

# Application of portfolio optimization using pension and other 6 stocks

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**Abstract.** This paper uses the Capital Asset Pricing Model and Fama-French Three-Factor Model (FF3F) to construct optimal portfolios with maximized Sharpe ratio, aiming to help retired firefighters decide between the pension's annual benefits and the lump sum of the pension. Firstly, this paper selects six well-diversified assets from different industries. Secondly, this study estimates the expected return of each asset using the CAPM and FF3F Model, respectively. Thirdly, this paper constructs two 'With Pension Portfolio's using the expected return from the two models. Fourthly, this paper also builds two portfolios using the same results from the two models but keeping the pension's weight to zero. By comparing the maximized Sharpe ratios from 'With Pension Portfolio's and 'Without Pension Portfolio's, we conclude that the firefighter should choose to take the benefits of the pension. The results in this paper can serve as a reference for retired people when they want to allocate their nest egg and pension better.

**Keywords:** CAPM; Fama-French Three-Factor Model; Pension; Sharpe ratio.

## 1. Introduction

The aging of industrialized societies happens everywhere in the world. For example, the median age in the European Union (EU), has increased vastly between 1985 and 2019, and this trend has been replicated elsewhere in the world [1]. At the same time, Zhu claims that people currently lack awareness of pension planning [2]. Therefore, there is a need for more research on pension investment to help retired people do pension planning.

Research on pension finance has snowballed these years. Deng and Li study the solution to China's pension finance problems by introducing a digital economy and providing a reference for formulating policies [3]. Wang, Rong, and Zhao find that the TBP model for loss-averse participants effectively provides a steady and continuable pension account [4]. Boonen and Waegenaere model intergenerational risk sharing in closing funded pension plans and find that older generations benefit substantially from repeated bargaining [5]. Luis and Ranu study the influence of administrative charges or fees on balance or flow during the accumulation phase of a defined-contribution (DC) pension plan on individual welfare [6].

However, relatively little research has been done to help retired firefighters choose between pensions and reinvest in them. This article uses the promise of a pension in Florida as an example to help a retired doctor decide on a pension. The client has served as a firefighter for many years and is about to retire. He is in good health and has no serious diseases. The client has a deposit of around \$ 600,000 in addition to his pension. During his retirement, the state promises him an annual payment of \$ 100,000 plus an annual 3% cost of living increase until his dead. Meanwhile, the firefighter has another option, which is to receive a lump sum payment of \$ 1.450 million dollars upon retirement instead of the annual benefits. The lump sum payment is the amount with earnings of 7.75% annually through the period of Client X's life expectancy. Moreover, he is constrained to have a large percentage of his wealth tied up with the state pension fund. Therefore, he will have approximately  $w=70.73\%$  ( $w=1,450,000/2,050,000$ ) of his wealth invested in something with a risk profile similar to that of long-term State of Florida bonds. This paper aims to judge the attractiveness of the pension benefit relative to the buyout in the context of IRR equal to 7.75% and design a new portfolio with six other assets to reinvest with his nest egg and the lump sum payment.

In other words, this paper's ultimate goal is to help Client X decide between two scenarios:

1) Client X takes the lump sum payment and invests it and his nest egg in the newly designed portfolio.

2) Client X takes the pension benefits and invests his nest egg in the newly designed portfolio.

To help the retired firefighter make a better choice, the first step of this research is to select appropriate data from well-diversified assets. Assets are selected based on the Sharpe ratio, standard deviation, correlation coefficients, and diversification in several aspects. Secondly, this paper calculates the estimated expected returns for each asset using the data from January 2017 to January 2020 through the CAPM and Fama-French Three-Factor Model methods. Thirdly, this paper uses the expected returns to construct two ‘With Pension Portfolio’s and two ‘Without Pension Portfolio’s. Lastly, this paper reaches a conclusion by comparing the maximized Sharpe ratios of ‘With Pension Portfolio’s with the maximized Sharpe ratios of ‘Without Pension Portfolio’s.

