# Application of the Markowitz model in Construction of Optimal Portfolio in the US stock market

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**Abstract.** The optimal portfolio has always been a popular topic in the financial field, and there are many different models to choose from. The Markowitz model is one of the commonly used models. This paper aims to find the optimal portfolios by applying the Markowitz model to a group of ten different stocks from three different sectors and comparing the model's performance under different constraints. In this paper, the Markowitz model is adopted for analysis, and the model is compared under the two constraints by calculating the minimum variance frontier and the highest expected return, and finally finds the best investment portfolio. The study concludes that the model behaves better under the SPX index constraint and has a higher expected return. Therefore, the SPX index should be considered when constructing an optimal portfolio of multiple different stocks to obtain a higher expected return.

**Keywords:** Markowitz model; Optimal portfolio; Different constrain.

## 1. Introduction

### 1.1 Background

How to build the optimal investment portfolio to maximize the return has always been a topic of discussion in the financial market. For different stock markets, people often choose different models to help build the optimal portfolio. Among these models, the Markowitz model and the single exponential model are the ones that are often used. Both models have excellent performance in different markets, so they are recognized as the regular model for optimal. However, in real life, investors often do not invest in only one market. To maximize returns, they must invest in stocks in multiple markets at the same time. Currently, we need to consider the investment in multiple different markets and how to construct the optimal portfolio in this situation.

### 1.2 Related research

Plessis et al. applied the model to 11 years of data through JSE and Markowitz model to compare the index calculated from these two models. The research concluded that a trading strategy based on the Markowitz model performed better than the market and that still works with short-selling or less than 10% in nearly all single security. It proved that the Markowitz model provides a strong base for trading rule strategy which benefits the trading area [1]. The research conducted by Xin et al. expanded the Markowitz model by considering the assumption that the investment portfolio does not involve tax, does not consider the transaction costs of the portfolio investment processes, and investors only earn a certain amount of capital and do not hold any securities when making decisions. The research discovered a new version of Markowitz model and got the conclusion which is the efficient frontier of the portfolio in the model is convex [2]. To applicate the model to events other than trading and finance, Singh et al. applicated the Markowitz model to cricket teams by considering a team as a portfolio and calculating the expected return and risk of each team, the teams were being evaluated. The research raised the question of whether the cricket team's performance in the knockout stage should be more important than in the finals [3].

As the Markowitz model is a popular choice to analyze the return and risk portfolios the index model is also very popular. Hristache et al. researched that giving the assumption of (Yi, Xi), i = 1, ... is generated by the index model then investigate the model from the two problems of this model which are the unknown faction f(x) and the index value 0*. The research developed a math model to solve for the index model and concluded that when the model is not significantly violated, the direct estimate system works well, also the iterative procedures need some minimal observation to start working and the value increases as the noise value increases [4]. Bilbao-Terol et al. applicated the index model with fuzzy compromise programming in choosing the optimum portfolio by using fuzzy betas from both statistical data and expert knowledge and taking the data into the index model to select the optimum portfolio. The research compared the model with alternative classical portfolio selection models with Thirty-one Spanish mutual funds and concluded that the model in the research includes data reflecting more information [5]. There is also research that only applies the single index model conducted by Lillo and Mantegna by applicate the single index model to a variety of situations that includes normal and extreme days. The result shows that empirical data shows that market imbalances increase more during rallies than during crashes [6].

Chakraborty and Kumar construct an optimal portfolio on Nifty50 Stocks based on both the Markowitz model and single index mode. The research compared the two optimal portfolios by expected return and risk. The research concluded that constructing a single index optimal portfolio model for Nifty50 is more comfortable while using the Markowitz model will be more stable [7].

The fluctuations in the stock market are also attractive topics to research. Lakshmi and Radha researched Adobe's stock price by using the ARIMA model for the time series. The research concluded that there is a 12.179 % increase in stock prices from September 2021 to February 2022 [8]. Baker investigates whether the profits from the Citi group stocks can be a financing stimulus. The research discussed how the government profits $8 billion from selling the Citi groups stocks and raised some questions about the future development of Citi groups, Inc [9]. Atems and Yimga researched how seriously the US airline stock price is being affected by the covid-19 pandemic. The research compared the trading frequency of the US airlines' stock before and after the pandemic. The result shows that Covid-19 did affect the US airlines' stock price by decreasing the demand for the airlines [10].

