

Spread Option Pricing Based on Black-Scholes Model and Monte Carlo Simulation Under the Situation of Russia-Ukraine

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Abstract. In 2022, under the current situation of Russia and Ukraine as well as the outbreak of epidemics, the price of crude oil and gasoline are fluctuating dramatically. On this basis, investors face great uncertainty in the future markets. To find out a way to help investors benefit from the war, we used the current information to estimate future trends. By applying the Monte Carlo model to generate random numbers, Black-Scholes model is applied to price the future option based on the data from Yahoo Finance. In addition, the benchmark simulation is employed to obtain a more accurate option value. Spread option has both strengths and weaknesses. Specifically, it can help investors to earn their profits and minimize the risks, while will fluctuate the price of the option when the two assets are highly correlated thus reducing the value of the option. According to sensitive analysis, the positive relationship is clarified between the option pricing as well as the volatility and time period. These results shed light on the best strategy for the clients to make more profits. It is concluded that customers should hold their stocks for the long term and sell them for a higher profit when the war is over.

Keywords: Monte Carlo model; Black-Scholes model; Benchmark simulation; Spread option.

1. Introduction

Option contracts are versatile financial derivatives that give buyers the right to purchase or sell an underlying asset at an agreed price and expiration date. The contracts build the foundation for a broad spectrum of strategies designed for hedging, income, and speculation. Among all kinds of options, they can be divided into vanilla options as well as exotic options. This thesis will give a brief introduction concerning the research on spread options as one of the exotic options. According to the definition, a spread option is a type of option that derives its value from the difference, or spread, between the prices of two or more assets. Some types of commodity spreads enable the trader to gain exposure to the commodity's production process, specifically the difference between the inputs and outputs. The most notable examples of these processing spreads are the crack, crush, and spark spreads, which measure profits in the oil, soybean, and electricity markets, respectively [1].

Black-Scholes Model is a common basic pricing model, which has been created by Fisher Black and Myron Scholes in 1973. The most important assumption for this model is that there are no arbitrage opportunities [2]. The formular is based on certain assumptions [3]:

- No arbitrage.
- The asset price follows a geometric random walk.
- No dividends are paid out.
- The risk-free rate and volatility of the underlying assets are known and constant.
- No transaction costs

To derive the Black-Scholes PDE, we assume that no dividends are paid out during the lifetime of the option. Additionally, we will have prices of random walk following a geometric Brownian motion [4]. The option contract will be executed at strike price K and maturity T . In mathematical notation, it can be written as:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1)$$

Where W_t denotes a standard Brownian motion, μ is percentage shift while σ denotes the volatility of the underlying asset. Since the free interest rate is constant, the cash invested at time 0 will be worth $B_t = e^{rt}$ at time t . Supposing that $C(S, t)$ denotes the value of the call option at time t and according to Ito's lemma, one derives

$$dC(S, t) = \left(\mu S_t \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 C}{\partial S^2} \right) dt + \sigma S_t \frac{\partial C}{\partial S} dW_t \quad (2)$$

By applying a self-financing replicating strategy, we will have x_t units of cash in the cash account as well as y_t units of the stock. Then, the replicating strategy values

$$P_t = x_t B_t + y_t S_t \quad (3)$$

It can also be expressed with the following equation:

$$\begin{aligned} dP_t &= x_t dB_t + y_t dS_t = r x_t B_t dt + y_t (\mu S_t dt + \sigma S_t dW_t) \\ &= (r x_t B_t + \mu y_t S_t) dt + \sigma y_t S_t dW_t \end{aligned} \quad (4)$$

Based on the no-arbitrage assumption, the value of the asset must equal the cost of replicating strategy. With this in mind, by combining equations

$$y_t = \frac{\partial C}{\partial S} \quad (5)$$

$$r S_t B_t = \frac{\partial C}{\partial t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 C}{\partial S^2} \quad (6)$$

as well as substituting Eqs. (5) and (6) into (3), one gets

$$r S_t \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 C}{\partial S^2} - r C = 0 \quad (7)$$

