the variance. The Sharpe rate is then an effective tool to help investors find the optimal return-to-risk ratio.

2.2.2 CAPM（Capital assets pricing model）

The Capital Asset Pricing Model (CAPM) is used to study the relationship between expected returns on assets and risky assets in the securities market and the way equilibrium prices are formed [10]. The essence of the CAPM is discussing the correlation between risk and return on capital assets, i.e., high risk is usually accompanied by high return [11]. However, the model is only a one-factor model considering only the market portfolio, while the CAPM model also makes the following five assumptions.

- Investors are rational mean-variance optimizers
- Information is costless and available to all investors
- There are homogeneous expectations
- Individual investors are price takers
- No taxes and transaction costs

The model equation is shown below.

$$E(r) = R_f + \beta_i * (E(R_m) - R_f). \quad (5)$$

E(r) represents the expected return. R_f is the risk-free rate. β_ii represents the beta value of the investment. The return of the market portfolio is R_m, and $E(R_m)$ is the expected market return. Since β_i is positive in almost all cases, it can be assumed to reflect the sensitivity of the asset to market volatility. Therefore, when β_i >1, the asset has a higher risk relative to the market. Conversely, when β_i<1, the asset has a lower risk relative to the market.

2.2.3 FF3F(Fama-French three-factor model)

As the market developed, the CAPM model did not work as well as expected in the empirical evidence. Later, Fama and French in 1992, while studying stocks in the U.S. capital market, found that the beta of the stock market could not

fully explain the differences between the returns of different stocks. However,

through analysis, it was concluded that stock returns could be explained by decomposing them into three factors: the market capitalization of the listed company, the book-to-market ratio, and the excess market risk taken [12]. The model can be expressed as follows.

$$E(R_p - R_f) = \beta_p * (E(R_m) - R_f) + s_p(E(SMB)) + h_p(E(HML)) \quad (6)$$

R_p denotes the expected portfolio return, R_f denotes the risk-free rate, and $E(R_p - R_f)$ denotes the expected portfolio excess return. $E(R_m)$ denotes the expected market return, and $E(R_m) - R_f$ denotes the market risk premium. SMB is the simulated portfolio return for the market capitalization factor at time t, and HML is the simulated portfolio return for the book-to-market ratio factor at time. β_p s_p, h_p are all factor coefficients.

3. Results

Assuming that the portfolio without pension buyout is Portfolio I and the portfolio with pension buyout are Portfolio II, the maximum Sharpe rate portfolio can be found for each of the two portfolios using Mean-Variance Analysis, and the results are as follows.

Table 2. Results for maximum Sharpe ratio portfolio I optimization under Mean-Variance Analysis

	DIS'	MC.PA'	AAPL'	SBUX'	FL GO BOND
MVE portfolio weight	0.0993	0.4929	0.0810	-0.3832	0.71
Portfolio return	0.0163542344				
Portfolio variance	0.000466215				
Standard deviation	0.02159202				
Sharp Ratio	0.757420303				

As the weight of FL GO BOND is fixed, the largest weight other than it is 0.4929, which appears in 'MC.PA', while 'SBUX' has the smallest share of -0.3832. This result is because LVMH in recent years this result is since LVMH has made a lot of mergers and acquisitions in recent years, and has rapidly expanded its market share and developed its business well. However, Starbucks has been squeezed by many emerging coffee brands in recent years, and its own branded products are no longer competitive.

Table 3. Results for maximum Sharpe ratio portfolio II optimization under Mean-Variance Analysis

	DIS'	MC.PA'	'AAPL'	SBUX'
MVE portfolio weight	-0.3089	0.7443	0.4383	0.1263
Portfolio return	0.031348581			
Portfolio variance	0.003892104			
Standard deviation	0.062386727			
Sharp Ratio	0.502487993			

In the case of not considering FL GO BOND, 'MC.PA' still accounted for the highest ratio: 0.7443. However, in this case, the lowest ratio appeared in Disney, it accounted for -0.3089. This is the industry turmoil that led to its prospects are not good. Specifically, due to the impact of the epidemic in recent years leading to the decline in tourism and the slump in the film industry, Disney's two pillar industries have been hit one after another, with insufficient revenue. In addition, the estimation results obtained from the CAPM model are as follows.

Table 4. Results for maximum Sharpe ratio portfolio I optimization under CAPM model

	DIS'	MC.PA'	AAPL'	SBUX'	FL GO BOND
Portfolio weight	0.1848	0.1879	0.0869	-0.1695	0.71
Portfolio return	0.00347				
Portfolio variance	0.000428				
Portfolio standard deviation	0.020695				
Portfolio Sharp Ratio	0.12528				

Considering the risk premium, both Disney and LVMH account for a higher percentage of portfolio I at 0.1848 and 0.1879, respectively, because these two companies have a higher risk premium as the top companies in the industry. According to the theory of the CAPM model, the higher the risk premium, the higher the contribution of the asset in the portfolio. This theory also explains why Starbucks, which has a low-risk premium, only has a -0.1695 share in the portfolio.