### 1.3 Objective

This paper selected 9 stocks, three each from Industrials, Technology, and Financial Services sectors. By calculating the optimal portfolio of these 9 stocks through the Markowitz model and applying two different constraints then analyzing the results to determine the optimal portfolio. Since the data are from when the Covid-19 pandemic happened, the research has also been affected by the pandemic. Therefore, it also shows the performance of the Markowitz model for the pandemic.

## 2. Method

### 2.1 Markowitz model

Markowitz model will be used in this paper as the method. It concerns minimizing the risk and maximizing the return of the investment portfolio.

First, create an investment portfolio including risky and non-risky assets. The purpose of doing this is to lower and even minimize systematic risk. Second, use the historical data of those selected assets to plot a minimal variance frontier of the portfolio.

To plot the minimal variance frontier, a calculation of the daily return rate of each asset included in the portfolio will be needed at the beginning.

$$\mathrm{r} = \mathrm{R_t}/\mathrm{R_{t-1}} - 1 \tag{1}$$

The daily rate of return is represented by "r", the return of the stock on the trading day "t" is represented by "$R_t$", and the return of the stock on the trading day before the trading day "t" is represented by "$R_{t-1}$".

After calculating the daily return rate of assets, the annualized average return rate, which was considered the expected return has to be calculated next.

Then, it comes to the calculation of the portfolio. The portfolio should satisfy this equation:

$$x_0 = \sum_{i=1}^{n} w_i\, x_0 \tag{2}$$

Where is the total number of assets, $x_0$ means the initial Budget for this investment, $w_i$ presents the weight of money spend on stock "i"

These constraints are due to the prohibition of short selling which should be followed.

$$w_i \geq 0 \tag{3}$$

$$the\ \sum_{i}^{n} w_i \leq 1 \tag{4}$$

$$the\ \sum_{i}^{n} w_i \geq 0 \tag{5}$$

The calculation of the total expected return of the portfolio

$$R_t = \sum_{i=1}^{n} w_i\, E(r_i) \tag{6}$$

Where the portfolio's total expected return of is represented by $E(r_t)$, the average annualized rate of return of stock divided by the annualized expected rate of return "i" is represented by $E(r_i)$. The next step is to calculate the variance of the portfolio.

$${\sigma_p}^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{7}$$

Where in $\sigma_{ij}$ is the covariance of asset "i" and asset "j","i" ≥1, "j" ≤n, N: Total amount of risky assets including in the portfolio. Portfolio with the least variance is the global minimum variance portfolio. These calculation results will allow us to find the available minimum variance frontier.

Provided the minimum variance frontier, and the global minimum variance portfolio. Investors can start plotting the efficient frontier. It is the part of the minimum variance frontier above the global minimum variance point on the minimum variance frontier.

Continue, the capital allocation line (CAL) with the highest reward-to-volatility (Sharpe) ratio should be plotted.

$$S=(E(r_i)-r_f)/\sigma_i \tag{8}$$

Where S is the sharpe ratio (reward-to-volatility), $E(r_i)$ is the annualized expected return rate of the portfolio, $r_f$ is the return rate of risk-free assets and $\sigma_i$ is the standard deviation of the portfolio

Finally, the point of the optimal risky portfolio on the efficient frontier that investors need can be found at where the CAL is tangent to the minimum variance frontier.

### 2.2 Data

ADBE: Adobe is an office software provider.

IBM: IBM is a IT company.

SAP: SAP is a German company providing firms with effective managing software SAP.

BAC: Bank of America Corp is a Bank holding company (BHC) and financial holding company.

C: Citigroup is a financial service holding company.

WFC: Wells Fargo & Co is a financial services company.

TRV: TRV is an insurance company.

LUV: Southwest Airlines Co (LUV) operates Southwest Airlines.

ALK: Alaska Air Group Inc. operates two airlines, Alaska and Horizon, and also involve in travel agent.

HA: Hawaiian Holdings Inc is a holding company that conducts scheduled air flight between the Hawaiian Islands and certain cities in the United States, the South Pacific, Australia, New Zealand, and Asia.

## 3. Results and Discussion

As shown in Table 1, after calculating the annualized average return, annualized standard deviation, beta, annualized alpha, and annualized residual standard deviation of all instruments selected in the portfolio, the calculation of the correlation coefficient of those instruments begins. The correlation between assets shows the relativity between their return. When the correlation coefficient is equal to 1. It represents two perfectly correlated stocks. For example, if assets "c" and "d" have a correlation coefficient equal to 1. Then, the return of stock "c" has the same increase as the return of stock "d" When the two assets have a correlation coefficient that equals 0. It means that two stocks are non-related. When two assets have a correlation coefficient equal to -1. Then, two assets are inversely correlated. If the return of asset "c" is 5%, the return of asset "d" will be -5%.

