

Progress of the Study on the Models of Option Pricing

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Abstract. Option pricing, a core part of options trading, has been fruitfully researched over the years. This article reviews the history of the emergence and advancement of option pricing in terms of a thorough classification of the widely used option pricing models and their empirical studies that follow. The three option pricing models are summarized, including the Black-Scholes pricing model, the tree diagram model, and the Monte Carlo simulation techniques, which have all represented significant progress in the field of option pricing theory. Moreover, the differences between various pricing models are analyzed and compared to show their applications and to provide an outlook on future work in this theory. It is vital to carry out some research on option pricing in order to better suit the preferences of investors. In order to meet the continuous development of financial markets, valuation of financial derivative securities is the key to effective investment in risky assets.

Keywords: Options; Option Pricing; Black-Scholes (B-S) model; Binomial Tree Model; Monte Carlo Method.

1. Introduction

One of the pillars of contemporary finance is the notion of option pricing. The most significant, popular, and challenging models and techniques for pricing options are found in studies of financial derivatives. Modern options emerged in the 1970s. However, options have a very long history. The first form of option trading was developed during the ancient Greek and Roman eras, and the first known instance of an option contract in history can be found in Aristotle's account of the tale of Thales and his creation of a financial instrument. The Black-Scholes (B-S) model, also referred to as the "Second Revolution on Wall Street," was formally presented in 1973 and served as the foundation for contemporary option pricing research [1]. When the Chicago Board of Trade started conducting organized futures trading in 1984, option theory was a subject of research.

A contract between two parties known as an option gives the option buyer (holder) the right to purchase or sell the underlying asset at a defined period in the future (the expiration date) or earlier, at a predetermined price, and in a predetermined quantity.

Options are a very important class of financial derivatives that essentially price rights and obligations separately. The obligor must complete its commitments within a certain amount of time and the right's assignee has the choice to exercise its rights. It enables the buyer to both prevent negative consequences and profit from positive outcomes. The market price of the underlying asset, the option exercise price, the option's volatility, the risk-free interest rate, the remaining time until expiration, etc. are the key variables influencing the option price. One of the most important uses of options is to manage risk, and in order to manage risk effectively, options must be properly valued. The growth of the options market has been significantly influenced by the invention and improvement of option pricing theories [2].

Yale University's famous professor Stephen Ross said, "Option pricing theory is the most effective theory in economics overall, not only in the field of finance." It has been applied to many financial and non-financial fields, including the pricing of various derivative securities, corporate investment decisions, and so on. In the financial securities market, option pricing theory provides investors with reasonable option prices and optimal implementation periods to maximize profits with minimal investment [3].

The tremendous progress in the academic field has led to a rapid development in the practical field. The most fundamental factor in the creation of worldwide financial products and financial markets is the expertise of option pricing. The development and growth of option pricing theory have been

facilitated by the recent success of the sector involved in the design and pricing of financial products. Therefore, the idea of option pricing is of great importance, both from the theoretical and practical needs. Numerous scholars have conducted in-depth studies on the option pricing problem, and this paper reviews the theoretical studies of major scholars on the option pricing problem in order to have a clearer and more explicit understanding of the option pricing problem.

2. Historical development of option pricing theory

2.1 Early Option Pricing Theory Studies

The Code of Hammurabi, written in 1800 B.C., was the idea of options first emerged, and the first examples of options trading may be found in 1200 B.C. in Greek commerce. Options became popular during the 17th-century Dutch "tulip bubble" and the 18th-century agricultural trade in the United States. However, the history of theoretical models of option pricing is relatively short[4]. The Theory of Speculation, a work written by the French mathematician Bachelier, developed equilibrium models for option pricing. Bachelier suggested that the process of stock price fluctuations is comparable to a drift-free standard arithmetic Brownian motion. However, this option pricing model was flawed in terms of economics and mathematics, but it also indicated various ways to study options using equilibrium principles. This discovery lay dormant for fifty years before it was accepted by the financial community and became known as the "random walk" or "drunkard's walk".

Bachelier derived an expectation formula for the expiration date of the call option by connecting the Brownian motion and heat conduction equations via a Gaussian probability density function.

$$C(S, T) = SN\left(\frac{S-X}{\sigma\sqrt{T}}\right) + \sigma\sqrt{T}n\left(\frac{X-S}{\sigma\sqrt{T}}\right) \quad (1)$$

Where, in turn: S is the stock price; X is the option strike price; T is the option expiration time, N(.) is the probability density function, and n is the distribution function of the common normal distribution. The model's applicability is constrained by the assumption that the stock price is an absolute Brown movement, which ignores the value of money and permits a negative number of shares. However, the inclusion of the concept of stochastic wandering and the use of Brownian motion to explain stock price changes are both novel ideas. Bachelier 's option pricing theory marked the birth of financial mathematics.

