

Different Models and Their Influences on Portfolio

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Abstract. In the modern era, people's lives and economic activities become increasingly related. People use financial models to make their financial decisions for investing. With these financial models, investors can analyze the portfolio, find the optimal weights of each stock to maximize return and minimize risks, and ultimately make wise decisions. Markowitz Model put forward in 1952 by Harry Markowitz and Index Model developed by William Sharpe in 1963 are two important modelling methods in the field of portfolio management. In this paper, ten stocks from four different industries are used to build a portfolio with low overall risk. After that, Markowitz Model and Index Model are compared and contrasted through constructing minimum variance portfolios and maximum Sharpe portfolios. In addition, the analysis is applied on three different constraints might be provided by the clients. During the process, the data analysis and model construction are through Microsoft Excel. When constructing the Minimum Variance Portfolio, the Markowitz Model has larger return, standard deviation and Sharpe, but when constructing the Maximum Sharpe Portfolio, the Index Model has larger return, standard deviation and Sharpe.

Keywords: Portfolio Management; Markowitz Model and Index Model.

1. Introduction

As people's lives and economic activities become more closely related, people use financial model to make important financial decisions. With the help of models, people could have their business ideas and risks to be estimated in a cost-effective way. To better achieve this purpose, different financial models has been introduced and continuously improved in the recent decades. Markowitz Model put forward in 1952 by Harry Markowitz and Index Model developed by William Sharpe in 1963 are two important modelling methods in the field of portfolio management.

Markowitz Model has been put forward by Harry Markowitz in 1952 [1]. The merits of using a specific security relationship model in the practical application of Markowitz's portfolio analysis techniques have been described. In this article, the Index Model was developed by William Sharpe [2]. Different portfolio models, focusing on their corresponding or related optimization problems has been discussed [3]. The practical limitations of Markowitz model have been suggested and how Index Model simplified these processes has been introduced [4]. It has been shown that how the Markowitz's mean-variance portfolio selection model, which is known as the Markowitz Model, minimizes investment risk under some level of return and maximizes investment return under some level of risk. [5]. Optimal portfolios have been determined by using the Markowitz Model theory from the data inputs of the prices of in total 50 stocks from the Bulgarian Stock Market, and the optimal portfolio that could maximize rate of return of portfolios regarding the risk preference has been selected [6]. How the Sharpe Model construct an optimal portfolio for 15 securities and evaluate their performance has been shown [7]. An optimal equity portfolio has been constructed by the assistance of Sharpe index model for twenty companies from media and entertainment sectors [8]. The research utilizing Sharpe Index Model to see the risk and return of 18 stocks from three distinct large sectors, the optimum portfolio to be formed, and the number of shares to be purchased each stock has been shown [9]. The optimum portfolio through the Sharpe's Index Model for ten companies selected from both the sugar and metal sectors has been constructed [10]. The methods of constructing an optimal portfolio utilizing Sharpe's Index Model with the NSE NIFTY Shares has been shown [11]. New insights into the dilemmas which are identified in routine project portfolio management has been offered by alternative perspectives and avenues for resolving them are opened up, success in project portfolio management has been promoted [12]. The Markowitz model using

only series notation has been developed and simulation-based approaches to the Markowitz model as well as classroom aspects of the various ways of approaching this important topic has been discussed [13]. Markowitz portfolio theory has been explained in an understandable and correct way, and implementation of the Markowitz model called Capital Asset Pricing Model has been discussed [14]. By analyzing the capital asset pricing model and the Markowitz model, managers has been able to insert their systematic approach to changing market interest rates and associated risk elements. [15]. The comparison of the results of optimal portfolio formation by utilizing Markowitz models and index models in the covid 19 pandemic in companies listed in the LQ Index 45 has been found out [16].

In this paper, a portfolio with 10 stocks from four different industries is built. After constructing the Markowitz Model and Index Model separately and find their corresponding minimum variance and maximum Sharpe portfolios, patterns could be found through the results. When constructing the Minimum Variance Portfolio, the Markowitz Model has larger return, standard deviation and Sharpe, but when constructing the Maximum Sharpe Portfolio, Index Model has larger return, standard deviation and Sharpe.

2. Data

In this paper, the portfolio is formed by 10 stocks, covering 4 different industries: consumer cyclical, technology and financial service and industrial industry.

The first industry is consumer cyclical. Consumer cyclical is a category of stocks rely heavily on the economic conditions and business cycle. Among the companies in the portfolio, Amazon.com, Inc (AMZN) belongs to this industry.