In the binomial model, the call option price is given by

$$C(S, t) = E^Q[e^{-r(T-t)} \max(S_t - K, 0)] \quad (8)$$

Let $S \rightarrow \infty$, one derives:

$$C(S_t, t) = S_t \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2) \quad (9)$$

Here,

$$d_1 = \frac{\log\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma \sqrt{T-t}} \quad (10)$$

$$d_2 = d_1 - \sigma \sqrt{T-t} \quad (11)$$

The Black-Scholes model can also be employed for the pricing spread option. However, the set-up is considered too general for accurate computations that will only be appropriate for a rough approximation [5]. More advanced and popular pricing models are also proposed, e.g., Markovian Models and Partial Differential Equations, Trinomial Trees, Monte Carlo Computations, and approximation via Fourier Transform [6], which will be discussed further in this essay.

At present, the fuzzy jump-diffusion model is mostly used on binary options [7]. The economist Robert Merton combined the Black-Scholes model with the binomial pricing model to get our jump-diffusion model. Stock prices can fluctuate attributed to various reasons, which is called jump. Numerous factors are not very accurate in the market (e.g., the interest rate, volatility, jump intensity, and other parameters). In order to investigate the volatility of the stock market, the fuzzy jump diffusion model is always used. This model can simulate a future price trend and design an effective method to optimize investment revenue.

Recently, a rather special case occur that affects the worldwide economy. On February 24, 2022, with Russian President Vladimir Putin's declaration of war against Ukraine, the Russia-Ukraine war began. The prices concerning gas and other products in the energy markets are under significant upward pressure, thus imposing a great uncertainty generated by the Energy agreement (Insight. p3) inflation, political risks, and sanctions toward Russia [8]. The prices of the inputs (e.g., crude oil) and the outputs (e.g., natural gas heating oil) have fluctuated and risen drastically since February 2022. As Russia is one of the world's largest exporters of gasoline and crude oil, the war will have a huge impact on oil futures.

Therefore, the challenging political and economic situation can be a good opportunity to further explore the distinct features of spread options as well as different models for pricing spread options in the financial market. We chose the spread option for three reasons. First, the traditional crack spread is used to hedge against risks. Second, the two assets have a strong correlation, which means the trend would be stable, last is the current situation in Russia and Ukraine. The war presents “substantial uncertainties” for international oil markets. Energy markets are likely to be hit if tensions turn into further conflict, which can be a challenge for the investors in the oil market. Besides, it is hope to offer a way to benefit from the fluctuations.

The rest part of the thesis is organized as follows. Section II will show contracts between the spread option and other kinds of options and we will introduce the BS model. Then in section III, a review of the former method conceived for price spread options will be given, after which the methodology will be described as a sample simulation based on the Black-Scholes model for a 3:2:1 crack spread. Afterwards, a discussion regarding the limitations of the spread option will be delivered in Section VI. Eventually, a brief summary will be given in the last section.