The idea that the dynamic process of stock prices fulfills a log-normal distribution and that stock prices have fixed means and variances, which led Sprenkle to develop the option pricing formula in 1961 in "Option Prices as Indicators of Expectations and Preferences."

$$C(S, T) = e^{\rho T} SN(d_1) - (1 - A)XN(d_2) \quad (2)$$

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S}{X}\right) + \left(\rho + \frac{\sigma^2}{2}\right) T \right] \quad (3)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (4)$$

Where: ρ denotes the average growth rate of stock prices and A denotes the degree of risk aversion. This formula's application is influenced by the average pace of stock price rise and the matching level of risk aversion.

Boness proposed a similar model in 1964 while assuming a steady logarithmic distribution for stock returns and acknowledging the importance of risk insurance. He made the simplistic assumption that "investors care little about risk." Using this presumption as support, he justifies dividing the projected return on the stock by the expected value of the final option. His final model is:

$$V(S, t) = S\Phi(d_1) - e^{-\alpha(T-t)}K\Phi(d_2) \quad (5)$$

The latter Black-Scholes equation and this one have the exact same form. Boness would have arrived at the Black-Scholes equation if he had substituted the presumption that investors do not care about risk for the logical conclusion that alpha equals r.

Samuelson proposed a put option pricing mechanism in the manner of Europe in 1965. He suggested that options should have a greater average return because the expected returns of options and stocks are different according to risk characteristics. His proposed pricing model is:

$$V = e^{(\alpha-\beta)T}S \cdot N\left[\frac{\ln\left(\frac{S}{K}\right) + \left(\alpha + \frac{1}{2\sigma^2}\right)T}{\sigma\sqrt{T}}\right] - e^{-\beta T}K \cdot N\left[\frac{\ln\left(\frac{S}{K}\right) + \left(\alpha - \frac{1}{2\sigma^2}\right)T}{\sigma\sqrt{T}}\right] \quad (6)$$

Boness model is a special case of Samuelson's model (when $\alpha = \beta$)

A pricing model for European call options was presented by Samuelson in the article "A Rational Theory of Option Pricing" in 1965. He argued that because there are variations between the risk characteristics of options and stocks, the expected return on options is higher. Samuelson unifies the previous findings in a single model, but still contains subjective parameters. The pricing equation is:

$$C(S, T) = Se^{-(\rho-\alpha)T}N(d_1) - Xe^{-\alpha T}N(d_2) \quad (7)$$

The option pricing theories prior to the 1973 Black-Scholes Model lacked practical value and were referred to as "incomplete models"[5]. The absolute Brownian motion assumption, which is the Bachelier model's major flaw, opens the door to the potential of negative equity, which runs counter to the firm's limited liability. Both Sprinkle and Samuelson have two subjective variables in their formulas, which are difficult to prove empirically. Boness is one step removed from the B-S-M model that is assumed to be risk neutral. They set the groundwork for the ensuing B-S-M model, but the advancement of option pricing theory is the same.

2.2 Black-Scholes Model

The Pricing of Options and Corporate Liabilities, a 1973 research by Fisher Black and Myron Scholes, is one of the most well-known studies on option pricing[6]. In the study, the conventional Black-Scholes option pricing formula was presented. In the same year, R.C. Merton of MIT independently proposed a more generalized model with a series of improvements and refinements. The Black-Scholes option pricing equilibrium model, whose success is attributed to option's ability to be realized through synthetic means, founded modern option theory.

This model creates an equilibrium model of option pricing while avoiding restricting assumptions about personal risk preferences and the equilibrium price structure of the market. Black and Scholes show that it is possible to put together an options-compliant portfolio that includes equities and risk-free asset positions and is completely lucrative over brief periods of time. Additionally, Black and Scholes demonstrate precisely how the portfolio's composition evolves over time in response to changes in stock prices and time, ensuring that its earnings remain consistent with those of the options. In this model, the price of the options is independent of what appears to be a significant investor's expectation of profits on changes in stock price. The equilibrium value of the options in this model is obtained by building a portfolio of hedging securities that produce a firm income independent of future stock prices. As a result, the payoff of this portfolio of hedging securities is unaffected by variations in stock returns. Using the Black-Scholes model, investors may control risk and assess the value of options with ease. All of the model's variables are immediately observable with the exception of alpha. The model is both an aesthetically pleasing and sizable portrayal of the relationship between risk and value[7]. The investor is aware of the stock and option's present prices, as well as their performance prices, expiration dates, and interest rates. Based on this information and the option's

pricing, the computer can forecast the implied volatility of the stock price. It is then up to the investor to determine whether the market's prediction of stock volatility is too low, too high, or moderate.