The summary of the remaining sections is as follows. Section 2 is about the process of choosing good assets’ data to design the portfolio. Section 3 depicts the model and analysis method used in this paper. Section 4 shows the results, and the last section is the conclusion.

## 2. Data

This paper derives data from Yahoo finance (<https://finance.yahoo.com/>). Assets are selected based on three criteria: diversification, Sharpe ratio, and correlation between assets. Ten assets from various industries are chosen, including the internet, entertainment, commodities, airline, etc. At the same time, we focus on some indicators that would help us do better diversification, such as EPS (TTM), Market Cap, and Beta (5-year month). Finally, the ten selected assets’ basic data information is shown in Table 1.

**Table 1.** First Selected Assets

|              | Asset description                | EPS<br>(TTM) | Beta (5Y<br>Monthly) | Market<br>Cap | Sharpe<br>Ratio | Standard<br>deviation |
|--------------|----------------------------------|--------------|----------------------|---------------|-----------------|-----------------------|
| <b>AAL</b>   | American Airlines Group Inc.     | -3.67        | 1.53                 | 8.403B        | -0.0672         | 0.1226                |
| <b>AMC</b>   | AMC Entertainment Holdings, Inc. | -2.05        | 1.67                 | 4.918B        | 0.1486          | 0.7828                |
| <b>AMD</b>   | Advanced Micro Devices, Inc.     | 2.28         | 1.96                 | 132.907B      | 0.3423          | 0.1594                |
| <b>APRN</b>  | Blue Apron Holdings, Inc.        | -4.29        | -2.80                | 196.596M      | 0.0317          | 0.5218                |
| <b>BABA</b>  | Alibaba Group Holding Limited    | 2.16         | 0.58                 | 245.492B      | -0.0630         | 0.0993                |
| <b>GLD</b>   | SPDR Gold Shares                 | N/A          | 0.05                 | N/A           | 0.1592          | 0.0376                |
| <b>GOOGL</b> | Alphabet Inc.                    | 5.23         | 1.08                 | 1.488T        | 0.2406          | 0.0703                |
| <b>INTC</b>  | Intel Corporation                | 4.66         | 0.62                 | 139.399B      | 0.0909          | 0.0753                |
| <b>TSLA</b>  | Tesla, Inc.                      | 8.12         | 2.18                 | 930.943B      | 0.2968          | 0.2067                |
| <b>SBUX</b>  | Starbucks Corporation            | 3.55         | 0.95                 | 97.988B       | 0.2365          | 0.0650                |

Table 1 shows that the assets we selected are well-diversified from the above aspects. AMC and APRN are eliminated because of their high standard deviation which represent high risks. Next, the correlation coefficient shown in Table 2 indicates the direction of change between two assets also needs to be considered. A smaller correlation coefficient facilitates diversification. Two assets are strongly correlated if the absolute value of the correlation coefficient is above 0.8; between 0.4 and 0.8 is considered moderately correlated; below 0.4, we believe a weak or even no correlation [7]. In

conclusion, we screen out the final six assets: ‘AAL’, ‘AMD’, ‘BABA’, ‘INTC’, ‘SBUX’, and ‘TSLA’.

**Table 2.** Correlation coefficients of the final selected assets

|      | AAL     | AMD     | BABA    | INTC    | SBUX    | TSLA |
|------|---------|---------|---------|---------|---------|------|
| AAL  | 1       |         |         |         |         |      |
| AMD  | 0.14306 | 1       |         |         |         |      |
| BABA | 0.18424 | 0.28387 | 1       |         |         |      |
| INTC | 0.47598 | -0.1493 | 0.10619 | 1       |         |      |
| SBUX | 0.4108  | 0.17987 | 0.04293 | 0.23515 | 1       |      |
| TSLA | 0.21726 | 0.18444 | 0.2287  | 0.13705 | 0.14647 | 1    |