Table 5. Results for maximum Sharpe ratio portfolio II optimization under CAPM model

	DIS'	MC.PA'	'AAPL'	SBUX'
Portfolio weight	0.2982	0.2617	0.2860	0.1541
Portfolio return	0.007574			
Portfolio variance	0.003164			
Portfolio standard deviation	0.056251			
Portfolio Sharp Ratio	0.119061			

When FL GO BOND is removed from the portfolio, Disney has the largest share in the portfolio: 0.2982. However, Apple, LVMH, and Disney all have similar shares in this portfolio. The reason for this result is that the first three companies are the top companies in their respective industries and have a high level of risk premium. Therefore, their contribution to the portfolio is also higher. On the contrary, Starbucks is affected by the competition in the industry, the arrival of a large number of coffee chains in the market, and the rise of boutique coffee, which leads to a decline in its position in the industry and a lower level of risk premium, resulting in a lower share in the portfolio. Next, the results of the maximum Sharpe rate portfolio were obtained according to the Fama-French three-factor model.

Table 6. Results for maximum Sharpe ratio portfolio I optimization under FF3F model

	DIS'	MC.PA'	AAPL'	SBUX'	FL GO BOND
Portfolio weight	0.1623	0.3212	0.0334	-0.2269	0.71
Portfolio return	0.006961858				
Portfolio variance	0.000384894				
Portfolio standard deviation	0.019618724				
Portfolio Sharp Ratio	0.310145832				

In the results of the FF3F model, LVMH continues to have the highest share: 0.3212, while Starbucks continues to have the lowest share - 0.2269. Part of the reason could be that 71% of the assets in this portfolio are fixed as FL GO BOND, which is a low-return and low-risk asset. To increase the Sharpe rate of the portfolio, the only way to achieve the goal is to maximize the return on the remaining assets, provided that the majority of the assets are fixed as low-risk assets. Therefore, in this portfolio, the high-yielding LVMH is more favored, while the low-yielding Starbucks is not favored to choose for short-selling operations.

Table 7. Results for maximum Sharpe ratio portfolio II optimization under FF3F model

	DIS'	MC.PA'	'AAPL'	SBUX'
Portfolio weight	0.3773	0.4151	0.1224	0.0852
Portfolio return	0.00747701			
Portfolio variance	0.003166411			
Portfolio standard deviation	0.056270869			
Portfolio Sharp Ratio	0.117286557			

LVMH continues to have the highest percentage of the portfolio, being in the luxury industry, with a stable target audience and a high brand premium. As a result, it is a low-risk, high-margin industry, resulting in it accounting for about half of the portfolio. On the contrary, Starbucks, which is a restaurant chain, is facing price competition and cost increases in the economic downturn, leading it to gradually become a high-risk, low-revenue business. It ended up with only 0.0852 in the portfolio.

4. Conclusion

According to the current information statistics, the portfolio I has a higher Sharpe rate in either model calculation. Based on the previous assumptions, the portfolio I would be a better choice for this firefighter. On the other hand, however, the higher Sharpe rate for the portfolio I is since 71% of the assets in portfolio I are fixed in very low-risk municipal bonds. The low level of risk affects the higher Sharpe rate. Therefore, when comparing the returns of the two portfolios, Portfolio I is significantly lower than Portfolio II. However, a portfolio with a sufficiently high Sharpe rate at lower risk is more suitable for a retiree. However, there is no denying that these methods still have shortcomings, for example, there are more factors affecting the portfolios that are probably not considered.

References

[1] Zhu, W. S. A study on the investment management of U.S. pensions under the financial crisis. Journal of the Central Party School of the Communist Party of China, 2010, (06): 105-107.

[2] Novy-Marx, R., Rauh, J. D. The liabilities and risks of state-sponsored pension plans. Journal of Economic Perspectives, 2009, 23(4): 191-210.

[3] Nora. Nora: Current status and experience of the U.S. pension market and pension investment. China Pension Finance 50 Forum Shenzhen International Summit Conference Presentation Materials 2017: 14-17

[4] Li, T. L. Implications of U.S. pension investment and operation for China. Wisdom and wealth era, 2018, (05): 25.

[5] Gao J. The development and choice of the U.S. pension system under the aging of the population - the example of private sector employer-provided pension plans. Journal of Heilongjiang Academy of Political and Legal Management Cadres, 2020 (06): 71-76.

[6] Feng, Y L. Implications of U.S. pension market entry for China. China Economic Report, 2017, (08): 94-97.

[7] Wei J. Pension system for college teachers in the United States. China Institute of Education Finance Science, Peking University. 2017

[8] Zola. A study of portfolio strategy based on mean-variance model. Research in Financial Theory, 2021, (05): 9-16.

[9] Ma, Sweet. Mean-variance model theory and its application in China's stock market. Wealth Times, 2022, (01): 148-150.

[10] Ban D L. Risk analysis of Luzhou Laojiao Co. based on CAPM model. Journal of Economic Research, 2022, (08): 114-116.

[11] Yang Z H. A test of CAPM model and Fama-French three-factor model for China's A-share market. Journal of Economic Research, 2022 (08): 117-119.

[12] Shi H T. An investigation of the applicability of Fama-French three-factor model and the impact of risk warning on portfolio excess return--an empirical study based on SSE main board data. Modern Business Industry, 2022 (01): 118-122.