AMZN is a multinational technology company from the United States focusing on e-commerce, online cloud computing, digital streaming and the artificial intelligence known as AI.

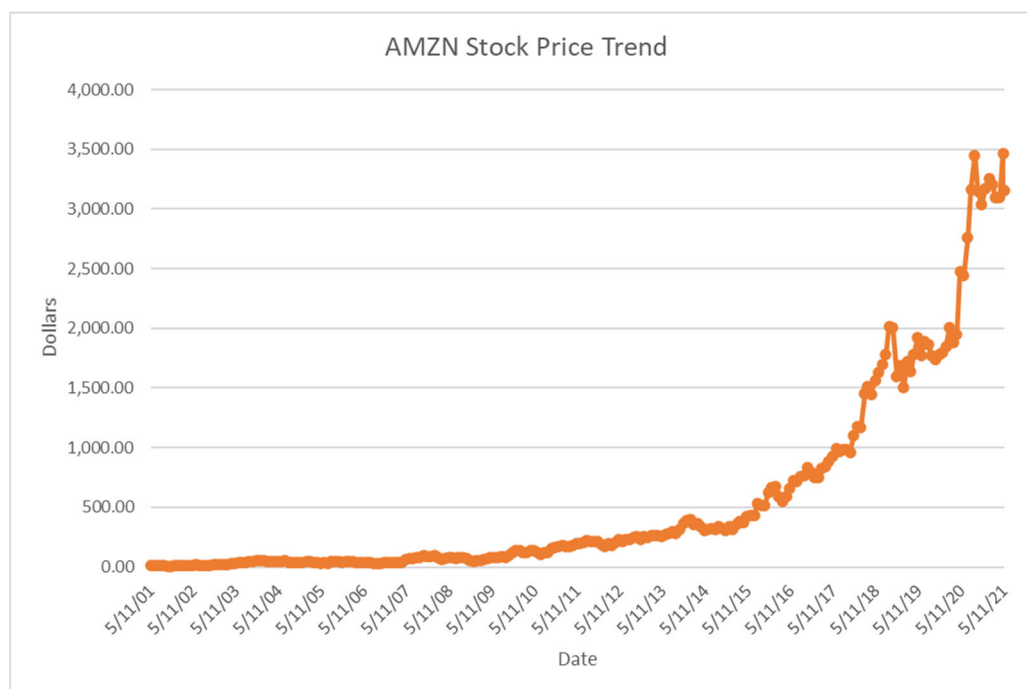


Fig. 1 AMZN Stock Price Trend
Source: Yahoo Finance [17]

The second industry is technology. The technology sector is the category of stocks related to the development, research, or the distribution of goods and services which as technologically based. Apple Inc.(AAPL) and Citrix Systems, Inc.(CTXS) belong to this industry.

AAPL is a multinational technology company from the United States. It specializes in its consumer electronic devices, software, and its online services. This Company is headquartered in Cupertino, California, United States.

