# Application of portfolio optimization

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**Abstract.** Portfolio optimization, as the basis for quantitative investment finance, has become a fundamental topic in modern finance. Consequently, the portfolio constructing, and optimization is always a hotspot in the modern finance area. This paper concentrates on the application and analysis of portfolio optimization. Asset diversification plays a crucial role in examining how to improve the performance of a portfolio. To achieve portfolio diversification, this paper calculate the correlation coefficient matrix of the seven selected assets and look for smaller correlation coefficients. By referring to the Interpretation of Pearson's Correlation Coefficient and the process of constructing portfolios with assets from multiple industries, this paper constructs portfolio within the framework of mean-variance analysis and using the Capital Asset Pricing Model (CAPM) to initially demonstrate portfolio performance. Finally, this paper adopts the Fama-French three-factor model in terms of expected return, Sharpe ratio and variance analysis to evaluate the financial performance of the portfolio.

**Key words:** Portfolio optimization, Fama–French three-factor model, mean-variance, diversification, Capital Asset Pricing Model (CAPM).

## 1. Introduction

In 1952, Harry Markowitz published the revolutionary "Portfolio Selection" for the financial community [1]. In addition, the introduction of mean-variance theory became the basis for the development of modern financial research, providing rational asset managers with the basic concepts of optimal portfolios by maximising investment yield for a given degree of risk, or minimises exposure to investment risk for a stated degree of return. [2]. The core idea of mean-variance theory is defined as follows: the returns of a single asset are treated as stochastic variables and the expectancy and variance are used to quantitate returns and risks of an investment. And the concept of efficient frontier came out, it can be explained as a summary plot of standard deviations and expected returns for all active assets (portfolios) above the global minimum variance portfolio. Generally speaking, the idea of diversification is one of the most powerful ideas in Finance, and probably the only “free lunch” available to investors [3]. One of the driving forces behind the development of the modern portfolio optimization is the ability of a diversified portfolio to effectively reduce investment risk and, to a certain extent, enhance profitability, in comparison to individual assets.

The application of portfolio optimization has increased in recent decades, for example, in the study of medicine, team management (strategic decision making), IT project portfolio management, projects selection, and evolutionary algorithms selection, defense and so on. In drug discovery, IrynaYevseyeva handle the problem of selecting drug candidates from a supplier database is reshaped into a portfolio selection problem [4]. By using the principle of portfolio optimization, the portfolio used to simulate the selection of a subset of molecules for a drug discovery as a combination scenario. In addition to taking into account the (bio)activity of the molecules selected in the combination, the model also takes into account the diversity of such combinations. For dense institutions, they can the components of defense system as assets in portfolio, like such as studies, training programs, equipment acquisition, upgrades or maintenance, and apply portfolio optimization. [5].

The aim of this paper is to establish a diversified portfolio and achieving the aim of portfolio optimization, after that, make an analyze of the portfolio’s performance. The diversified portfolio is constructed with 7 shares from multiple industries. To evaluate the performance of the portfolio, primally, CAPM in framework of mean-variance theory will be processed. For more comprehensive

illustration of the portfolio performance, Fama-french three factors model will be used to demonstrate the expected return, variance and Sharpe ratio.

The construction of the essay as follows, Part 2 shows the selected shares price data from multiple industries, descriptive data and the correlation analysis between the portfolio assets. Part 3 and 4 explained the theory of establishing the model and figuring the portfolios which are reached the standard of optimal portfolios. Part 5 is a brief discussion, and the final Part will be the conclusion and further discussion of the essay study.

