Difference Between Risk Pooling and Risk Sharing in the Diversification of the Portfolio

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Abstract. In this era of stock speculation, knowledge of stocks has become very important. Risk management can help achieve more excellent benefits at the lowest possible cost, while the improper use of the approaches can lead to a counterproductive result. Risk pooling and risk sharing are present in many domains, with entirely different definitions. Many people apply purposes of these two means from other fields to the financial area, without knowing, or just ignoring, their difference. This article uses numerical results to demonstrate the difference between them by building portfolio models to perform calculations. The true impacts of applying these two methods to reduce risk can be found by calculating respectively. Although the two methods do not sound different, they reflect other effects when applied to the same portfolio. By understanding the different roles of the two methods, investors will have a better understanding of portfolio risk management and can avoid the severe consequences of misusing them.

Keywords: Risk pooling, Risk sharing, Diversification, Risk management

1. Introduction

Diversification is the most common way of risk management in finance. It means reducing the risk of the whole portfolio by increasing the assets invested, and extending the portfolio. Most people consider diversifying risk through the extension of the period. From their perspective, long-run investment is more secure than short-run investment. However, they are likely to be still confused with the actual meaning of diversification in finance; in other words, this approach has 0 relations to diversification. If the extension of the investment period is considered to have the same function as diversification, how should the risk be defined? Besides, the difference between risk pooling and risk sharing in reducing risk is also less acknowledged. In such an era of stock popularity, fatal mistakes could be made by misunderstanding the difference between approaches. A typical example is the insurance industry. A subtle error of estimation on the policy or the correlation between different policies could make the whole firm go out of business or even make their collaborators get involved.

Berman et al. ,by contemplating risk pooling in the problem of newsvendor in different locations and determining the sensitivity of the benefits of operating risk pooling to the demands standard deviation at other areas and the number of regions, reveal that the growth of absolute saving synchronizes with the development of demands standard deviation when the variation coefficient less than approximately 0.5 [1]. In addition, Cai and Du by proposing a new mathematical model, which adjusted the former risk pooling models to make them coordinated, introduced the phenomenon of decreasing marginal returns of risk pooling under some particular constraints and also proved that the incorrectness of the higher standard deviation, the more benefit of risk pooling [2]. Moreover, Robert et al. analyzing Chaebols, the Korean consortium, during the period of Korean financial crisis through a newly introduced method, in this technology, they estimated the equilibrium debt determination model for the firm. Divided the equilibrium debt rate into demand, supply, and firm-specific factors, they find that the risk pooling mechanism, which is effective on increasing the efficiency of firms in a normal situation, makes the consortium weak in the financial crisis [3].

Taking account of risk sharing, Chan and Kwok, by investigating the effects of risk-sharing and other factors to re-estimate the price of equities during the liberalization of the Shanghai Stork Exchange, indicate that risk-sharing is a vital mechanism for repricing the equities to become eligible for international purchasing [4]. Besides, for Japan, Khanna and Yafeh, by analyzing theories of economy, empirical work, which is mainly done in Japan, and unverified evidence, suggest that risk sharing seems to be one of the most critical functions of the business conglomerate. Besides, they also investigate how the composition of the industry of business changes along with the evolution of time in a long time series, which further highlights the reasons and results of operational risk sharing in a business collaboration [5]. Furthermore, Deverux and Smith build a growth model of multi-countries. In this model, an agent lives permanently in each country, and for each country, there is a country-specific income risk; however, no aggregate uncertainty exists as a whole in the world. By studying the effect of risk sharing in such an effect, they conclude the impact of risk sharing on welfare, both for the state and for individuals at home, is vague with uncertain endowments, but always positive with uncertain yields [6].

This paper will use financial formulas to calculate the numerical changes on the portfolio after applying risk pooling and risk sharing, respectively. By analyzing and comparing data obtained, the actual changes will be demonstrated and further the use of each method in reality.

2. Risk pooling

Diversification is widely used to manage the risk of the portfolio. [7] Risk pooling is one of the approaches to risk diversification. It reduces the risk of portfolios by diversifying, and combing the uncorrelated risky assets. It is widely used in the insurance industry, finance industry, etc. It is taking a rich trader, W, for example. W holds a portfolio P, including two assets only. One is risky asset A; the other is risk-free asset S. There is another risky asset B, which is non-correlated with A. The process for the trader adding asset B into his portfolio P to hedge the risk of A is called risk pooling.