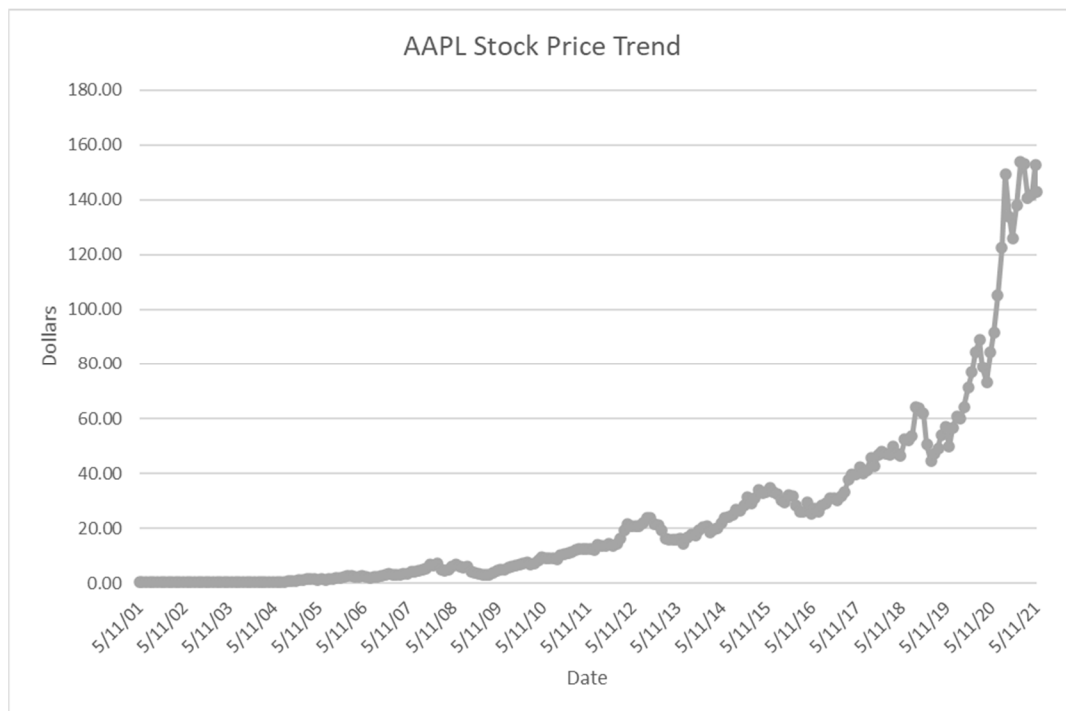


Fig. 2 AAPL Stock Price Trend
Source: Yahoo Finance [17]

CTXS is a multinational cloud computing and virtualization technology company from the United States which provides application, software as a service (SaaS), desktop virtualization, networking, and the online cloud storage technologies.



Fig.3 CTXS Stock Price Trend
Source: Yahoo Finance [17]

The third industry is financial services, which is an important section in the economy contained with institutions and firms that provide commercial and retail customers with financial services. There are three companies included in the portfolio: JPMorgan Chase & Co. (JPM), Berkshire Hathaway Inc. (BRK/A), and The Progressive Corporation (PGR).

JPM is a multinational investment bank, and it is also a company of financial services from the United States incorporated in Delaware and headquartered in New York City.

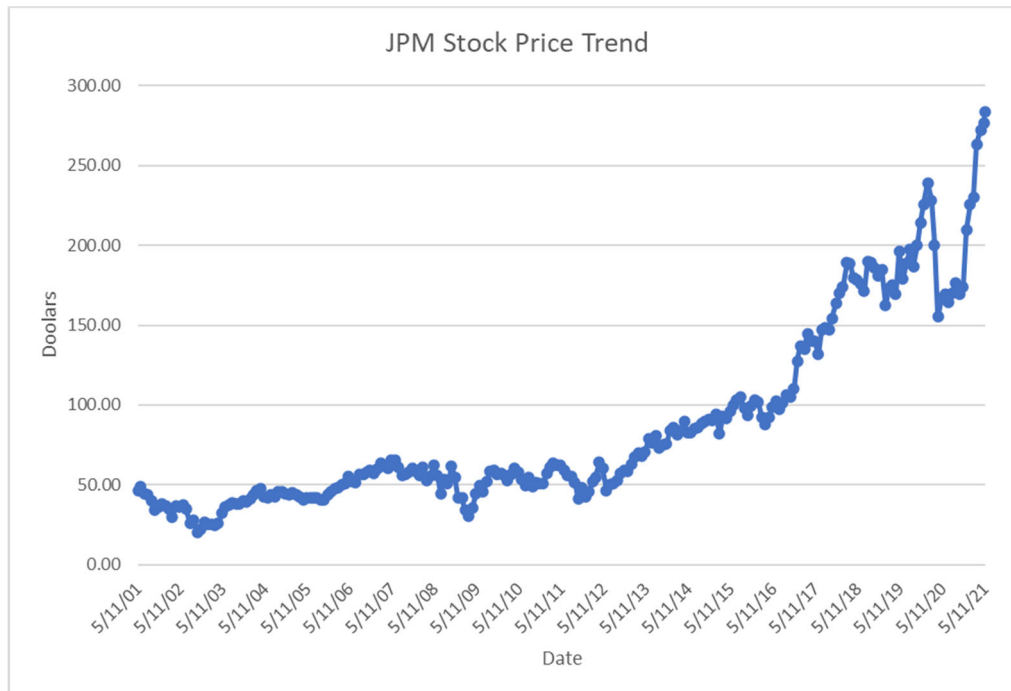


Fig. 4 JPM Stock Price Trend
Source: Yahoo Finance [17]

BRK/A is a multinational conglomerate holding company from the United States headquartered in Omaha, Nebraska, United States.



Fig. 5 BRK/A Stock Price Trend
Source: Yahoo Finance [17]

PGR is an insurance company from the United States, the third largest insurance carriers and the top one commercial auto insurer in America.