## 2. Data

The raw data (share price) in this article is obtained from Yahoofinance ((https://finance.yahoo.com/)). To construct a diversified investment portfolio, we choose 7 less correlated stocks by referring to the Interpretation of Pearson's Correlation Coefficient. The stocks are, LVMUY, COST, LULU, UKOG.L, BABA, MRVL, IBM. Closing price of each stock monthly in 5 years from 1st June 2017 to 1st May. After gathering the raw data of each stock, we can calculate the rate of return by Formula [6].

$$Rate\ of\ return\ for\ share = \frac{Share\ price\ this\ month}{share\ price\ previous\ month} - 1 \quad (1)$$

**Table 1.** Descriptive data for selected data

| | Minimum | Maximum | Mean | Std. dev | Variance |
|---|---|---|---|---|---|
| LVMUY | -0.1405873 | 0.22617656 | 0.01920044 | 0.066932259 | 0.00447993 |
| COST | -0.1217586 | 0.14495911 | 0.01955587 | 0.058383602 | 0.00340864 |
| LULU | -0.1746609 | 0.34284952 | 0.03611746 | 0.109730771 | 0.01204084 |
| UKOG.L | -0.464 | 1.70731707 | 0.00756325 | 0.340111916 | 0.11567612 |
| BABA | -0.2268098 | 0.22922592 | 0.00101094 | 0.100756199 | 0.01015181 |
| MRVL | -0.1900711 | 0.25791872 | 0.02649682 | 0.10243571 | 0.01049307 |
| IBM | -0.2366246 | 0.18254596 | 0.0057633 | 0.073345582 | 0.00537957 |

From Table 1, this paper visualizes the descriptive data of the raw data of the selected share, in the view of average return, LULU has the highest average return (0.0361) and COST has the less fluctuations (0.05838) during this half of decade. UKOG.L is the most risk asset in this portfolio with relatively wide range from minimum share price (-0.464) to maximum share price (1.70731707).

In the selection of assets, we need to calculate the correlation coefficient to make sure the correlation between each portfolio asset should be weak positive or weak/strong negative, in order to achieve the diversification of portfolio, the correlation between each portfolio asset should be weak positive or weak/strong negative.

**Table 2:** Correlation coefficient between selected assets

| | LVMUY | COST | LULU | UKOG.L | BABA | MRVL | IBM |
|---|---|---|---|---|---|---|---|
| LVMUY | 1 | | | | | | |
| COST | 0.26382658 | 1 | | | | | |
| LULU | 0.36759374 | 0.46338562 | 1 | | | | |
| UKOG.L | 0.08983489 | -0.130109 | 0.26201148 | 1 | | | |
| BABA | 0.29005919 | 0.05985428 | 0.26270184 | 0.21883043 | 1 | | |
| MRVL | 0.45341905 | 0.43444582 | 0.32581976 | -0.1038653 | 0.23887878 | 1 | |
| IBM | 0.4307363 | 0.26667803 | 0.26736447 | 0.02718608 | 0.27786754 | 0.38148307 | 1 |

The scatterplots demonstrate the evidence of LVMUY has moderate positive correlation with MRVL and IBM, UKOG.L has the least correlation with other assets, and negative correlation shows between UKOG.L with COST and MRVL, which means that select UKOG.L into the portfolio can reduce the risk of the investment portfolio by achieving the diversification.

# 3. Methodology

## 3.1 CAPM Based on Mean-variance Analysis

As the foundation theory of modern portfolio area, mean-variance analysis is a mathematical model for assets manager to achieve the portfolio optimization standards which are maximizing the expected return and minimizing the risk of the portfolio.

$$\sum_i Weight_i = 1 \tag{2}$$

Sum of the weights of the respective investment assets of the portfolio ($Weight_i$) equal to 1. To calculate the return and variance of a specific portfolio, the formula would be as follows.

$$R_P = \sum Weight_i \; x \; R_i \tag{3}$$

Where $R_p$stands for the return of the portfolio and the $Weight_i$ is the weight of asset and $R_i$ is defined as the return of the individual asset.

$$\sum_i Weight_i \; x \; ER_i = ER_p \tag{4}$$

According to Formula 4, expected return of the investment portfolio equals to the weight of individual asset multiple by the expected return of the individual asset.

$$Var(portfolio\ return) = \sum_{i,j} weight_i \; x \; weight_j \; x \; Cov[Asset\ return_i \; x \; Asset_j] \tag{5}$$

For the calculation of the portfolio's variance, equals to the weight of one of the portfolio asset multiples by weight of another portfolio asset and the result will be covariance of the assets return, then we can get the portfolio variance.

$$Sharpe\ Ratio = \frac{E(R_p) - R_f}{\sigma_P} \tag{6}$$

Sharpe ratio can be defined as a mathematics expression for the proposition that over time, excess returns may imply more volatility. It compares the return of the portfolio with its risk and $\sigma_p$ is the portfolio's standard deviation, $R_f$ represent the rate of risk-free, and $E(R_p)$ in term of the portfolio's expected return.