**Table 3.** Descriptive statistics of the selected assets

|          | AAL     | AMD     | BABA    | INTC    | SBUX    | TSLA    |
|----------|---------|---------|---------|---------|---------|---------|
| Mean     | -0.74%  | 5.54%   | 1.00%   | 1.06%   | 1.62%   | 7.10%   |
| Variance | 1.50%   | 2.54%   | 0.97%   | 0.52%   | 0.42%   | 3.78%   |
| Max      | 25.27%  | 47.18%  | 22.92%  | 19.46%  | 16.72%  | 74.15%  |
| Min      | -35.77% | -41.05% | -22.68% | -20.22% | -15.79% | -22.43% |

Lastly, as the descriptive statistics shown in Table 3, ‘TSLA’ shows the highest arithmetic average return, while the lowest average return is -0.74% from ‘AAL’. To compare their variances, ‘TSLA’ shows the highest variance while ‘SBUX’ offers the most down. Additionally, ‘TSLA’ has the highest maximum monthly return, and ‘SBUX’ has the lowest maximum monthly return. Moreover, ‘SBUX’ has the highest minimum monthly return, and ‘AMD’ has the lowest.

### 3. Methods

#### 3.1 Sharpe Ratio Analysis

Novel laureate William F. Sharpe developed the Sharpe ratio and used it to help investors make a better trade-off between the return of an investment and its risk [8]. Zhang claims that the Sharpe ratio is directly proportional to the risk-adjusted return, which means the higher the Sharpe ratio, the more appealing the risk-adjusted return [9]. This paper uses the maximal Sharpe ratio as the primary measurement to judge the attractiveness of the ‘with pension portfolio’ related to the ‘without pension portfolio’. The Sharpe ratio is calculated in the following formula:

$$\text{Sharpe ratio} = E(R_i - R_f) / \sigma(R_i - R_f) \quad (1)$$

Where  $R_i$  is the expected return of ‘asset i’ and  $R_f$  is the risk-free rate.  $E(R_i - R_f)$  is the excess return, and  $\sigma(R_i - R_f)$  is the standard deviation of excess return.

#### 3.2 Capital Asset Pricing Model

Initially, Markowitz proposed the Modern portfolio theory, introducing the mean-variance analysis on portfolio optimization [10]. After that, American economists Lintel, and Mosin first put forward

the CAPM in 1964 on the basis of portfolio theory and capital market theory. This model mainly studies the relationship between a single asset's expected return rate and the asset market portfolio's return rate [11]. Song claims that the CAPM model has been dominant in the field of financial pricing model, and it applies diffusely in corporate finance [12]. The calculation formula of CAPM model is exhibited as follows:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \quad (2)$$

Where  $R_i$  is the expected return of 'Portfolio i' and  $R_f$  is the risk-free rate and  $\beta_i$  is the  $\beta$  coefficient of a certain portfolio. The beta coefficient indicates the systematic risk and it is used in the CAPM model to express the relationship between the return of investment in an asset and the return of market portfolio.  $E(R_m)$  is the expected return of the market portfolio and  $E(R_m) - R_f$  is denoted by the expected return of the market minus the risk-free rate. It is also known as market risk premium.

### 3.3 Fama-French Three-Factor Model

After the famous CAPM's development, Banz found out that the assets' returns correlate not only with the market risk premium but also with their market value [13]. Then, more relevant factors have been proposed. Fama and French developed the three-factor model in 1993. Fama-French Three-Factor Model is an extension of the CAPM, which adds two new considerations based on the CAPM, namely SMB and HML. SMB and HML refer to two different factors: the market value factor and the book-to-market ratio factor. The model can be displayed as:

$$E(R_p) - R_f = \beta_{mkt}(E(R_m) - R_f) + \beta_{SMB}(SMB) + \beta_{HML}(HML) \quad (3)$$

Where  $E(R_p)$  is the expected return of the portfolio,  $R_f$  is the risk-free rate, and  $(E(R_m) - R_f)$  is the market risk premium.  $SMB$  is the size risk premium (small minus big),  $HML$  is the value risk premium (high minus low) and  $\beta_{mkt}$ ,  $\beta_{SMB}$ ,  $\beta_{HML}$  are the three beta coefficients.