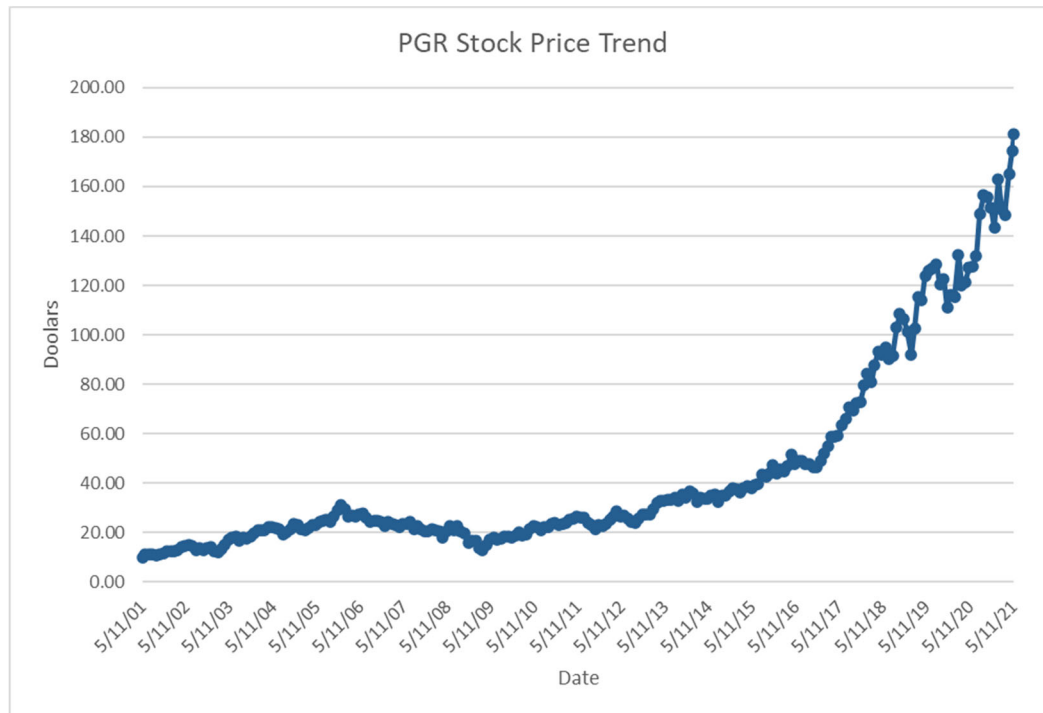


Fig. 6 PGR Stock Price
Source: Yahoo Finance [17]

The fourth industry is the industrial industry. The industrial goods sector is a category consists of companies that make or sell equipment, machinery, or supplies used in construction and manufacturing. This industry includes four of the companies in the portfolio: United Parcel Service, Inc. (UPS), FedEx Corporation (FDX), J.B. Hunt Transport Services, Inc.(JBHT), Landstar System, Inc.(LSTR).

UPS is a multinational shipping, supply chain management, and receiving company from the United States founded in 1907.

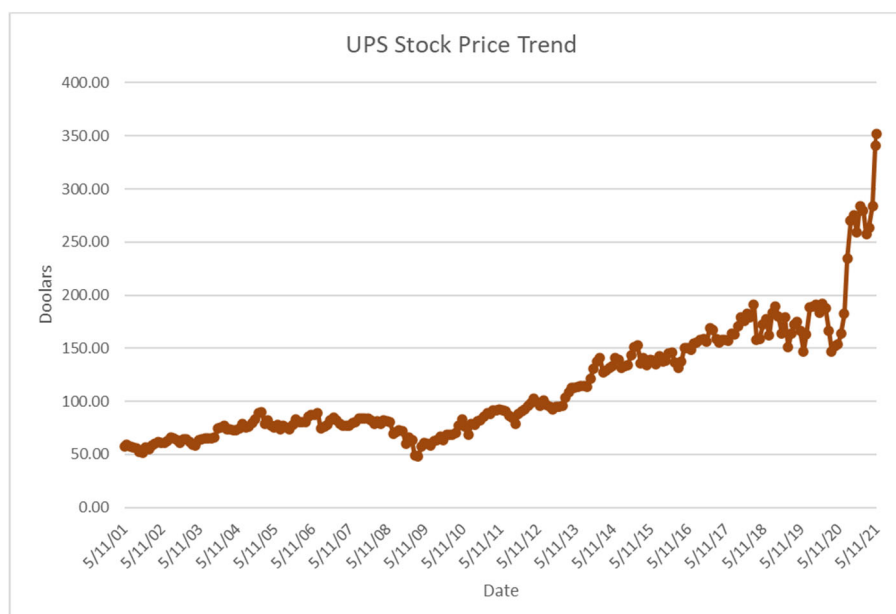


Fig. 7 UPS Stock Price Trend
Source: Yahoo Finance [17]

FedEx Corporation is a multinational company of conglomerate holding from the United States. Its business scope is mostly focused in the field of transportation, on e-commerce and services.

