Portfolio Optimization Analysis in American Industry

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Abstract. Portfolio optimization, as a process of reorganizing investment and dispersing risks by considering the established target return and risk tolerance, is widely used in the financial field. This paper analyzes the asset allocation of six kinds of assets. It uses three methods, namely, mean-variance analysis, CAPM, and the FF3F model, to optimize the investment portfolio. At the same time, the proportion of assets in the portfolio should be considered under the conditions of maximizing the return or minimizing the risk at a specific risk level. The results show that in the mean-variance model, the AEP and Gold ETF have the most considerable weight in the combination of maximum Sharpe ratio and minimum variance, respectively. In the FF3F model, AT&T and Gold ETF have the most prominent weight in the combination of maximum Sharpe ratio and minimum variance, while in CAPM, Google and Gold ETF have the most significant weight in the combination of maximum Sharpe ratio and minimum variance respectively. This research may help potential investors interested in power, communication, science and technology, gold, and automobile industries.

Keywords: Mean-variance; CAPM Model; FF3F Model; Sharp ratio.

1. Introduction

American economist Markowitz initially proposed the portfolio theory and carried out systematic, comprehensive, and persuasive research that year, earning him the Nobel Prize in economics. The efficient portfolio border model and the mean-variance analysis approach are two crucial parts of the theory. The Markowitz portfolio theory has been proven to be practical in the established stock market and is widely used to asset pricing and portfolio management.

People choose their investments based on the ambiguity of the risks and advantages. Every financial investor's first priority is to maximize returns while reducing dangers. Portfolio theory defines both of the primary elements as mean and variance. The so-called mean value, which is the weighted average of the predicted rate of return of a single security, represents the expected rate of return of the portfolio, and the weight is the appropriate investment proportion. Undoubtedly, dividends and capital gains are included in the income from stocks. The portfolio's return rate variance is referred to as the "so-called variance." We refer to the measure of risk in a portfolio as the volatility of the standard deviation of the rate of return. How should consumers decide how much income and risk to take on while investing in securities? The main problem in portfolio theory research is this .

Portfolio theory research has been ongoing and is constantly progressing. Portfolio, capital asset pricing, APT, efficient market, and behavioral finance theories are all part of modern portfolio theory. Their evolution has dramatically altered traditional investment management practice, which previously relied primarily on fundamental analysis, and has caused modern investment management to develop in a systematic, scientific, and combinatorial direction.

In general, Markowitz's portfolio theory offers an analytical framework for how to properly diversify investments as well as a theoretical foundation for diversified investment. The Markowitz approach, however, has several restrictions and challenges in real-world use. The expected return on securities, variance, and covariance between two securities, for instance, are some of the main inputs needed by the Markowitz model. The usage of Markowitz is constrained when the number of securities is considerable because the major input necessitates extensive estimation. Consequently, the Markowitz model is mostly used when deciding on the best asset allocation strategy. The unpredictability of answers brought on by data inaccuracies is another illustration. The predicted

return on investments, the expected standard deviation, and the anticipated correlation between investments must all be used as known data as inputs into Markowitz's model. If there is no estimating error in these data, the Markowitz model can guarantee a successful portfolio. However, as the anticipated data must be statistically estimated because it is unknown. Due to statistical estimation's inaccuracy, some asset classes' investment proportions will be excessively high and others excessively low.It can be noted that portfolio management between certain stocks and mutual funds is limited. Therefore, this has aroused the interest of in-depth research on this issue. This paper selects the closing prices of six companies from August 2017 to August 2022: AT&T, American Electric Power Co Inc., Google, Tesla, Apple, and Gold ETF. The research process can be summarized as follows. First, this paper selects the closing prices of these six companies from different fields from August 2017 to August 2022. Moreover, the data needs to be sorted out within a specified period. Second, this paper lists three analysis methods to find the optimal portfolio: mean-variance analysis, capital asset pricing model, and Fama-French three-factor model, and calculates the maximum Sharpe ratio portfolio and the minimum variance portfolio, specifically Third, for these three portfolio analysis methods, we do data analysis of each method and compare the differences between them, because it is impossible that those methods still have shortcuts. The mean-variance model relies too much on historical data and cannot judge the market trend well. Based on the market prediction of future earnings, the latter two models may not consider all the factors affecting earnings, which will cause specific errors.

The following is a summary of the rest. Part 2 shows the data, Part 3 describes the method, Part 4 shows the results, and Part 5 shows the conclusion.