## 4. Results

### 4.1 CAPM

Firstly, this paper calculates the expected return for each asset using the CAPM model. The data for market risk premium comes from Ken French Data Base. This article estimates the average market risk premium value from July 1926 to May 2022. Then we used the excess return of each asset from January 2017 to January 2022 to perform a regression analysis on the Market excess return from January 2017 to January 2022 to obtain the Market beta of each asset. According to the following formula, the expected return of each asset is shown in the following table.

**Table 4.** CAPM Model: Expected returns

|                 | AAL    | AMD    | BABA   | INTC   | SBUX   | TSLA   |
|-----------------|--------|--------|--------|--------|--------|--------|
| <b>MKT beta</b> | 1.6148 | 1.7627 | 0.8965 | 0.5093 | 0.7580 | 1.9938 |
| <b>E(r)</b>     | 0.0136 | 0.0146 | 0.0087 | 0.0061 | 0.0078 | 0.0162 |

### 4.2 Fama-French Three-Factor Model

Similarly, this paper uses the Fama-French Three-Factor Model to calculate the expected return of the six selected assets. Data for the Market risk premium, SMB risk premium, and HML risk premium

are from Ken French Data Base. This paper uses the average monthly risk premium from July 1926 to May 2022. Next, this paper uses the excess return for each asset and Market Excess return, SMB Excess Return, and HML Excess Return to perform regression analysis and obtain the beta coefficients of each asset. The data for excess returns are from January 2017 to January 2022. According to the following formula, the expected return of each asset is shown in Table 5.

**Table 5.** Fama French 3 Factor Model: Expected returns

|                      | AAL    | AMD     | BABA    | INTC   | SBUX    | TSLA    |
|----------------------|--------|---------|---------|--------|---------|---------|
| <b>FF3F Mkt beta</b> | 1.3196 | 2.0492  | 0.9630  | 0.4050 | 0.8062  | 2.1747  |
| <b>FF3F SMB beta</b> | 0.5982 | -0.4542 | 0.2198  | 0.4497 | -0.4476 | -0.1853 |
| <b>FF3F HML beta</b> | 1.2640 | -1.3668 | -0.6769 | 0.1830 | 0.1806  | -0.9752 |
| <b>FF3F E(r)</b>     | 0.0173 | 0.0108  | 0.0072  | 0.0069 | 0.0079  | 0.0135  |

### 4.3 With Pension Portfolios

This paper selects the monthly returns of Florida government bonds from January 2017 to January 2022 as pension proxy returns to calculate the covariance matrix between pension and other six assets. The covariance matrix is shown in Table 6.

**Table 6.** Variance-Covariance Matrix (FL GO BOND)

|                | Pension | AAL     | AMD      | BABA    | INTC     | SBUX    | TSLA    |
|----------------|---------|---------|----------|---------|----------|---------|---------|
| <b>Pension</b> | 0.0146% | 0.0066% | 0.0550%  | 0.0397% | 0.0124%  | 0.0099% | 0.0373% |
| <b>AAL</b>     | 0.0066% | 1.4987% | 0.2791%  | 0.2227% | 0.4189%  | 0.3267% | 0.5173% |
| <b>AMD</b>     | 0.0550% | 0.2791% | 2.5398%  | 0.4466% | -0.1711% | 0.1862% | 0.5717% |
| <b>BABA</b>    | 0.0397% | 0.2227% | 0.4466%  | 0.9746% | 0.0754%  | 0.0275% | 0.4391% |
| <b>INTC</b>    | 0.0124% | 0.4189% | -0.1711% | 0.0754% | 0.5169%  | 0.1098% | 0.1916% |
| <b>SBUX</b>    | 0.0099% | 0.3267% | 0.1862%  | 0.0275% | 0.1098%  | 0.4219% | 0.1850% |
| <b>TSLA</b>    | 0.0373% | 0.5173% | 0.5717%  | 0.4391% | 0.1916%  | 0.1850% | 3.7828% |