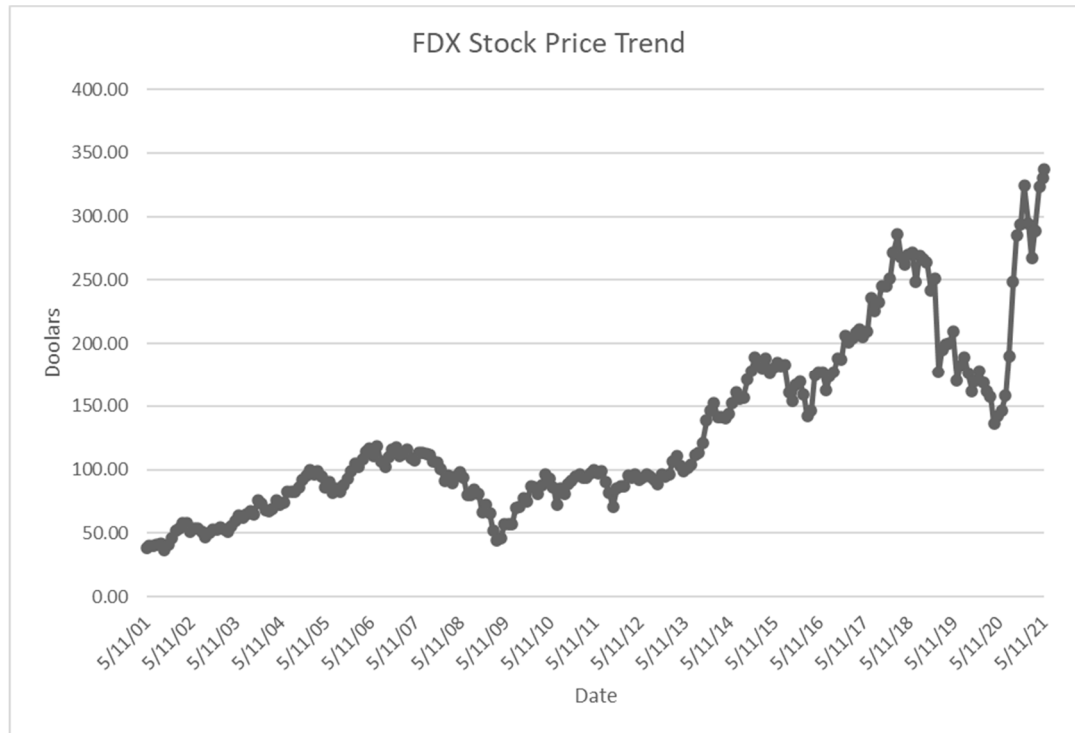


Fig. 8 FDX Stock Price Trend
Source: Yahoo Finance [17]

JBHT is a logistics and transportation company from the United States based in Lowell, Arkansas.