Taking account of risk pooling in the insurance industry, risk pooling is primarily for selling uncorrelated policies, which is entirely acknowledged as the principle of insurance.

3. Risk sharing

Risk sharing is another way of risk diversification. The central concept of risk sharing is merging risky assets without correlation, while keeping the overall exposure. It always has the same Sharpe ratio as pooling, while lower volatility (absolute risk). It is widely used by the insurance companies, securities companies, etc. Assume the same wealthy gentleman W; he holds the same portfolio P, which includes risky asset A and risk-free one S. There is also another risky asset B without any correlation with A. W is willing to reduce the risk of his portfolio by purchasing B into it, whereas this time, he is going to sell part of A and replace it with the same amount of B. To sum up, the total quantity of risky assets in portfolio P does not change.

4. Comparison

Both the risk pooling and risk sharing aim to reduce the portfolio risk by introducing other risky assets with no correlation. While risk pooling changes the ratio of risky assets and risk-free assets within a portfolio, in contrast, risk sharing maintains the same. The conventional philosophy identified risk pooling as risk reduction and has became the driving force behind risk management in the insurance industry. However, after a short reflection, a more profound problem could be recognized. How could the exposure reduce by applying a new ‘gambling’? As it could be seen, sometimes, people may misuse the principle of insurance in long-run investment --- They estimate the total return of a portfolio by applying the property of average return.

Such a phenomenon could be verified by a simple calculation.

Assuming the original portfolio P, which holding by trader W, contains y proportion of risky asset A, therefore, the proportion of risk-free asset S is 1-y. R(a) is the risk premium of asset A, and σ(a) stands for the standard deviation of A. The risk premium of portfolio P could be found by [8]

$$R(p) = wR(a)+w\cdot 0 = yR(a) \quad (1)$$

Besides, also the standard deviation [9],

$$\sigma(p) = \sqrt{w^2\cdot\sigma^2(a) + w^2\cdot 0} = \sqrt{y^2\cdot\sigma^2(a)} = y\sigma(a) \quad (2)$$

Sharpe ratio of P [10],

$$S(p) = yR(p)/y\sigma(p) = R(a)/\sigma(a) \quad (3)$$

Supposing there is a new risky asset B with the same risk premium and standard deviation as A, and the correlation of A and B is 0. Now, W is willing to introduce B into his portfolio P to reduce the risk by diversifying it. He finally purchases the same amount of B as that of A. Here is the new portfolio P1: y proportion of A, y proportion of B, and 1-2y proportion of S. By applying the same calculation, results could be attained:

$$R(p1) = yR(a)+yR(b)+(1-2y)\cdot 0 = 2yR(a) \quad (4)$$

$$\sigma(p1) = \surd\ y^2\cdot\sigma^2(a)+y^2\cdot\sigma^2(b)+(1-2y)\cdot 0 = \surd 2y\sigma(a) \quad (5)$$

$$\text{and } S(p1) = R(p1)/\sigma(p1) = 2yR(a)/\surd 2y\sigma(a) = \surd 2(R(a)/\sigma(a)) = \surd 2S(p) \quad (6)$$

The good news is the √2 times increase of the Sharpe ratio. However, standard deviation also increases by √2 after the introduction of risky asset B. By simply repeating the former calculation, the following statement could be attained: When W has α risky assets, which all have the same risk premium and standard deviation but no correlations between them, in his portfolio, the risk premium,

$$R(p\alpha 1) = yR(1)+yR(2)+...+yR(\alpha)+0 = \alpha yR(1) \quad (7)$$

standard deviation,

$$\sigma(p\alpha 1) = \surd\ y^2\cdot\sigma^2(1)+y^2\cdot\sigma^2(2)+...+y^2\cdot\sigma^2(\alpha)+0 = \surd\alpha y\sigma(1) \quad (8)$$

Sharpe ratio,

$$S(p\alpha 1) = R(p\alpha 1)/\sigma(p\alpha 1) = \alpha yR(1)/\surd\alpha y\sigma(1) = \surd\alpha S(1) \quad (9)$$

Sharpe ratio of the portfolio increases by √α times, whereas, meanwhile, standard deviation also increases by √α times. Such a result reveals the limitation of reducing risk by simply risk pooling. It increases the scale of investment, and it does not decrease absolute risk.