#### 4.3.1 CAPM

The monthly and annual expected returns calculated through the CAPM are summarized in Table 7.

**Table 7.** Monthly and Annual Expected Returns (CAPM)

|                | Risk-free rate | Pension | AAL      | AMD      | BABA     | INTC    | SBUX    | TSLA     |
|----------------|----------------|---------|----------|----------|----------|---------|---------|----------|
| <b>Monthly</b> | 0.2666%        | 0.6240% | 1.3600%  | 1.4601%  | 0.8737%  | 0.6115% | 0.7799% | 1.6166%  |
| <b>Annual</b>  | 3.2471%        | 7.7500% | 17.5977% | 18.9994% | 11.0027% | 7.5895% | 9.7702% | 21.2208% |

Initially, this paper uses the method of compound interest to convert the committed annualized IRR of pension into the monthly interest rate. Next, under the assumption that short-selling is not allowed and combined with the monthly expected return calculated by the CAPM, this paper finds

the maximized Sharpe ratio while keeping the pension weight equal to 70.73%. The weight allocation and the maximized Sharpe ratio are shown in Table 8.

**Table 8.** CAPM: With pension portfolio

|                    | Pension | AAL   | AMD   | BABA  | INTC  | SBUX   | TSLA     |
|--------------------|---------|-------|-------|-------|-------|--------|----------|
| Portfolio weight   | 70.73%  | 2.71% | 2.50% | 3.37% | 6.68% | 12.58% | 1.42%    |
| Expected return    |         |       |       |       |       |        | 0.7061%  |
| Standard deviation |         |       |       |       |       |        | 1.9800%  |
| Sharpe ratio       |         |       |       |       |       |        | 22.1948% |

From the table, we can conclude that under the CAPM, the maximized Sharpe ratio for the ‘With Pension Portfolio’ is 22.1948%. The highest weight is allocated to ‘SBUX’ and the lowest weight is allocated to ‘TSLA’.

#### 4.3.2 Fama-French Three-Factor Model

The monthly and annual expected returns calculated through the FF3F Model are summarized in Table 9. Similarly, under the assumption that short selling is not allowed, this paper uses the monthly expected return calculated by Fama-French Three-Factor Model to construct the ‘With Pension Portfolio’. By keeping the portfolio weight for pension as 70.73%, the maximized Sharpe ratio is 22.5951%. The highest portfolio weight is distributed to ‘SBUX’ and the lowest weight is distributed to ‘TSLA’.

**Table 9.** Monthly and Annual Expected Returns (FF3F)

|         | Risk-free rate | Pension | AAL      | AMD      | BABA    | INTC    | SBUX    | TSLA     |
|---------|----------------|---------|----------|----------|---------|---------|---------|----------|
| Monthly | 0.2666%        | 0.6240% | 1.7279%  | 1.0766%  | 0.7168% | 0.6917% | 0.7928% | 1.3533%  |
| Annual  | 3.2471%        | 7.7500% | 22.8235% | 13.7123% | 8.9485% | 8.6236% | 9.9398% | 17.5046% |

**Table 10.** FF3F: With pension portfolio

|                    | Pension | AAL   | AMD   | BABA  | INTC  | SBUX   | TSLA     |
|--------------------|---------|-------|-------|-------|-------|--------|----------|
| Portfolio weight   | 70.73%  | 6.05% | 1.26% | 2.53% | 6.23% | 12.39% | 0.80%    |
| Expected return    |         |       |       |       |       |        | 0.7298%  |
| Standard deviation |         |       |       |       |       |        | 2.0498%  |
| Sharpe ratio       |         |       |       |       |       |        | 22.5951% |