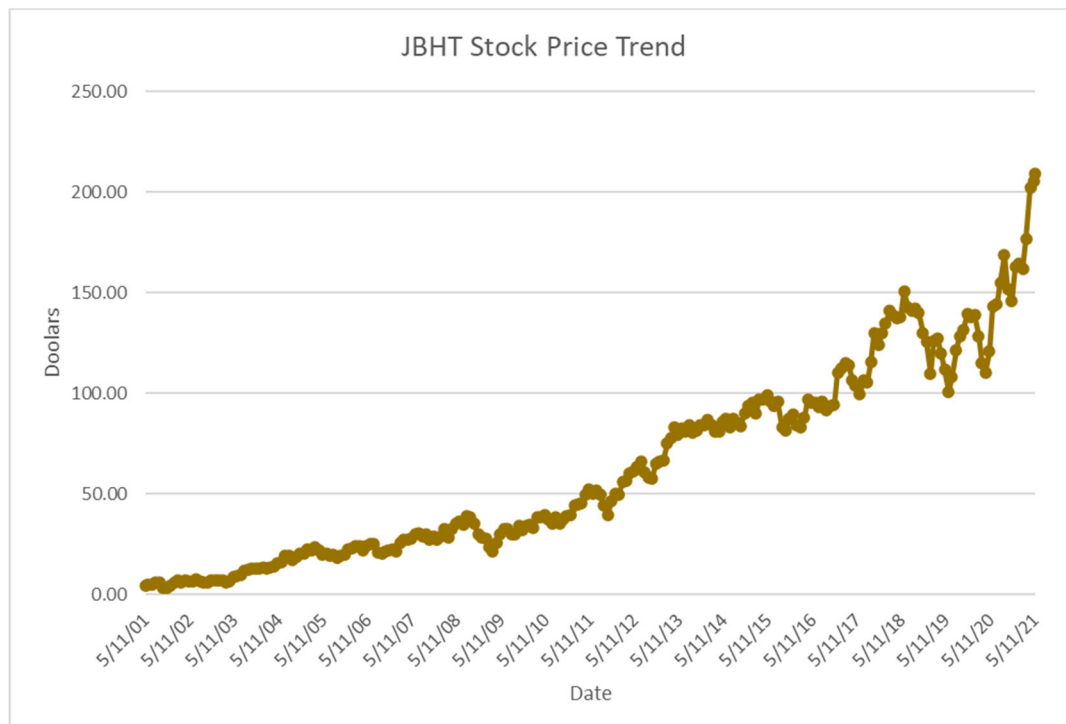


Fig. 9 JBHT Stock Price Trend
Source: Yahoo Finance [17]

LSTR is a transportation services company from the United States specializing in logistics and more specifically, third-party logistics.



Fig. 10 LSTR Stock Price Trend
Source: Yahoo Finance [17]

Besides the stock price trend of each separate companies, Annualized Average Return, Annualized Standard Deviation, Beta, Alpha and Annualized Residual Standard Deviation are important for further calculation and analysis. In addition, the correlation between companies reflects the overall risk of this portfolio.

Table 1. Annualized Average Return, Annualized Standard Deviation, Beta, Alpha and Annualized Residual Standard Deviation

	SPX	AMZ N	AAP L	CTX S	JPM	BRK/ A	PGR	UPS	FDX	JBHT	LSTR
Annualized Average Return	7.542 %	33.79 6%	34.01 6%	15.63 5%	11.86 5%	9.002 %	15.40 6%	9.850 %	12.95 2%	22.52 4%	17.38 7%
Annualized Standard Deviation	14.85 0%	41.41 0%	34.45 2%	41.51 4%	29.01 3%	16.24 3%	21.05 9%	21.43 7%	26.69 0%	30.65 5%	23.92 3%
beta	1	1.351 3	1.256 89	1.220 55	1.360 8	0.572 13	0.712 05	0.829 62	1.103 63	1.076 3	0.797 56
alpha	0.000 %	23.60 4%	24.53 6%	6.429 %	1.601 %	4.687 %	10.03 5%	3.592 %	4.628 %	14.40 6%	11.37 2%
Annualized Residual Standard Deviation	0.000 %	36.22 2%	28.95 7%	37.34 8%	20.81 8%	13.84 4%	18.21 1%	17.54 3%	21.06 6%	26.15 8%	20.78 5%

Table 2. Correlation Matrix Between Companies

Correlations	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
SPX	1	0.4846	0.54177	0.43661	0.69652	0.52306	0.50213	0.57472	0.61405	0.52139	0.49508
AMZN	0.4846	1	0.37698	0.21657	0.25183	0.11761	0.19977	0.29627	0.28003	0.30779	0.25649
AAPL	0.54177	0.37698	1	0.33173	0.24443	0.17347	0.23969	0.23144	0.32953	0.26844	0.28701
CTXS	0.43661	0.21657	0.33173	1	0.32411	0.18091	0.27141	0.2643	0.33074	0.28983	0.25205
JPM	0.69652	0.25183	0.24443	0.32411	1	0.45212	0.39329	0.36054	0.43961	0.44154	0.37465
BRK/A	0.52306	0.11761	0.17347	0.18091	0.45212	1	0.26365	0.40407	0.3849	0.23906	0.23389
PGR	0.50213	0.19977	0.23969	0.27141	0.39329	0.26365	1	0.39164	0.3646	0.28038	0.28919
UPS	0.57472	0.29627	0.23144	0.2643	0.36054	0.40407	0.39164	1	0.67486	0.45888	0.44073
FDX	0.61405	0.28003	0.32953	0.33074	0.43961	0.3849	0.3646	0.67486	1	0.5367	0.482
JBHT	0.52139	0.30779	0.26844	0.28983	0.44154	0.23906	0.28038	0.45888	0.5367	1	0.58953
LSTR	0.49508	0.25649	0.28701	0.25205	0.37465	0.23389	0.28919	0.44073	0.482	0.58953	1