The correlation coefficient between "HA" and "SAP" is the smallest which is 0.14364. Thus, investing in "HA" and "SAP" together may result in relatively lower risk. The correlation coefficient between "C" and "BAC" is the highest which is 0.826222. This correlation is quite high. Therefore, investing "C" and "BAC" at the same time may result in high risk.

### 3.1 No constraint

The next step is finding the minimum variance frontier with no constraint. The minimum variance frontier consists of different portfolios. Those portfolios on the minimum variance frontier have the minimum standard deviation with given expected return. Thus, portfolios that lie on the frontier are the lowest risk portfolios for each given expected return. As shown in Fig. 1, the range of expected return set of this minimum variance frontier is from -0.3 to 0.5 which is -30% to 50%.

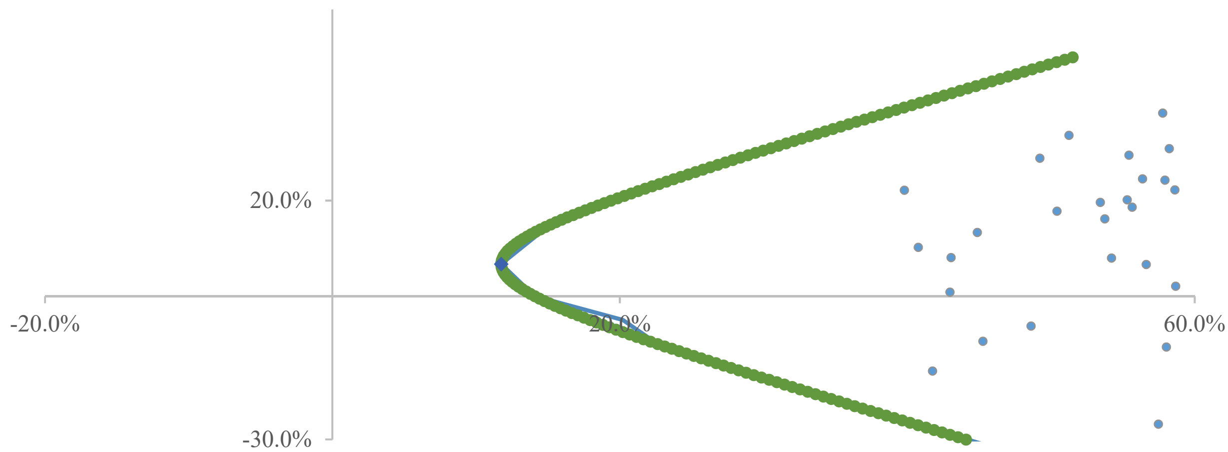


**Fig. 1** Minimum variance frontier with no constraint

The orange curve consisting of many orange dots is the minimum variance frontier.

**Table 2.** Data of three extreme portfolios with no constraint

| | Highest expected return portfolio | Global minimum variance portfolio | Lowest expected return portfolio |
|---|---|---|---|
| Expected return | 51.5% | 6.7% | -30.0% |
| Standard deviation | 50.0% | 11.7% | 44.5% |

As shown in Table 1, the highest expected return portfolio has an expected return of 50%. Plus, it has a standard deviation of 51.51%. The middle of the frontier is called the global minimum variance portfolio. The global minimum variance portfolio has an expected return of 6.7% and a standard deviation of 11.7%. The lowest expected return portfolio has an expected return of -30%. Also, it has a standard deviation of 44.50%.

In order to find the optimal risky portfolio, the plotting of efficient frontier is needed. It is part of the part of the minimum variance frontier that is above the middle of the minimum variance frontier in Fig. 2.