2. Data

The data in this paper comes from Yahoo Finance (https://finance.yahoo.com/). This paper selects the closing prices of six companies from August 2017 to August 2022: AEP, AT&T, Google, Tesla, Apple, and Gold ETF. These companies perform well and have a high rate of return. At the same time, according to the "modern portfolio theory" developed by Macovei in the 1950s, assets with low correlation in different fields such as power, communication, science and technology, gold, and automobile can improve the effectiveness of the portfolio and reduce the risk of portfolio without affecting the return of the portfolio. Table 1 shows the descriptive statistics of these assets.

Table 1. Descriptive statistics of the selected assets

	AEP	AT&T	GOOG	TSLA	AAPL	GLD
Geometric Mean	1.0094	1.0012	1.0156	1.0439	1.0253	1.0061
Variance	0.0032	0.0035	0.005	0.0427	0.0076	0.0014
maximum	0.142	0.1451	0.1651	0.7415	0.2144	0.1079
minimum	-0.1435	-0.1723	-0.1767	-0.2243	-0.184	-0.0715
Value of risk	-0.0877	-0.0864	-0.0985	-0.1978	-0.114	-0.0448

As seen from the table1, the average yield of Tesla is 1.0439 at the highest, and that of AT&T is 1.0012 which is the lowest. As for the variance value of risk measurement, Tesla enterprises have the highest risk of 0.0427, and the Gold ETF, as an ETF, has a lower risk of 0.0014. When comparing the maximum yield of each asset, Tesla has the highest peak, reaching 0.7415. For the minimum return rate of each asset, the lowest lower limit for Tesla is -0.2243. At the same time, we also calculated the performance of each asset's average monthly income in the 5% worst month. Tesla achieved the worst performance, which also reflected that Tesla's stock income fluctuated wildly.

3. Methods

3.1 Mean-Variance Analysis

Mean-variance analysis is a mathematical framework for assembling a portfolio of assets such that the expected return is maximized for a given level of risk. What is more, the mean-variance analysis is known to be most fundamental infrastructure in financial economics. According to Ismail and Pham, using Markowitz's efficient portfolio strategy is a popular approach because of its straightforward calculation method. As one of the essential financial tools, it is concise and practical. The following is the relevant formula:

$$\sum_{i} Weight_i = 1 \tag{1}$$

$$\mathrm{E}(R_P) = \sum_{i=1}^{n} w_i r_i \tag{2}$$

The weight of total capital invested in asset i is denoted by $\mathrm{Weight_i}$, while the expected return for asset i is expressed by $\mathrm{r_i}$. The expected return of total assets is expressed as E(RP), the sum of the expected returns of all assets in the portfolio.

$$\text{Variance of Portfolio} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j Cov(r_i, r_j) \tag{3}$$

It shows the variance-covariance between expected return for asset i and expected return for asset j.

$$\text{Sharpe ratio} = \frac{\mathrm{E}(R_P) - R_f}{\sigma_P} \tag{4}$$

Sharp ratio, as an asset performance evaluation index that comprehensively considers income and risk, calculates the additional income of each unit of risk undertaken by the asset portfolio. The formula is (annualized rate of return-risk-free interest rate) annualized volatility of the portfolio, where $\mathrm{E}(R_P)$ is the expected rate of return of assets, R_f is the risk-free interest rate, and σ_P is the standard deviation of asset return rate, which depicts the volatility of asset return.

3.2 Capital Asset Pricing Model

Based on asset portfolio theory and capital market theory, American academics William Sharpe, John Lintner, Jack Treynor, and Jan Mossin created the capital asset pricing model (CAPM) in 1964. This model now still "remains central to economic theory" and has also been widely used in the area of corporate finance practicing from the research of Graham and Harvey. It primarily investigates how the equilibrium price is established and the relationship between the expected return of assets and risk assets on the securities market. It is the cornerstone of contemporary financial market price theory and is extensively applied in both corporate finance and investment decision-making.