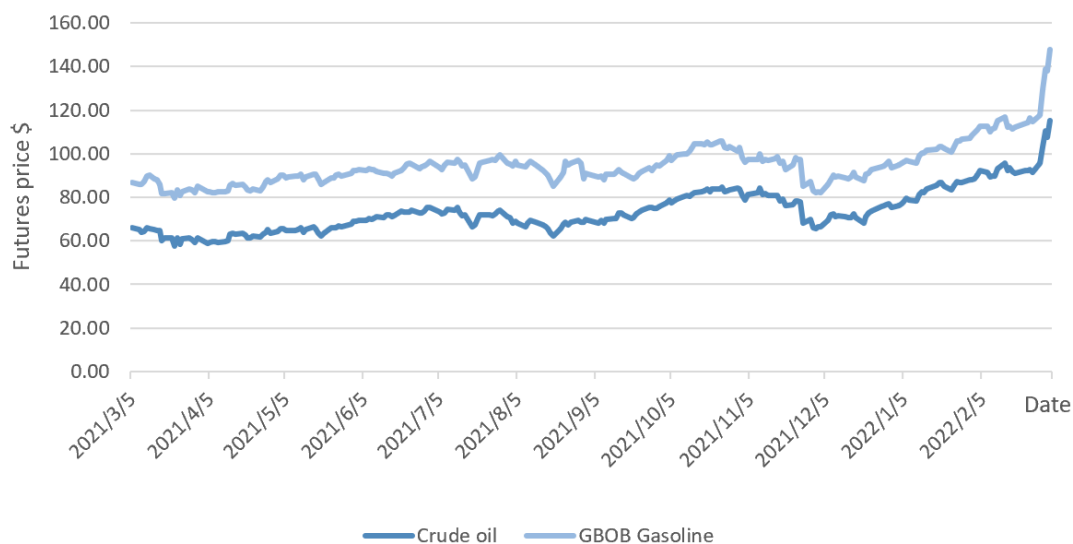


Figure. 1 Illustration of closing prices of the sample data.