#### 4.4 Without Pension Portfolios

##### 4.4.1 CAPM

By using the expected return summarized in Table 7, this paper calculates the maximized Sharpe ratio and weights of ‘without pension portfolio’ by setting the weight of the pension to zero. The most considerable weight is for ‘SBUX’ to be 31.93%, the most negligible weight is allocated to ‘TSLA’. The maximized Sharpe ratio is 12.6161%, less than 22.1948%.

**Table 11.** CAPM: Without pension portfolio

|                    | Pension | AAL    | AMD    | BABA   | INTC   | SBUX   | TSLA     |
|--------------------|---------|--------|--------|--------|--------|--------|----------|
| Portfolio weight   | 0       | 18.11% | 15.34% | 13.95% | 11.43% | 31.93% | 9.24%    |
| Expected return    |         |        |        |        |        |        | 1.0605%  |
| Standard deviation |         |        |        |        |        |        | 6.2922%  |
| Sharpe ratio       |         |        |        |        |        |        | 12.6161% |

#### 4.4.2 Fama-French Three-Factor Model

Similarly, this paper calculates the portfolio without pension using the expected return from Table 9. The result is shown in Table 12. The largest weight is allocated to ‘AAL’, and the lowest weight is allocated to ‘INTC’. The maximized Sharpe ratio is 13.1058%, which is also smaller than 22.5951%.

**Table 12.** FF3F: Without pension portfolio

|                           | Pension | AAL    | AMD   | BABA     | INTC  | SBUX   | TSLA  |
|---------------------------|---------|--------|-------|----------|-------|--------|-------|
| <b>Portfolio weight</b>   | 0       | 42.03% | 8.66% | 8.31%    | 5.35% | 29.16% | 6.48% |
| <b>Expected return</b>    |         |        |       | 1.2350%  |       |        |       |
| <b>Standard deviation</b> |         |        |       | 7.3888%  |       |        |       |
| <b>Sharpe ratio</b>       |         |        |       | 13.1058% |       |        |       |

#### 4.5 Sensitivity Analysis

This paper sets N (Estimated Expected Life Expectancy) and g (Growth Rate) as two independent variables and taking the Present value of the pension (PV) as the dependent variable, it has the following calculation formula.

$$PV = \frac{K}{r - g} \left( 1 - \left( \frac{1 + g}{1 + r} \right)^N \right) \quad (4)$$

Where the annual payment from the pension (K) is known to be \$ 100,000, and the IRR (r) is also known as 7.75%. By using this formula, this paper changes G from 0.020 to 0.030 (with an interval of 0.001) and N from 16 to 30 (with an interval of 1). PV values are calculated for each g and N. The detailed results are shown in Table 13. From the table, we can see that if the growth rate ranges from 0.024 to 0.030, only a life expectancy longer than 25 years has the probability of providing a present value larger than \$ 1,450,000.