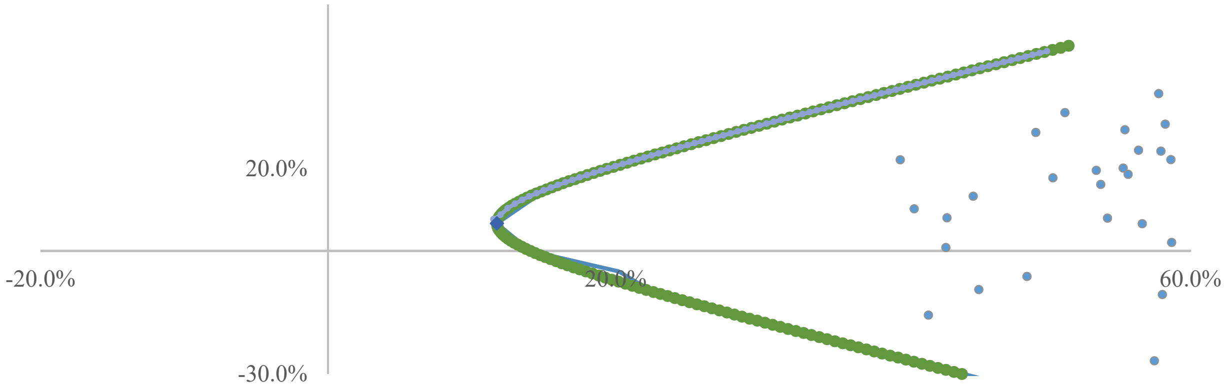


**Fig. 2** Efficient frontier with no constraint

The efficient frontier is the light blue curve on the minimum variance frontier that provides the best risk-return combination. Investors can ignore the minimal return frontier which is part of the minimum variance frontier under the global minimum variance portfolio in Fig. 3.

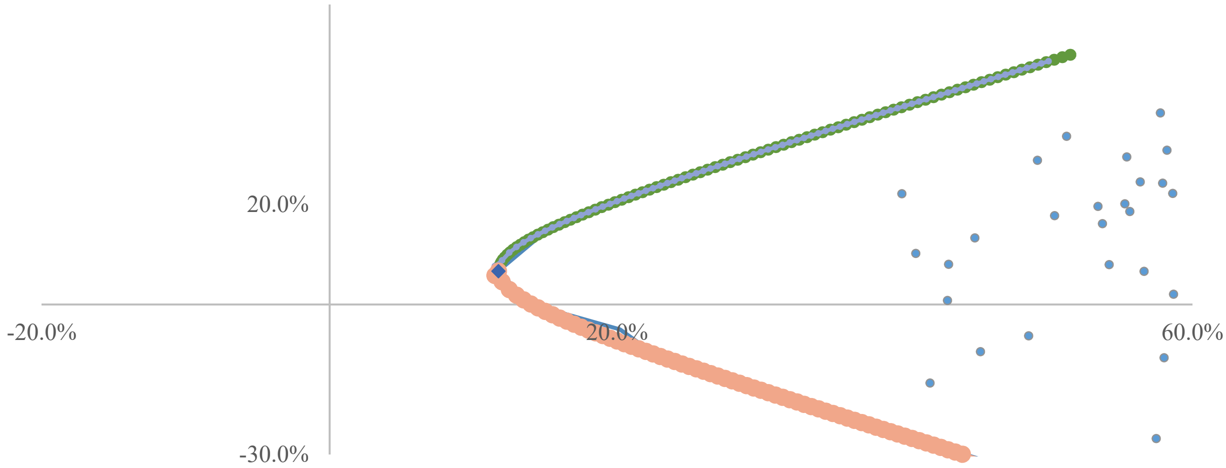


**Fig. 3** Minimal return frontier with no constraint

It lies on the minimum variance frontier and consists of green dots. This frontier has no use for investment because the portfolios lying on the frontier will bring lower returns with similar standard deviations compared with the portfolios lying on the efficient frontier.

In order to find the optimal risky portfolio, the plotting of the capital allocation line will be the last step. In this step, as shown in Fig. 4, the addition of a risky free asset to the portfolio is needed. The risk-free rate, the expected return, and the standard deviation of the portfolio are needed in the function of the CAL.