The capital asset pricing model makes the supposition that all investors choose their investments in accordance with Markowitz's asset selection theory and that the estimates of anticipated return, variance, and covariance are the same for all investors. Loans are unrestricted for investors. Based on this supposition, research on capital asset pricing models focuses on examining the quantitative link between the return of risky assets and the risk, i.e., how many return investors should receive to offset a specific level of risk. The simple conclusion drawn by CAPM is that there is only one way for investors to increase returns: by purchasing high-risk stocks. Despite the fact that many researches have cast doubt on the CAPM's accuracy, the financial industry still uses this model extensively. The capital Asset Pricing Model is a single-factor model that only considers the market portfolio. In other words, a stock's beta value is 1 if its price is in line with the price volatility of the market as a whole. The daily market returns and daily price returns of individual equities over the same time period are statistically analyzed to determine beta. The related equation is shown below:

$$\mathrm{E}(R_i) = R_f + \beta_i(\mathrm{E}(R_m) - R_f) \quad (5)$$

3.3 Fama-French Three-factor Model

In 1992, Fama and French conducted research on the variables that affect stock returns in the American stock market. They found that while market value, book to market value ratio, and P/E ratio of listed companies can all explain stock return variations, the beta value of the stock market was unable to do so. According to Fama and French, the above-mentioned excess return has an impact on CAPM's "Compensation for risk variables not reported." They discovered that the two variables of stock market value and book-to-market value ratio can substitute the functions of other risk indicators (such as E/P) in explaining fluctuations in the majority of stock prices. In their paper in 1993, they constructed three factors by simulating market risk, market-to-market risk, and book-to-market value ratio risk to explain the changes in stock returns. It expands on CAPM by adding another two risk factors. When evaluating a portfolio's performance compared to market returns, the Fama-French three-factor model is always applied [10].

Fama and French noted that a three-factor model might be developed to account for the stock return in 1993. The model contends that the exposure to three factors—market asset portfolio (RM RF), market value factor (SMB), and book-to-market ratio factor—can explain the excess return of an investment portfolio, including a single stock (HML). The following is an example of the multi-factor equilibrium pricing model:

$$\mathrm{E}(R_i - R_f) = \beta_i(\mathrm{E}(R_m) - R_f) + s_i(\mathrm{E}(\mathrm{SMB})) + h_i(\mathrm{E}(\mathrm{HML})) \quad (6)$$

SMB and HML refers to two different types of firm's condition, i.e., market capital and size, respectively.

4. Results

4.1 Mean-variance

According to the table2, the absolute value of covariance among most assets calculated by the correlation function is less than 0.01, which indicates that there is little correlation among assets. It can effectively reduce the risk of the asset portfolio and increase the stability of asset investment.

Table 2. Variance-covariance matrix

	AEP	AT&T	GOOG	TSLA	AAPL	GLD
AEP	0.0032	0.0008	0.0008	-0.0001	0.0005	0.0005
T	0.0008	0.0035	0.0007	0.0005	0.0009	-0.00005
GOOG	0.0008	0.0007	0.0050	0.0063	0.0030	0.0002
TSLA	-0.0001	0.0005	0.0063	0.0427	0.0101	0.0019
AAPL	0.0005	0.0009	0.0030	0.0101	0.0076	0.0006
GLD	0.0005	-0.00004	0.0002	0.0019	0.0006	0.0014

Under the mean-variance model, we solve the portfolio with the most significant Sharpe ratio.

Table 3. Results for maximum Sharpe ratio

		AEP	AT&T	GOOG	TSLA	AAPL	GLD
MVE portfolio weight	1	0.3123	0	0.1075	0.0881	0.2657	0.2265
MVE portfolio return	0.0201						
MVE portfolio variance	0.0024						
MVE Standard deviation	0.0494						
MVE Sharp Ratio	0.4079						

According to table3, AEP occupies the highest proportion, followed by Apple, Gold ETF, Google, and Tesla, among which AT&T occupies a minuscule proportion, accounting for 0. As profit-making companies, AEP have relatively stable stock prices with minor fluctuations, occupying a high weight in the investment portfolio. As high-end technology industries, Apple and Google still show a good development trend. When global finance fluctuates significantly, the Fed's interest rate hike is expected to be realized, and the price of gold shows up, gold, as a convenient and flexible exchange investment gold variety, once again attracts investors' attention, and it may have better returns according to its past performance. However, AT&T may be affected by the epidemic situation, and the historical data does not show an encouraging development trend, so it occupies a small weight. Finally, the maximum Sharpe ratio is 0.4079.