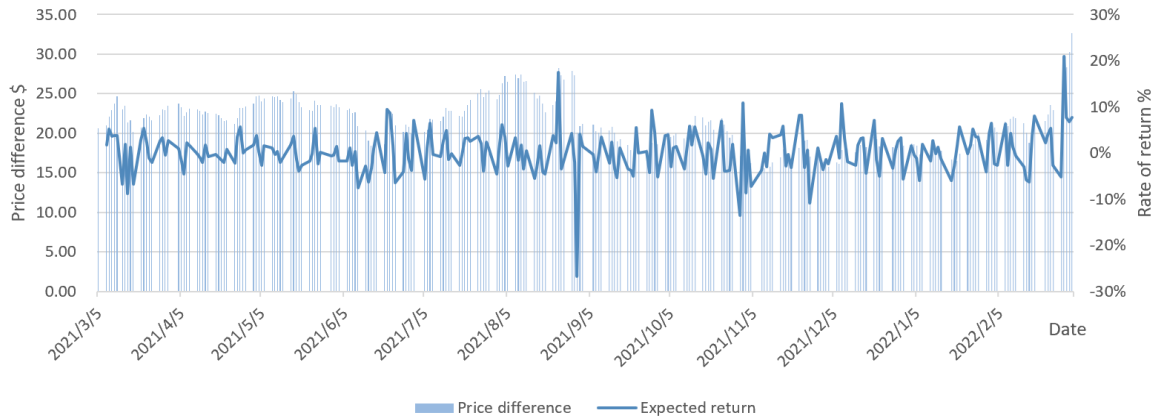


Figure. 2 Illustration of possible return of the spread option

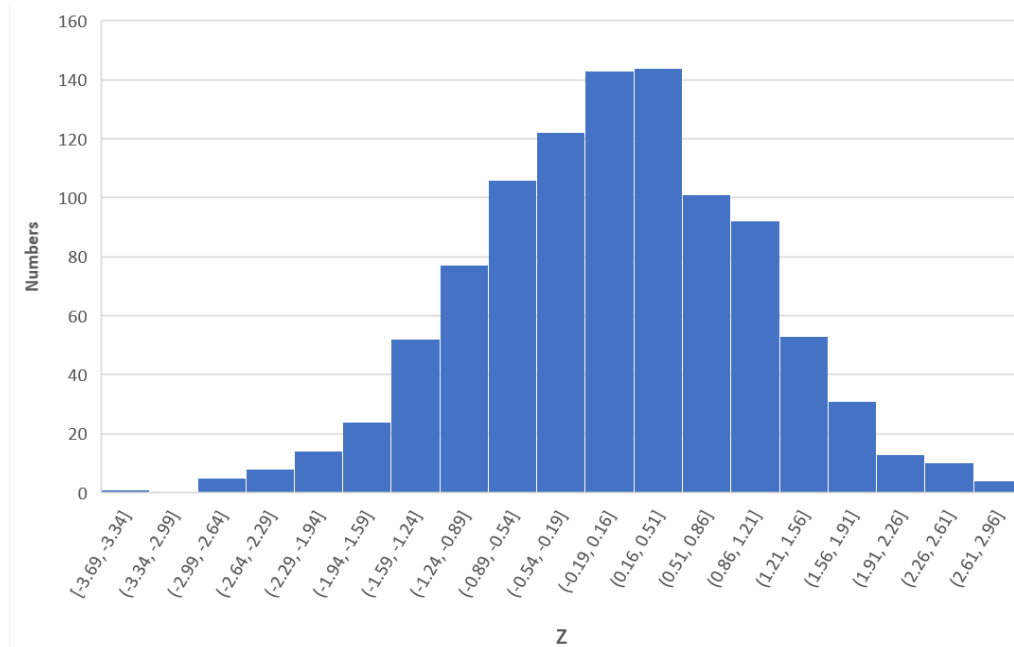


Figure. 3 Illustration of the distribution of value Z

2. Methodology

2.1 Black - Scholes model

In this paper, the analysis method is BS pricing model [9]. Given the value of an asset now is S_0 , the future value of an asset is S_T , e is the natural logarithm, T is the time period, (the parameters are expressed in annual terms, and T is computed in the unit of years). The expected return is α and standard deviation is σ . The formular is proposed as follows:

$$S_T = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)T + Z\sigma\sqrt{T}} \quad (12)$$

In order to get Z value, the Monte Carlo method is applied by using random numbers to calculate it, which can be realized by NORMSINV function in excel. The same procedure of generating the random numbers is repeated 1,000,000 times when carried out the simulations. By changing the time period, the formular can be used to compute a long-term future price or a path of shorter-term prices.

The sensitive analysis was chosen because it is an essential analysis method to find out the sensitivity factors as well as deliver a guideline to investors.

2.2 The data in the current market

By definition, a crack spread is conducted by purchasing or selling against buying or selling refined petroleum products. For the sake of illustration, we will first make a simulation of a European call spread option on crude oil and gasoline. The data is collected from Yahoo Finance, which is closing prices of crude oil futures and gasoline futures on daily basis from 2021/3/5 to 2022/3/4, as well as the return on the underlying assets. The results are shown in Fig. 1 and 2 [10, 11]. The futures prices of the two assets seem to co-move with each other, which indicates a strong correlation of 0.95. According to the assumption of the Black-Scholes model, the asset price is unpredictable and follows a log-normal distribution. Therefore, we can generate a standard normal distributed number Z , by using the formular: $Z = \text{NORMSINV}(\text{RAND}())$ as illustrated in Figure 3. Then we use the Black-Scholes model Eq. (12) to simulate the value of the underlying asset by using the information in Table. I. The average value of the spread option is derived after 1000,000 times of computation by using benchmark simulation. The simulation results are given in Table. II and depicted in Fig. 4.

Table. 1 Information needed in BS model for computation

Spot	Strike	Volatility	Holding period
32.63	35	0.74	0.25

Table. 2 Results of computation

Average	Stdev	Maxerr
37.31	0.93	0.06

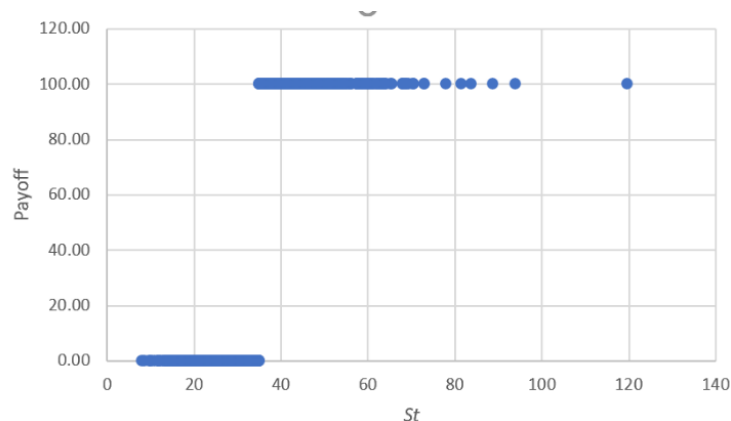


Figure. 4 Illustration of the possible payoffs for European binary call options.