Because almost all correlations are less than 0.5, the stock price of all the companies in the portfolio is not closely correlated. Therefore, due to the low correlation, the overall risk for the portfolio is low. Therefore, it is a well-established portfolio to minimize potential loss.

3. Model

The portfolio performance is based on different models. We are discussing the Markowitz Model and the Index Model in this paper.

In finance, the Markowitz mode is a portfolio optimization model put forward in 1952 by Harry Markowitz [1]. It helps to select the optimal portfolio by analyzing many possible portfolios based on the provided securities. Therefore, the Markowitz Model informs the investors how to reduce their risks by choosing securities that do not change exactly together. In addition, the Markowitz model is also called mean-variance model, because it is focused on the expected returns (also called mean) and the standard deviation (also called variance) of the different portfolios.

Markowitz Model portfolio return:

$$r_p = \vec{w} \cdot \vec{\mu}^T \quad (1)$$

Markowitz Model portfolio standard deviation:

$$\sigma_P = \sqrt{\vec{v}P\vec{v}^T} \quad (2)$$

The Index model is a basic model of asset pricing aimed to measure the stock returns as well as the risks. The model was developed in 1963 by William Sharpe [2] and because of its importance, it is commonly applied by investors in the finance industry today. These equations show the fact the market impact the stock returns (beta), the certain expected value(alpha) of a company and the certain unexpected component (residuals) of a company. The performance of each stock is associated with the performance of a market index. Security analysts often use the index models for some functions like calculating the stock betas, conducting event research, and evaluating stock-picking skills.

Index Model portfolio return:

$$r_p = \vec{w} \cdot \vec{\mu}^T \quad (3)$$

Index Model portfolio standard deviation:

$$\sigma_P = \sqrt{(\sigma_M \beta_P)^2 + \sum_{i=1}^n w_i^2 \sigma^2(\varepsilon_i)} \quad (4)$$

where we denoted $\beta_P = \vec{w} \cdot \vec{\beta}^T$ as the portfolio beta.

The analysis is applied on different constraints.

Constraint 1:

A "free" problem with no extra optimization constraints. This original constraint is aimed to illustrate what the region of the portfolio is allowed to look like in general, and in particular the efficient frontier without constraints

Constraint 2:

$$|w_i| \leq 1, \text{ for } \forall i \quad (5)$$

Clients can provide this extra optimization constraint, which is aimed to simulate some arbitrary "box" constraint on the stocks' weights of the portfolio.

Constraint 3

$$w_1 = 0 \quad (6)$$

Finally, we want to see if having a broad index in the portfolio would has some positive or negative impacts, so this additional optimization constraint is taken account for

4. Results

The Markowitz Model and the Index Model is compared and contrasted by the results of two portfolios: the minimum variance portfolio and the maximum Sharpe portfolio. The minimum

variance portfolio would only focus on reducing the risk, while the maximum Sharpe portfolio cares not only the lower risks, but also the higher returns.

The Minimum Variance Portfolio Focus on Reducing Risk, and here is the Minimum Variance Portfolio using Markowitz Model.

Table 3. Weights of Minimum Variance Portfolio using Markowitz Model

	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
Constraint#1	0.7224	-0.0235	-0.0385	-0.0105	-0.1847	0.3621	0.1391	0.0343	-0.1028	-0.0055	0.1075
Constraint#2	0.7224	-0.0235	-0.0385	-0.0105	-0.1847	0.3621	0.1391	0.0343	-0.1028	-0.0055	0.1075
Constraint#3	0.0000	0.0245	0.0419	0.0084	-0.0709	0.5631	0.2352	0.1141	-0.0805	0.0060	0.1582

Table 4. Return, Standard Deviation and Sharpe of Minimum Variance Portfolio using Markowitz Model

	Return	Standard Deviation	Sharpe
Constraint#1	0.0715	0.1224	0.5839
Constraint#2	0.0715	0.1224	0.5839
Constraint#3	0.1320	0.1339	0.9858

The Maximum Sharpe Portfolio Focus on Reducing Risk and Maximizing Return, and here is the Maximum Sharpe Portfolio using Markowitz Model.