**Table 13.** Sensitivity Analysis

|    | 0.020          | 0.021          | 0.022          | 0.023          | 0.024          | 0.025          | 0.026          | 0.027          | 0.028          | 0.029          | 0.030          |
|----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 16 | 1015930        | 1022281        | 1028685        | 1035144        | 1041657        | 1048225        | 1054849        | 1061529        | 1068265        | 1075059        | 1081910        |
| 17 | 1054523        | 1061484        | 1068507        | 1075594        | 1082744        | 1089959        | 1097239        | 1104585        | 1111997        | 1119476        | 1127023        |
| 18 | 1091057        | 1098631        | 1106278        | 1113997        | 1121791        | 1129659        | 1137603        | 1145623        | 1153719        | 1161894        | 1170147        |
| 19 | 1125641        | 1133831        | 1142103        | 1150459        | 1158899        | 1167425        | 1176038        | 1184737        | 1193525        | 1202403        | 1211371        |
| 20 | 1158379        | 1167184        | 1176083        | 1185076        | 1194165        | 1203351        | 1212635        | 1222019        | 1231503        | 1241088        | 1250776        |
| 21 | 1189371        | 1198789        | 1208312        | 1217942        | 1227680        | 1237527        | 1247484        | 1257553        | 1267735        | 1278032        | 1288445        |
| 22 | 1218708        | 1228736        | 1238882        | 1249146        | 1259531        | 1270037        | 1280667        | 1291422        | 1302303        | 1313313        | 1324453        |
| 23 | 1246480        | 1257114        | 1267877        | 1278772        | 1289800        | 1300963        | 1312264        | 1323703        | 1335283        | 1347006        | 1358874        |
| 24 | 1272770        | 1284003        | 1295378        | 1306899        | 1318566        | 1330383        | 1342350        | 1354471        | 1366748        | 1379183        | 1391778        |
| 25 | <b>1297657</b> | <b>1309482</b> | <b>1321463</b> | <b>1333603</b> | <b>1345904</b> | <b>1358369</b> | <b>1370999</b> | <b>1383798</b> | <b>1396768</b> | <b>1409911</b> | <b>1423231</b> |
| 26 | 1321216        | 1333625        | 1346205        | 1358957        | 1371885        | 1384991        | 1398278        | 1411750        | 1425408        | 1439256        | 1453297        |
| 27 | 1343518        | 1356502        | 1369672        | 1383028        | 1396575        | 1410316        | 1424254        | 1438392        | 1452733        | 1467280        | 1482038        |
| 28 | 1364629        | 1378180        | 1391930        | 1405882        | 1420040        | 1434408        | 1448988        | 1463785        | 1478802        | 1494043        | 1509512        |
| 29 | 1384614        | 1398721        | 1413042        | 1427580        | 1442340        | 1457325        | 1472540        | 1487988        | 1503674        | 1519601        | 1535775        |
| 30 | 1403533        | 1418185        | 1433066        | 1448180        | 1463532        | 1479126        | 1494966        | 1511057        | 1527403        | 1544009        | 1560880        |

## 5. Conclusion

This paper aims to help retired investors best allocate their nest eggs and better choose between the pension and the lump sum payment. Firstly, this paper chooses assets from various fields, such as the internet, beverage, civil aviation, etc. Secondly, this paper used two famous models, namely

CAPM and Fama-French Three-Factor (FF3F) Model, to calculate the expected return of each asset. The CAPM's beta coefficient is obtained by regression analysis of each asset's excess return and market excess return. For FF3F Model, the beta coefficients are obtained by regression analysis of the excess return of each asset, market excess return, SMB excess return, and HML excess return. After that, this paper uses the expected returns to construct two 'With Pension Portfolio's and two 'Without Pension Portfolio's. Based on the CAPM, the Sharpe ratio of the 'With Pension Portfolio' is 22.1948%, almost twice the Sharpe ratio of the 'Without Pension Portfolio'. Also, according to the FF3F Model, the Sharpe ratio of the 'With Pension Portfolio' is 22.5951%, almost twice the Sharpe ratio of the 'Without Pension Portfolio'. Additionally, this paper implements the sensitivity analysis between the Present Value (PV) of the pension and the Growth Rate (g), the Estimated Expected Life Expectancy (N). Based on the Sharpe ratio and sensitivity analysis, this paper suggests that the retired investor should keep the pension.

However, there is no doubt that the models used in this paper have shortcomings. Many other risk factors are not included in the CAPM and Fama-French Three-Factor model. Therefore, the estimated expected returns calculated in this paper are somewhat inaccurate. Future studies can consider other risk factors, such as the RMW factor and the CMA factor, to make the estimated expected return more precise.

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