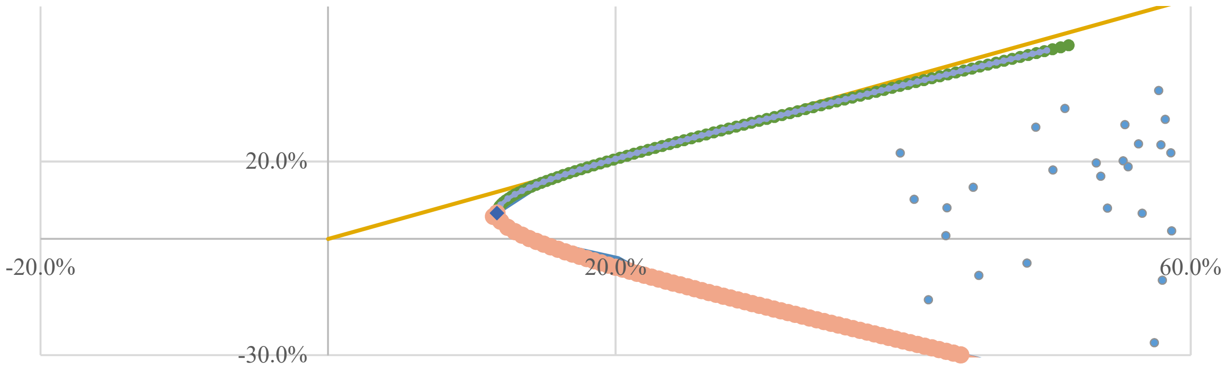


**Fig.4** Capital allocation line (CAL) with no constraint

As shown in Fig. 5, the orange straight line starts from the origin is the capital allocation line.

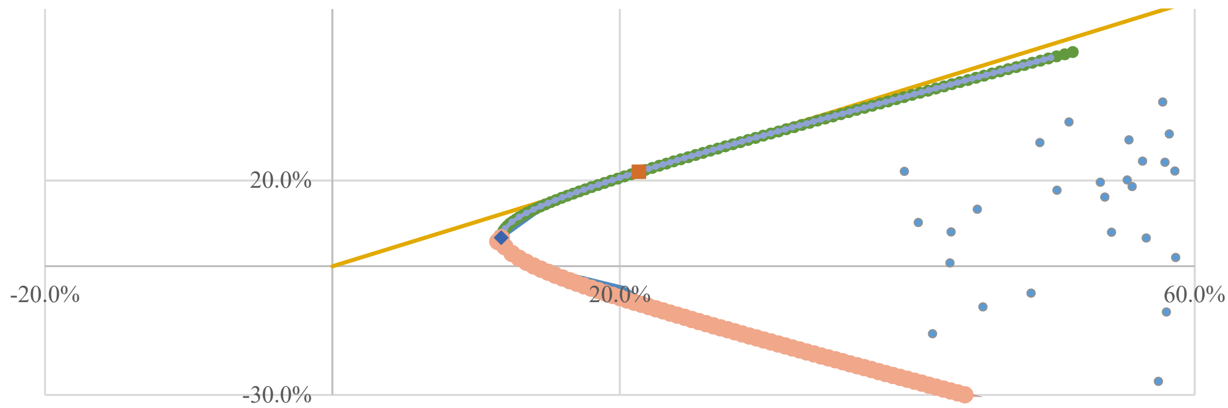


**Fig. 5** Optimal risky portfolio with no constraint

The red point where the capital allocation line and the efficient frontier intersect is the optimal risky portfolio with the highest reward-to-volatility (Sharpe) ratio. The expected return of the optimal risky portfolio is 22.1% and the standard deviation is 21.3%.

The finding of the optimal risky portfolio using the Markowitz model with no constraint ("Free" problem) is now finished. Then, it comes to the "no index" problem. The "no index" problem will be used to illustrate what effect the inclusion of a broad equity index as one of the risky instruments has. In specific, an additional optimization constraint: w1 =0 which means letting the weight of the "SPX" index be zero in the portfolio.

### 3.2 Constraints

The same method used for the construction of the minimum variance frontier with no constraint will be used to construct the minimum variance frontier, as shown in Table 3 and Fig. 6.

**Table 3.** Weight of all instruments in the portfolio with constraint

| | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Weight | 0.0% | -6.5% | -4.2% | 8.7% | 7.1% | -6.1% | 18.7% | 39.1% | 5.8% | 13.9% | 23.5% |

After eliminating the "SPX" index from the portfolio, the weight distribution didn't change a lot. The only significant change is the weight of "HA" changed from 4.6 to 23.5%.