Table 4. Results for minimum variance

		AEP	AT&T	GOOG	TSLA	AAPL	GLD
MVP portfolio weight	1	0.1024	0.2176	0.1026	0	0.0019	0.5755
MVP return	0.0076						
MVP variance	0.0009						
MVP standard deviation	0.0297						
MVP sharp ratio	0.2562						

According to table4, Gold ETF occupies the highest proportion, followed by AT&T, Google, AEP, and Apple, among which Tesla occupies a minuscule proportion, accounting for 0. What needs to be emphasized is that minimum variance portfolio is the portfolio with the least risk in a series of portfolios, which is suitable for risk averse investors. Because of the equivalence of risk and return, the return of this investment method is also the lowest. It only needs to estimate the risk and relevance of each risky asset. MVP is generated by solving a quadratic equation that uses the range of all risky assets and generates the portfolio with the lowest volatility. Finally, the Sharpe ratio of the minimum variance portfolio is 0.2562.

When a portfolio's minimum variance is calculated, assets with high anticipated returns and a quasi-standard deviation weight of a low-standard portfolio will be overvalued; conversely, securities with low expected returns and a high standard deviation weight would be undervalued. Therefore, the estimate mistake in the anticipated return or covariance matrix will result in fraud in the selection of the optimum portfolio weight for the portfolio. As a result, the promotion process is frequently mocked as "incorrect profit maximisation". In the mean standard deviation optimum control, it is worse for the estimate error of anticipated return than the covariance matrix entity model.

4.2 CAPM Model

However, the mean-variance model is highly dependent on the historical data, so we further improve it and adopt the CAPM model to re-plan and solve it. We calculate the portfolio with the most significant Sharpe ratio using the CAPM model.

Table 5. Results for maximum Sharpe ratio under CAPM model

		AEP	AT&T	GOOG	TSLA	AAPL	GLD
CAPM portfolio weight	1	0.0305	0.2786	0.4361	0.0331	0.1958	0.0258
CAPM portfolio return	0.007						
CAPM portfolio variance	0.0027						
CAPM portfolio standard deviation	0.052						
CAPM portfolio sharp ratio	0.1192						

According to the table5, Google, AT&T, and Apple stocks have a high weight in the investment portfolio, while Tesla, American Power Company, and Gold ETF have a low weight. The new generation of digital technologies, represented by science and technology, has the power to empower a bright future, and it is expected to promote all-factor intelligent innovation of economic and social development and release development dividends. Therefore, it is still an industry with a relatively good development trend in the future. As high-tech industries, Google and Apple are expected still to occupy relatively advantageous development positions in the future market. According to the model's prediction of future earnings, AEP, as profitable enterprises, may have relatively stable earnings in a short period. However, they cannot have as large earnings growth expectations as growth-oriented enterprises, so they occupy a small weight. Finally, the maximum Sharpe ratio is calculated to be 0.1192. Also, this paper finds the result of minimum variance portfolio.

Table 6. Results for minimum variance portfolio under CAPM model

		AEP	AT&T	GOOG	TSLA	AAPL	GLD
CAPM portfolio weight	1	0.1024	0.2176	0.1026	0	0.0019	0.5755
CAPM portfolio return	0.0029						
CAPM portfolio variance	0.0009						
CAPM portfolio standard deviation	0.0297						
CAPM portfolio sharp ratio	0.0706						

It can be seen in the table6 that the most significant weight of 0.5755 appears in the Gold ETF, and the proportion indicates that this asset fluctuates less, which can bring more stable returns to investors. The most negligible weight of 0 appears in Tesla, which suggests that the risk of this asset fluctuates enormously, so it occupies the lowest proportion when minimizing the portfolio variance. This result may be due to short-term risk aversion and long-term tight supply and demand. As a financial derivative that tracks the fluctuation of spot gold price, gold ETF has a high probability of moderate fluctuation, so the fluctuation is slight and gains the maximum weight. Tesla has received tremendous market influence and negative news effects. Therefore, it has the lowest ratio when the variance is the smallest. This result is also in line with the ranking of the expected risk value of each asset that we calculated at first. Finally, the Sharp ratio of the portfolio is 0.0706 by the CAPM model.

4.3 FF3F Model

we adopt the FF3F model for further optimization due to the single CAPM considerations. Through calculation, we get the data under the FF3F model:

Table 7. Results for maximum Sharpe ratio portfolio under FF3F model

		AEP	AT&T	GOOG	TSLA	AAPL	GLD
FF3F porfolio weight	1	0.0362	0.4735	0.4172	0.0231	0.05	0
FF3F portfolio return	0.0068						
FF3F portfolio return	0.0023						
FF3F portfolio standard deviation	0.0484						
FF3F portfolio sharp ratio	0.1233						