The B-S model's initial test was performed by Galai (1977) using data on stock options from the Chicago Board Options Exchange. Since then, numerous academics have conducted fruitful research in this area. In all, these tests produce some broad conclusions.

1. In particular, flat options with remaining effective maturities over two months and no dividend payouts are effectively evaluated by the algorithm.

2. For highly real or imaginary options, the model's pricing has a large bias, overestimating imaginary options and underestimating real options.

3. Large errors exist in the valuation of options close to their expiration dates.

4. When the dispersion is too large or too low, there is some pricing inaccuracy.

Despite its shortcomings, the B-S model is generally quite accurate with a lot of real-world applications. Most pricing models developed after B-S-M are advancements in a specific area, increasing parameters and complexity of the model, without substantial improvement in pricing efficiency and accuracy, and often fail to yield clear and applicable conclusions. As a result, the B-S-M model has an unrivaled fundamental position in option pricing.

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(1) Euclidean options.

(2) There are no tax-free or transaction-cost-free arbitrage opportunities.

(3) Transactions are continuous; generally stocks can be divided into arbitrarily small fractions, and stocks do not pay dividends.

(4) For all maturities, the risk-free rate is constant and the same.

(5) Geometric Brownian motion, a stochastic procedure, is what determines the stock's price. Under the same circumstances, it is presumably that the price of the underlying asset will follow a log-normal distribution:

$$c = S\Phi(d_1) - Xe^{-r(T-t)}\Phi(d_2) \quad (8)$$

$$d_1 = \frac{[\ln(\frac{S}{X}) + (r + \frac{\sigma^2}{2})(T-t)]}{\sigma\sqrt{T-t}} \quad (9)$$

$$d_2 = \frac{\ln(\frac{S}{X}) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \quad (10)$$

Where $\Phi(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^y e^{-\frac{x^2}{2}} dx$ is the common normal distribution function; c denotes the cost of the European call option; S denotes the stock price when the option is exercised; X denotes the option's exercise price; r denotes the risk-free interest rate (derived through continuous compounding); and s denotes the standard deviation of stock returns (i.e., stock price volatility).

The aforementioned analysis shows that since the Black-Scholes pricing model does not take investors' risk tolerance into account, it is not necessary to predict investors' expected returns, the amount of future cash dividends, or the direction of the company's growth, somewhat making up for the shortcomings of traditional stock pricing methods[8].

2.3 Binomial Model

American options are typically valued using the Binomial Model or Binomial tree, a relatively simple approach for pricing options created by John C. Cox, Stephen A. Ross, Mark Rubinstein, and William F. Sharpe[9].

This approach has the benefit of being relatively simple to use with little math knowledge. The continuous movement of asset prices is simulated using the binomial tree approach with a discrete model. Matching mean and variance yields the necessary parameters, which are then used to compute the option price backwards from the end of the binomial tree diagram.

The fundamental concept of binomial tree pricing is that the life of the option is divided into a number of arbitrarily brief time intervals. It is assumed that during each of these brief time intervals, the price of the underlying asset will move from the beginning of S to two new values, with only two possible directions for movement: up or down. After the pertinent parameters have been established by contrasting the expected and variance of asset returns, the option price is inverted starting at the end of the binomial tree.

$$S = e^{-r(T-t)}[Sup + Sd(1 - p)] \quad (11)$$

$$f = e^{-rT}[pf_u + (1 - p)f_d] \quad (12)$$

Divide the option effective period T into as small an interval as Δt , and at each Δt , the stock price changes from S to Sup or Sd . If the probability of an upward price movement is P , then the probability of a downward movement is $1-p$. Since the rate of change in the price of the underlying asset follows a normal distribution, the formula may be found using the risk-neutral pricing principle. The binomial tree option pricing formula for a two-period call option may be obtained based on the formula for a single-period call option, and it can then be expanded to the formula for an n -period call option. Consider the purchase of a call option for n periods, the current price of the stock is S , and the probability of the stock rising or falling in each period p , $1-p$, and u , d values are the same. There are two possible prices for each period, so that by the end of the n period, all possible values of the stock price are , $i = 0, 1, \dots, n$, in accordance with the idea of deriving the two-period model, starting from the n th period forward recursively. The binomial tree pricing formula for n -period look-alike options can be obtained as follows.

$$c = e^{-nr\Delta t} \sum_{i=0}^n \left[\frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} \max(Su^i d^{n-i} - X, 0) \right] \quad (13)$$

Due to its benefits of simplifying option pricing calculations and improving intuition, the binomial tree option pricing model has already established itself as one of the primary pricing standards for major stock exchanges all over the world. The trinomial tree strategy, which Jarrow, Hull, and Boyle approximated, examines the drawbacks of the binomial tree approach and adjusts it to make it more accurate in the trinomial tree method. Adaptive refinement of tree shape and larger dimensionality freely leads to a more efficient pricing.