A more popular type of option is the 3:2:1 crack spread, which involves 3 option contracts of crude oil, two contracts of gasoline and one contract of heating oil [12]. At any time, t the spread can be written as:

$$[CS]_t = \frac{2}{3}[GS]_t + \frac{1}{3}[HO]_t - [CO]_t \quad (13)$$

Noting that crude oil is quoted in "dollars per barrel", however, gasoline and heating oil are quoted in "dollars per gallon", so a conversion is required that there are 42 gallons per barrel. The basic information for prices and return are shown in Fig. 5 and 6.

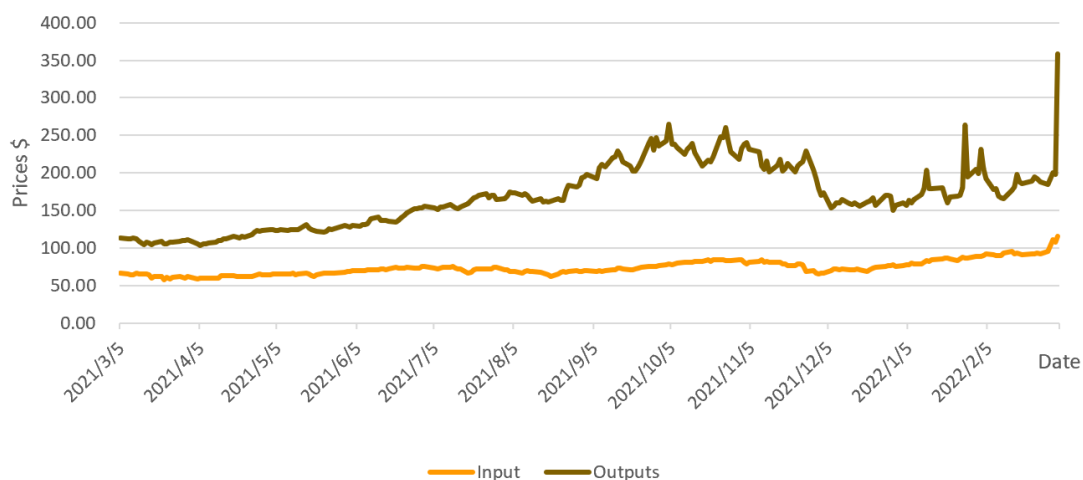


Figure. 5 Illustrations of closing prices of the sample data.

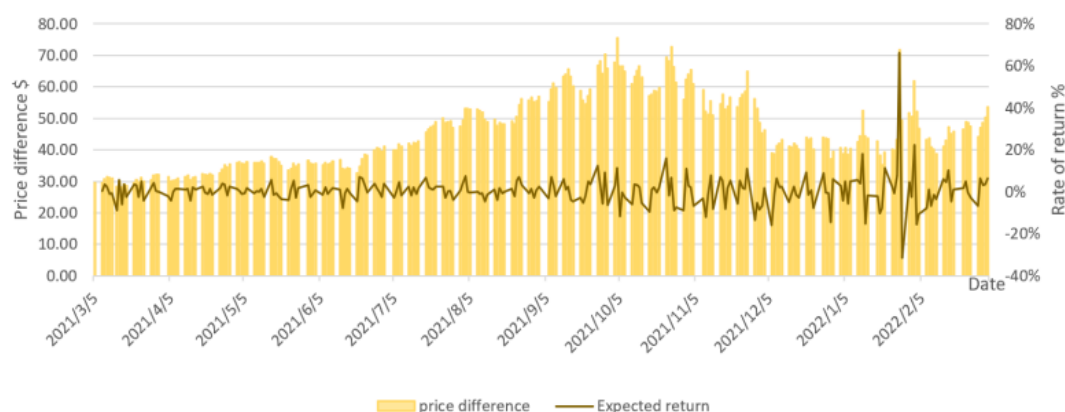


Figure. 6 Illustration of possible return of the spread option.