Table 5. Weights of Maximum Sharpe Portfolio using Markowitz Model

	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
Constraint#1	-2.3752	0.3706	0.6540	0.0042	0.1730	0.9158	0.6819	0.0127	-0.0868	0.3097	0.3402
Constraint#2	-1.0000	0.2233	0.3976	-0.0120	-0.0050	0.6247	0.4601	-0.0311	-0.1056	0.2088	0.2392
Constraint#3	0.0000	0.1465	0.2673	-0.0344	-0.1554	0.3634	0.3199	-0.1216	-0.1321	0.1783	0.1681

Table 6. Return, Standard Deviation and Sharpe of Maximum Sharpe Portfolio using Markowitz Model

	Return	Standard Deviation	Sharpe
Constraint#1	0.4961	0.3225	1.5385
Constraint#2	0.3318	0.2211	1.5007
Constraint#3	0.2389	0.1802	1.3261

Table 7. Weights of Minimum Variance Portfolio using Index Model

	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
Constraint#1	0.6579	-0.0414	-0.0474	-0.0244	-0.1287	0.3451	0.1342	0.0856	-0.0361	-0.0172	0.0724
Constraint#2	0.6579	-0.0414	-0.0474	-0.0244	-0.1287	0.3451	0.1342	0.0856	-0.0361	-0.0172	0.0724
Constraint#3	0.0000	-0.0155	-0.0081	-0.0011	-0.0502	0.4799	0.2165	0.1782	0.0345	0.0282	0.1376

Table 8. Return, Standard Deviation and Sharpe of Minimum Variance Portfolio using Index Model

	Return	Standard Deviation	Sharpe
Constraint#1	0.0646	0.1243	0.5200
Constraint#2	0.0646	0.1243	0.5200
Constraint#3	0.1147	0.1321	0.8685

When observing the result of the two models, patterns could be found through the differences. When constructing the Minimum Variance Portfolio, the Markowitz Model has larger return, standard deviation and Sharpe, but when constructing the Maximum Sharpe Portfolio, Index Model has larger return, standard deviation and Sharpe. This result means that if the Minimum Variance Portfolio is

expected, Markowitz Model could bring higher return with higher risks, and if the Maximum Variance Portfolio is expected, Index Model could bring higher return with higher risks.

Table 9. Weights of Maximum Sharpe Portfolio using Index Model

	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
Constraint#1	-3.2979	0.4302	0.6996	0.1102	0.0883	0.5848	0.7235	0.2791	0.2494	0.5034	0.6293
Constraint#2	-1.0000	0.2184	0.3662	0.0309	-0.0879	0.3332	0.3998	0.0952	0.0562	0.2497	0.3383
Constraint#3	0.0000	0.1856	0.3169	-0.0047	-0.2123	0.1167	0.2766	-0.0530	-0.0540	0.1857	0.2425

Table 10. Return, Standard Deviation and Sharpe of Maximum Sharpe Portfolio using Index Model

	Return	Standard Deviation	Sharpe
Constraint#1	0.6091	0.3816	1.5960
Constraint#2	0.3407	0.2240	1.5210
Constraint#3	0.2695	0.2024	1.3311

5. Conclusion

In this paper, by investigating the ten companies from four different industries (consumer cyclical, technology and financial service and industrial industry) and their stock price, we get to know the correlation matrix among the stocks. From the correlation matrix, we learn that the overall low correlation would minimize the risk and potential loss of this portfolio. Then from the two models constructed, the Markowitz Model and the Index model and their corresponding minimum variance and maximum Sharpe portfolio, we can find out the pattern that when constructing the Minimum Variance Portfolio, the Markowitz Model has larger return, standard deviation and Sharpe, but when constructing the Maximum Sharpe Portfolio, Index Model has larger return, standard deviation and Sharpe. Therefore, if investors just hope to minimize risk through the minimum variance portfolio, they can use Markowitz Model to have the weights of the portfolio that have larger return along with higher risks; however, if investors hope to find the portfolio that have low risk and large return at the same time, they can use Index Model to obtain the weights. However, in the present days, some other portfolio modelling methods like Capital Asset Pricing Model are also widely used by investors. Therefore, based on different real cases, the most optimal model may not be discussed in this paper. In real-life applications, more factors and more modelling methods should be taken into consideration.

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