2.4 Monte Carlo Method

For the purpose of valuing European derivative assets, the Monte Carlo simulation method is used. This method's basic idea is to divide the effective term of the option into a number of brief time intervals, and then use a computer to simulate the stochastic movement of both the price of the underlying asset and the corresponding option return[10]. The expected value of this option is equal to the arithmetic mean of discounted returns computed from several random samples. If the price distribution function of the underlying asset is known, the effective term of the option is split into a number of discrete time periods. To determine a final value of the option as a random sample, a

straightforward arithmetic average of random samples is calculated after repeating the simulation millions of times and represents the expected return of the choice. According to the no-arbitrage pricing concept, the anticipated return is decreased by the risk-free rate, which is the option's current price:

$$P = e^{-rT} E(X_T) \quad (14)$$

where P denotes the price of the option; r denotes the risk-free rate; and $E(X_t)$ is the expected return of the option at time T .

The advantage of the Monte Carlo simulation method is that it operates more successfully and can handle the complex scenario of expected asset returns and volatility functions. Additionally, the simulation time grows linearly with the number of variables. It does not require a high level of mathematical preparation and much work to transform the model.

3. Comparison of Different Models

The various pricing methods described above can be split into analytical and numerical methods from the standpoint of problem-solving. Black-Scholes and traditional option pricing techniques are included in the former; Monte Carlo simulation and binomial tree approaches are included in the latter. Since its inception, the B-S model has attracted the attention of many economists and has been widely used in the financial practice, making a groundbreaking addition to the theory and practice of option pricing, and it significantly affected the market for financial derivatives. The B-S model provides investors with a convenient pricing formula for any derivative security applicable to stocks, but its derivation and application are also subject to various constraints. Obviously, the overly strict assumptions undermine the use of the original pricing formula, making it flawed in theory and application. For example, some of the input variables in the B-S option pricing model inevitably require manual subjective estimation, which is the basis for calculating the option price, and its relevance and reliability are lacking. The later the option expiration date, the more volatile the market is likely to be, making investor disagreement and pricing relatively difficult. Also, the level of expected stock dividends in the market may differ from the current level. Trading prices are also influenced by market supply and demand, investor psychological expectations and other factors.

Numerical calculation methods have their own advantages and disadvantages. The Monte Carlo simulation approach has the benefit of being able to handle more complicated situations and having a high relative calculation efficiency since it derives the option value at a future time from the original value. However, it cannot be used to calculate American-style options which are exercised prior to the signing of a contract; it can only be used to compute European-style options. It is possible to compute American options using the binomial tree approach and the finite difference method since they are predicated on the future value of the option going back to its initial value. These two methods are not only computationally intensive and inefficient, but also challenging to calculate the complex situation of the option depending on the historical path of the state variable. In a non-complete market situation, the price of a derivative asset is not a definite value, but an interval. In the case of a complete financial market, this interval degenerates to a point. Deterministic arbitrage pricing methods, interval pricing methods and E-arbitrage pricing methods are all applicable to both complete and incomplete financial markets. Mathematical theories and tools are continuously applied to option pricing, such as common inverse stochastic differential equations, fast Fourier transforms, etc.

4. Conclusion

Option pricing theory is a significant component of finance, and its impact goes well beyond option trading. It is a useful tool for corporate decision-makers to analyze investment decisions; equity analysts use it to estimate corporate bonds; many economists have expanded its applications to include everything from analysis of currency fluctuations to wage negotiations; and it is also

frequently used in guarantee and insurance contracts. Over the past 20 years, especially since the 1990s, options have quickly evolved and risen to become one of the most well-liked financial derivative instruments. Options exhibit diversity, adaptability, and strong risk-averse, risk-investing, and value-discovery skills. The two main areas of option theory research today are how to create new options to fulfill the shifting investment needs of the market and how to calculate the value of these more complicated options. Theories and methods of option pricing are still being explored and developed, which will be applied to a wider range of financial derivative products and economic fields. This paper summarizes the previous research results on option pricing, which still has many prospects that can be expected by future researchers.

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