Taking into account another situation of applying risk sharing now. This time, before adding B into his portfolio, W will sell part of A while keeping the Sharpe ratio constant. Assuming W replaces half of A by the same amount of B. Now, his portfolio P2 will contain: y/2 of A, y/2 of B, and 1-y S. With the comparison of P1 and P2, the only difference could be found: Risk sharing replaces part of the former risky asset by the same amount of a new risky asset. It keeps the total amount of risky assets. Hence, the only thing that should be changed in the calculation is applying y/2R instead of yR. The following results could be obtained:

$$R(p2) = (y/2)R(a)+(y/2)R(b)+0\cdot(1-y) = yR(a) \quad (10)$$

$$\sigma(p2) = \surd\ (y/2)^2\cdot\sigma^2(a)+(y/2)^2\cdot\sigma^2(b)+0\cdot(1-y)^2 = (\surd 2y\sigma(a))/2 \quad (11)$$

$$S(p2) = R(p2)/\sigma(p2) = yR(a)/((\surd 2y\sigma(a))/2) = \surd 2(R(a)/\sigma(a)) = \surd 2S(p) \quad (12)$$

To sum up, the results show the same Sharpe ratio as that of P1(√2S(p)), while P2 has a lower standard deviation, and is also considered to have lower volatility. As for each risky asset A and B, it has a lower weight in portfolio 2 at y/2; taking the variation of A for example, the percentage impact for P2 is,

$$((y/2)/1)*100\% = 50y\% \quad (13)$$

For P1, the percentage impact is,

$$((y/1))*100\% = 100y\% \quad (14)$$

A fluctuation of risky asset A could have a double influence on P2 than P1.

In addition, by considering the situation for W adding infinite α same risky assets but with no correlations, the results will be,

$$R(p\alpha 2) = y/\alpha R(1)+y/\alpha R(2)+...+y/\alpha R(\alpha)+0 = yR(1) \quad (15)$$

$$\sigma(p\alpha 2) = \surd\ (y/\alpha)^2\cdot\sigma^2(1)+(y/\alpha)^2\cdot\sigma^2(2)+...+(y/\alpha)^2\cdot\sigma^2(\alpha)+0 = (\surd\alpha y\sigma(1))/\alpha \quad (16)$$

$$S(p\alpha 2) = R(p\alpha 2)/\sigma(p\alpha 2) = yR(1)/((\surd\alpha y\sigma(1))/\alpha) = \surd\alpha S(1) \quad (17)$$

What is demonstrated by the results is, by applying risk sharing, the decrease in absolute risk of the portfolio is directly proportional to the amount of non-correlated risky assets. In conclusion, diversifying the portfolio while fixing the investment on risky assets within a portfolio, could decrease the risk.

5. Conclusion

The introductions of two ways of diversification with an example in each are demonstrated at the beginning. Followed by applying risk pooling and risk sharing on the same portfolio, respectively. The conclusions are made based on the results:

For risk pooling, only when the increased rate of risky assets(without correlation) over the increased rate of risk, the profitability of risk pooling could increase. However, this is not an actual way to reduce risk; conversely this may even limit the potential profit of a growing combination of an insurance corporation. When increasing the quantity of non-correlated risky assets, though the Sharpe ratio of the portfolio will increase, the absolute risk also rises as the part of risky assets constitute more within a portfolio.

When it comes to risk sharing, the increase in the number of risky assets will directly make the risk of the portfolio drop. The use of risk sharing in the insurance industry is by selling policies to their clients to manage the risk as a whole; meanwhile, the Sharpe ratio will increase with the growth of the number of policies sold, further decreasing the risk of the individual customer.

In conclusion, most people confuse the reduction in the probability of loss with the fall of risk. Operating risk pooling could only decrease the probability of loss, rather than the absolute risk. Investors are not supposed to only to consider the mathematical meaning of diversification, but also the financial one.

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