According to the table7, AT&T and Google Company occupy a high proportion, and the predicted future earnings and risks have obvious advantages in this portfolio. Apple, AEP , Tesla Company, and Gold ETF all occupy a small weight, somewhat different from what we calculated using the mean-variance model. Although the market likes growth companies, it now pays a high return to high-growth companies with a higher P/E ratio. However, in the case of bad policies,

investors' interests may soon turn to value investment. Although A AT&T has not shown good performance in the past few years, when the market turns to a value-based company with an ultra-low P/E ratio and returns to a more defensive environment, the P/E ratio of AT&T will increase accordingly. Therefore, we still have a sizeable expected rate of return on this stock, and the maximum Sharpe ratio is calculated to be 0.1233. This paper also calculates the minimum variance of the portfolio.

Table 8. Results for minimum variance portfolio under FF3F model

		AEP	AT&T	GOOG	TSLA	AAPL	GLD
FF3F porfolio weight	1	0.1024	0.2176	0.1026	0	0.0019	0.5755
FF3F portfolio return	0.0029						
FF3F portfolio return	0.0009						
FF3F portfolio standard deviation	0.0297						
FF3F portfolio sharp ratio	0.0708						

As seen from the table8, Gold ETF has the most prominent weight, 0.5755, while TSLA has the most negligible weight, only 0. In the minimum variance portfolio, the higher proportion of Gold ETF may be due to the relatively stable gold assets and slight fluctuation variance, while Tesla has higher returns when the market is optimistic. As the Federal Reserve began to raise interest rates sharply and rapidly, the American stock market fell sharply, while Tesla's share price fluctuated wildly due to negative news such as bitcoin investment, supply chain shortage, and safety accidents. Finally, we calculated that the Sharp ratio was 0.0708 by the minimum variance portfolio calculation under the ff3f model.

5. Conclusion

In this paper, the research aims to help investors make the best investment decisions in diversified fields, taking into account more potential growth stocks and more stable income stocks. At the same time, the portfolio also considers Gold ETF to increase the stability of the portfolio. This paper lists three analysis methods to find the optimal portfolio: mean-variance analysis, capital asset pricing model, and Fama-French three-factor model, and calculates the maximum Sharpe ratio portfolio and the minimum variance portfolio, respectively. It can be known that when calculating the maximum return at a particular risk level, the enterprises with higher weight are generally enterprises with better return performance, while when calculating the minimum variance, the enterprises with higher advantages are generally enterprises with a stable return of Gold ETF, which is also in line with the expectation of high risk and high return. However, it is undeniable that these methods still have shortcomings. The mean-variance model relies too much on historical data and cannot judge the market trend well. The latter two models, based on the market prediction of future earnings, may not consider all the factors affecting earnings, which will cause specific errors.

References

[1] Tsaur, R.C., Chiu, C.L., Huang, Y.Y. Guaranteed Rate of Return for Excess Investment in a Fuzzy Portfolio Analysis. International Journal of Fuzzy Systems, 2020, 23(1): 94–106.

[2] Asawa, Y. S. Modern Machine Learning Solutions for Portfolio Selection. IEEE Engineering Management Review, 2022, 50(1): 94–112.

[3] Li, X., Vermeulen, F. High Risk, Low Return (and vice versa): The Effect of Product Innovation on Firm Performance in a Transition Economy. Academy of Management Journal, 2021, 64(5): 1383–1418.

[4] Trichilli, Y., Abbes, M. B. The impact of COVID-19 on the portfolio optimization. EuroMed Journal of Business, 2022

[5] Lai, T. L., Xing, H., Chen, Z. Mean–variance portfolio optimization when means and covariances are unknown. The Annals of Applied Statistics, 2011, 5(2A): 798–823.

[6] Ismail, A., Pham, H. Robust Markowitz mean-variance portfolio selection under ambiguous covariance matrix. Mathematical Finance, 2018, 29(1): 174–207.

[7] Levy, M. An inter-temporal CAPM based on First order Stochastic Dominance. European Journal of Operational Research, 2022, 298(2): 734–739.

[8] Graham, J. R., Harvey, C. R. The theory and practice of corporate finance: evidence from the field. Journal of Financial Economics, 2001, 60(2-3): 187–243.

[9] Nghiem, L. Risk-return relationship: An empirical study of different statistical methods for estimating the Capital Asset Pricing Models (CAPM) and the Fama- French model for large cap stocks. 2015.

[10] Shaabani, J., Jafari, A. A New Look to Three-Factor Fama-French Regression Model using Sample Innovations, 2020.