## 3.2 Fama-French Three Factors Model

Fama and French (1992) found CAPM as a single factor and single period model has lots of constrains. Therefore, they developed a more comprehensive pricing model by adding two factors, which are size risk factor and value risk factor. The Fama-French Three factors model is used to predict the expected return of the stocks, and the multiple models, can be expressed as the following formula.

$$E(r) = Rf + \; MKT\ RP \; x \; Mkt\ beta + SMB\ RP \; x \; SMB\ beta + HML\ RP \; x \; HML\ beta \tag{7}$$

In this equation, SMB (small minus large) represents the spread between the returns of large and small company stocks, HML (high minus low) is the differential between the returns of stocks with

high and low book gearing, and betas is the sensitivity of the state variable to the factors. $R_f$ is the risk free rate, $MKT\ RP$ is known as the market risk premium. Meanwhile, Fama-French three factors model is widely applicated in modern financial area. For instance, Jin Xu and Shaojun Zhang in 2013, wrote a study about the application in Chine stock market [6].

## 4. Results

In this section, the investment portfolios with maximum Sharpe ratio and minimum variance will be expressed within the framework of mean-variance analysis, CAPM and Fama-French 3 factor.

In the recent years, the industries in financial world have become more correlated. By referring the correlation coefficient between any two different companies, this paper selected the less correlated companies, and this result can be supported by Table 2.

According to Table 2, all the correlation coefficients are below 0.5 which explains the relationship between any two different companies moderate or small correlated with each other. Besides, the correlation coefficients are both negative and positive value, which means that an upward or downward movement in the price of one stock will result in an upward or downward movement in the price of another stock. What's more, all the correlation coefficients appear non-zero, which consistent with the statement that all the portfolio assets are correlated.

**Table 3.** Maximum Sharpe ratio portfolio under mean-variance analysis

| | LVMUY | COST | LULU | UKOG.L | BABA | MRVL | IBM |
|---|---|---|---|---|---|---|---|
| MVE weight | 0.29582844 | 0.44025524 | 0.1987278 | 0 | 0 | 0.06518852 | 0 |
| MVE return | 0.02319445 | | | | | | |
| MVE variance | 0.00299153 | | | | | | |
| MVE Std dev | 0.05469486 | | | | | | |
| Sharpe ratio | 0.42406993 | | | | | | |

**Table 4.** Minimum variance portfolio under mean-variance analysis

| | LVMUY | COST | LULU | UKOG.L | BABA | MRVL | IBM |
|---|---|---|---|---|---|---|---|
| MVE weight | 0.22925661 | 0.49821733 | 0 | 0.01776569 | 0.101189782 | 0 | 0.15357059 |
| MVE return | 0.01526664 | | | | | | |
| MVE variance | 0.00206471 | | | | | | |
| MVE Std dev | 0.04543906 | | | | | | |
| Sharpe ratio | 0.33598053 | | | | | | |

From Tables 3 and 4, this paper can notice that, for the purposes of maximum Sharpe ratio or minimum variance, the weight of assets in portfolio management will be the only controllable variables to change by the individual asset managers or an investment institution.

When building the portfolio with the purpose of maximizing the Sharpe ratio, the weight of UKOG.L, BABA and IBM stocks can be zero and for the minimum variance portfolio, the weight of LULU and MRVL stocks can be weighted zero. Furthermore, the common point of these two portfolios is the weight of the high-quality assets, like LVMUY and COST always stand on the top two occupation in these two portfolios.