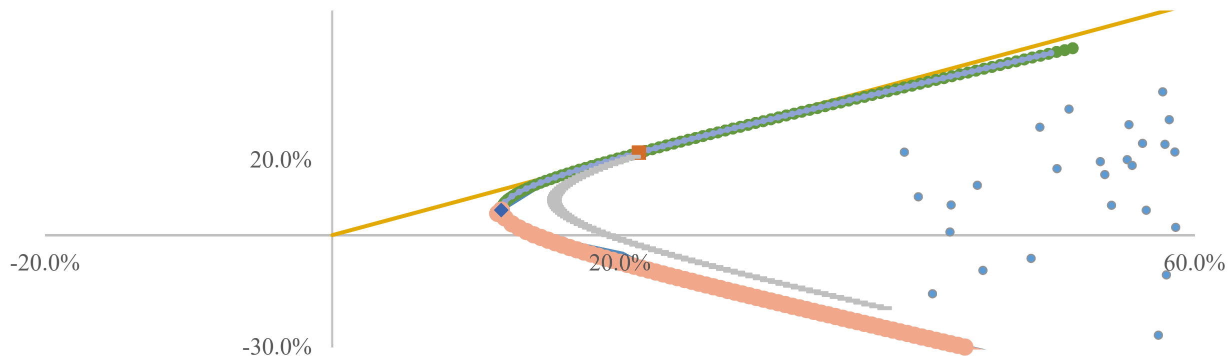


**Fig. 6** Minimum variance frontier with constraint

The gray curve consists of a grey dotted line, the minimum variance frontier of the "no index" problem. It is seen that the expected return is higher compared with the portfolio that includes the "SPX" index. Therefore, the theory of an effective investment portfolio can be proven to be n investment portfolio that can provide the greatest expected return under a certain risk. This result can also prove that the "SPX" index hurts the whole portfolio performance.

An issue related to the effectiveness of the Markowitz model had been found during our implementation. The use of the Markowitz model requires obtaining and calculating a large amount of data. Investors need to obtain the average value of returns over a long period to build the model and the use of a "solver table" in the "Excel" spreadsheet may cost a long time. Also, it requires the accuracy of the input data. In this situation, this time taken may affect the timeliness of the model, which may then fail to respond effectively to the latest changes in security returns. The investment will become riskier.

## 4. Conclusion

This paper uses the Markowitz model to analyze ten stocks in software, financial consultation, and aviation fields. Based on the calculation, the optimal portfolio gives aviation companies higher weights, whereas the weights in software corporations are lower. Under the pressure of the epidemic, people are locked down, and few choose to travel, leading to flight cancellations and reduced incomes for aviation companies. In this case, the higher weights of aviation stocks probably indicate that investors consider there will be more travel demands after the epidemic. Moreover, as the epidemic situation seems to turn around and people who used to work at home get back to the office, it is reasonable to have these lower weights for software companies. Besides, the lower weights may also suggest there is uncertainty in the stock market. This study would like to suggest investors put more attention on aviation stocks, which have the most weight in the obtained optimal portfolio, as well as to be vigilant of uncertain factors caused by the epidemic.

## References

[1] Plessis, A. J. D. , & Ward, M. . (2009). A note on applying the Markowitz portfolio selection model as a passive investment strategy on the jse. Investment Analysts Journal, 38(1), 39-45.

[2] Xin, K. L. , Zhou, X. J. , & Chen, R. . (2002). Markowitz model analysis of fund manager changing portfolio. Journal of Chongqing University(Social Sciences Edition).

[3] Singh, R. , Saikia, H. , & Bhattacharjee, D. . (2015). Application of Markowitz model for analyzing the performance of cricket teams in Indian premier league. Metamorphosis A Journal of Management Research, 14(1), 14-24.

[4] Hristache, M., Juditsky, A., & Spokoiny, V. (2001). Direct estimation of the index coefficient in a single-index model. Annals of Statistics, 595-623.

[5] Bilbao-Terol, A. , B Pérez-Gladish, & Antomil-Lbias, J. . (2006). Selecting the optimum portfolio using fuzzy compromise programming and sharpe's single-index model. Applied Mathematics & Computation, 182(1), 644-664.

[6] Lillo, F. , & Mantegna, R. N. . (2001). Empirical properties of the variety of a financial portfolio and the single-index model. EUROPEAN PHYSICAL JOURNAL B CONDENSED MATTER PHYSICS.

[7] Chakraborty, S. , & Kumar, A. . (2018). Construction of Optimal Portfolio Using Sharpe's Single Index Model and Markowitz Model An Empirical Study on Nifty 50 Stock JOURNAL OF GENERAL MANAGEMENT RESEARCH.

[8] Lakshmi, R. V. , & Radha, S. . (2021). Time series forecasting for the adobe software company's stock prices using arima (box-jenkin') model. JPhCS.

[9] Baker, D. (2010). Profits on Citigroup Stock: Can They Be the Basis for Financing Stimulus? (No. 2010-09). Center for Economic and Policy Research (CEPR).

[10] Atems, B., & Yimga, J. (2021). Quantifying the impact of the COVID-19 pandemic on US airline stock prices. Journal of Air Transport Management, 97, 102141.