The inputs and outputs of 3:2:1 also have a considerably strong positive correlation of 0.83. The calculation of log-normal distributed number Z follows the same method and the information is listed in Table. III. The average value of the spread option is derived after 1000,000 times of computation by using benchmark simulation. The simulation results are summarized in Table. VI and depicted in Fig. 7.

Table. 3 Information needed in BS model for computation

Spot	Strike	Volatility	Holding period
53.64	57	0.1.11	0.25

Table. 4 Results of computation

Average	Stdev	Maxerr
34.96	0.98	0.06

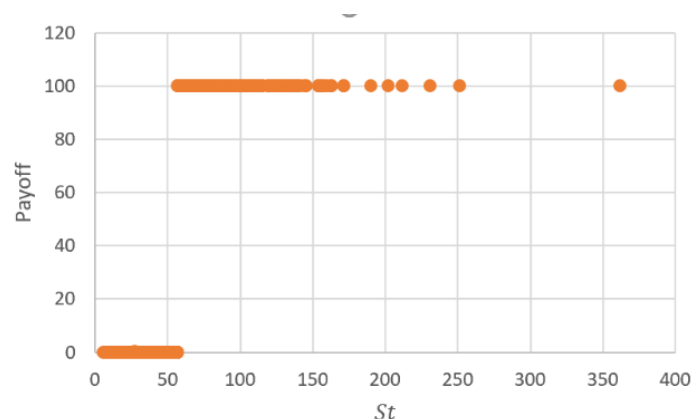


Figure. 7 Illustration of possible payoffs for European binary call options

3. Results & Discussion

The Fig. 8 shows the relationship between option pricing and volatility, noting that when option price increases volatility will increase. Based on What-if analysis, one sees that the option price is 22.63 when volatility is 0.74. The figure illustrates the positive slope which means a positive relationship between the two factors. As the option can be considered as a risk transfer, higher volatility corresponds to a higher risk. According to the Capital Asset Pricing Model, the option value has the same direction as the risk. In other words, high volatility leads to high returns. The Fig. 9 presents a positive relationship between the option pricing and time, where the option prices will go up as time goes on. The uncertainty increases over time; hence the risk will be higher, and the value will increase. Thus, investors may get a higher return by exercising the option in the future. Using the same method, it shows the option value is 22.63 at 0.25 years. Moreover, we also did a two-factors sensitive analysis (seen in the Table. V), the present value is 22.63, i.e., one can find T is 0.25 and the volatility is 0.74. A high positive correlation between the option price and these two factors is shown in our figures.

This paper will give some suggestions for investors who want to buy spread options. Spread options facilitate our investors to locking their profits, which can reduce their spread risk and make the transaction simpler, particularly during the Russia-Ukraine war. Furthermore, the spread option provides a more flexible way of hedging risks. For example, Refiners and investors can also implement crack strategies on multiple products. As we mentioned a more popular type of option 3:2:1 crack spread, the refiner could hedge the risk with the crack spread, in which the underlying asset is the prices differences between 2 barrels of gasoline plus 1 barrel of heating oil and 3 barrels of crude oil. By implementing this method, the investors can earn more profits. Apparently, one could set the ratio to X:Y: Z, depending on the customer's requirements. It is also true in the case of the products in other sectors. Moreover, the spread option offers investors an arbitrage opportunity required of a less initial cost. For instance, when a speculator believes that the relationship between two different assets currently trades significantly above its historical range. The trader can sell the spread, hoping that its value will soon return to normal. Otherwise, they can buy a put spread option to accomplish the same goal, but at a much lower initial cost.