**Table 5.** Outcome of maximum Sharpe ratio portfolio under CAPM

| | LVMUY | COST | LULU | UKOG.L | BABA | MRVL | IBM |
|---|---|---|---|---|---|---|---|
| CAPM weight | 0.24233014 | 0.23272857 | 0.10099003 | 0.01017416 | 0.092561504 | 0.101204 | 0.2200116 |
| CAPM return | 0.0070073 | | | | | | |
| CAPM variance | 0.00261117 | | | | | | |
| CAPM Std dev | 0.05109958 | | | | | | |
| Sharpe ratio | 0.12072439 | | | | | | |

According to Table 5, the two largest weightings in the portfolio with the largest Sharpe ratio under the CAPM are still occupied by LVMUY and COST. Meanwhile, these two stocks have the smallest variance, LVMUY (0.004405262) and COST (0.003351834), while the BABA stock has only about 9% weight in this portfolio, probably due to the unsatisfactory returns of the BABA stock.

**Table 6.** Estimations of FF3F parameters

| | LVMUY | COST | LULU | UKOG.L | BABA | MRVL | IBM |
|---|---|---|---|---|---|---|---|
| SMB RP | 0.00193001 | 0.00193001 | 0.00193001 | 0.00193001 | 0.001930009 | 0.00193001 | 0.00193001 |
| HML RP | 0.00352706 | 0.00352706 | 0.00352706 | 0.00352706 | 0.00352706 | 0.00352706 | 0.00352706 |
| FF3F SMB beta | 0.10545914 | -0.5094584 | 0.54433771 | 0.07788408 | -0.126087367 | 0.18735105 | -0.3045978 |
| FF3F HML beta | 0.0693996 | -0.4809106 | -0.7581381 | 0.48026234 | -0.293928176 | -0.5216693 | 0.3286173 |
| FF3F Mkt beta | 0.86576922 | 0.73552655 | 1.19418039 | 0.85537424 | 0.904907347 | 1.20525611 | 0.9718569 |
| FF3F E(R) | 0.00714609 | 0.00313685 | 0.00729702 | 0.00847166 | 0.005682613 | 0.00751703 | 0.00798695 |

For Fama-French 3 factors model, more factors are considered in evaluating the financial performance of a portfolio, including market excess return, size factor (SMB) and value factor (HML), and the estimations of FF3F parameter can be observed from Table 6.

**Table 7.** Maximum Sharpe ratio portfolio under Fama-French 3 factors model

| | LVMUY | COST | LULU | UKOG.L | BABA | MRVL | IBM |
|---|---|---|---|---|---|---|---|
| FF3F weight | 0.39745153 | 0 | 0.06072031 | 0.01818667 | 0.027656707 | 0.05841806 | 0.43756673 |
| FF3F return | 0.00752849 | | | | | | |
| FF3F variance | 0.00327761 | | | | | | |
| FF3F std devi | 0.05725045 | | | | | | |
| FF3F Sharpe ratio | 0.11685773 | | | | | | |

As Table 7 demonstrates, IBM stocks weighted the highest occupation in this portfolio, which is different with CAPM, following by LVMUY stock which weighted almost 40% and COST stocks are not invested in this portfolio.

## 5. Discussion

As quantitative techniques become more widespread in the investment industry, the importance of mitigating estimation and modelling risk in portfolio management is rising. [7-10]. For the practice of asset selection diversification theory, and the selection of different portfolio optimization models for demonstration. In this essay, by establishing portfolio with current data under CAPM and Fama-French three factors model, different portfolio assets weight and financial performance can be the evidence that both models are useful in the modern portfolio world, there is no good or bad model between these two.

## 6. Conclusion

In conclusion, this paper obtained 7 multiple industries stocks data from yahoo finance (LVMUY, COST, LULU, UKOG.L, BABA, MRVL, IBM) and constructing an investment portfolio based on the mean-variance analysis, CAPM and Fama-French three factors model, by maximizing the Sharpe ratio or minimizing the variance to achieve the portfolio optimization.

This paper mainly focuses on the application of portfolio optimization by using mean-variance theory, CAPM and Fama-French three factors model. And the selection of the assets are based on the concept of diversification, comparing the correlation coefficient, industries, market sectors and countries. After the construction of the CAPM and Fama-French three factors model portfolio, it can be the reference for individual portfolio manager or investment institution to make the decision making about the assets weighting and the portfolio diversification.

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