Anyway, the whole analysis still has some limitations and drawbacks to some extent. Since the spread options are multi-asset derivatives, their correlation is an important factor in valuation. If the two assets are highly correlated, the prices can co-move with each other resulting in limited fluctuation concerning the spread, thus reducing the value of the option.

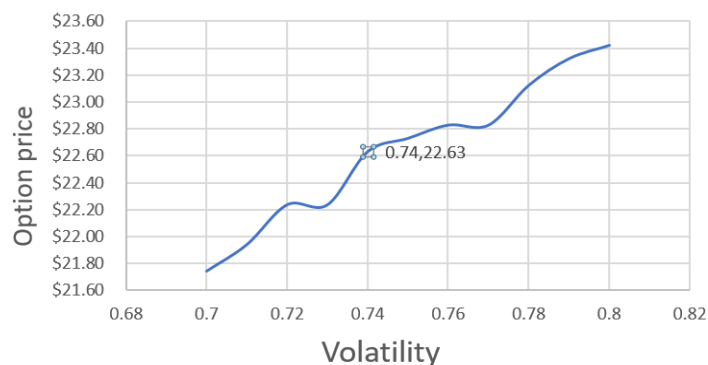


Figure. 8 The relationship between the option price and volatility.

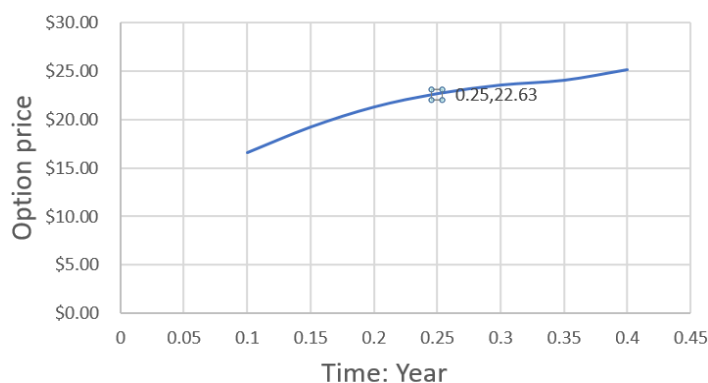


Figure. 9 The relationship between the option price and time period

Table. 5 Two-factors sensitive analysis

Volatility Option price	0.7	0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78
Time									
0.1	14.89	15.48	16.06	16.26	16.55	16.55	16.75	17.14	17.43
0.15	18.51	18.70	18.70	18.80	19.19	19.39	19.58	19.68	19.98
0.2	20.47	21.06	21.16	21.25	21.25	21.35	21.55	21.65	21.74
0.25	21.74	21.94	22.24	22.24	22.63	22.73	22.83	22.83	23.12
0.3	22.83	23.02	23.22	23.42	23.52	23.62	23.62	23.62	23.81
0.35	23.62	23.62	23.71	24.01	24.01	24.01	24.70	24.70	24.80
0.4	24.01	24.60	24.70	24.80	25.10	25.20	25.30	25.40	25.40

4. Conclusion

In summary, this paper investigates capital asset pricing based on the Black-Scholes model. Based on the analysis, the impacts of Russia-Ukraine war on the crude oil and gasoline market are discussed. Subsequently, the causation is analyzed, where the war has a great impact on futures prices. Specifically, the more popular type of option (3:2:1 crack spread) is selected as investigation and target and the formulae are demonstrated. This paper aims to assess the value of the chosen products and make some suggestions based on the sensitivity analysis. According to the results, time and volatility lead to a huge effect on our choice. However, one should not ignore the limitation of our products, if customers choose two products with high correlation, their price difference may not be large, which will reduce the value of the option. Overall, these results offer a guideline for the research on binary options and assets pricing.

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