Portfolio Investment Optimization based on Markowitz Model and Index Model Under 4 Constraints, a comparative approach

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Abstract. Since early 2020, global economy and financial markets have faced enormous turbulence due to the spread of COVID-19. Against this backdrop, this paper is drafted to help investors better understand portfolio management by finding how various constraints affect asset allocation under two dynamic modeling: Markowitz and Index. Raw data of ten stocks from four popular sectors and the SPX500 are collected as the source of this paper. By processing raw data and using the processed data, this paper reaches the conclusion that the stock market stays resilient to external shocks. As mentioned, there are two models that would be the focus of this paper: Markowitz Model and Index Model. Both of which would be analyzed under one non-constrain and four simulated constraints to examine portfolio performance. Comparing the results, Index Model generates higher numerical results with respect to associated risk than these produced based on the Markowitz Model. On the other hand, the portfolios constructed based on the results of the Index Model performs better under most of the constraints than the ones based on the Markowitz Model in terms of Sharpe ratio. Based on the concluded results, this paper meets by demonstrating stock investors a potential approach to mitigate risks during unexpected shocks with broad impacts.

Keywords: Optimal Markowitz portfolio; Markowitz Model; Index Model; Investment Constraints; Modern Portfolio Theory; Optimization.

1. Introduction

As of today, investors have many choices on venue of investment, whether it is to establish a portfolio from ground-zero or invest in mutual funds, index funds, even hedge funds. Behind this phenomenon is the soaring popularity of security-trading with hopes to 'beat' the market. Alongside the popularity of security trading comes with tightening of regulations as well as developing of systematic approach to make trading decisions. While investments have varying constraints due to regulatory requirements and investor's degree of risk-tolerance, the end of investment for any rational investor stays the same: minimizing risk while maximizing return or vice versa. For institutional investors, given that risk and return are in negative association, it is their responsibility to balance the two. On the other hand, individual investors with different degrees of risk preference are also encouraged to understand the concepts of diversification, portfolio optimization, and risk mitigation.

The construction of a portfolio as an intricate task of selecting securities and allocating relative weights (in terms of assets) to securities makes the world of finance a place that attracts many talents. Among them, in 1952, Harry Markowitz published a groundbreaking article that set the foundation of mean variance portfolio theory [1]. Then, Markowitz consolidated his portfolio optimization strategy with further details in a series of follow-up publications [2]. Based on Markowitz's work, William Sharpe invented the Capital Asset Pricing Model (CAPM), a financial model that has often been used to calculate equity value for investment valuation today [3]. The two models set the foundation of modern portfolio theory and open opportunities for a series of further development which eventually defines modern portfolio theory. For example, Frantz and Payne took modern portfolio theory to the next stage by attesting the concept that the associated risks (standard deviation) of investment decrease as the variety of equities increases. In other words, associated risks in average of a diversified portfolio are smaller than that of any individual stock [4]. Given this insight, investors should mitigate their exposed risks by comprising their investment portfolio with numerous stocks instead of individual security. In addition, the research also sheds light onto how "a truly diversified portfolio can improve returns and significantly reduce non-systematic risk," in which the market risk is determined based on the global macroeconomic factors, while company specific risk that affect individual equity is caused by micro determinants that "specifically affect a single asset or narrow group of assets" [5].

Developed on the former approaches which analyze individual securities, this paper emphasizes the statistical correlation of expected returns between securities based on the models. The Markowitz Model's portfolio highlights the correlation between different equities and their assigned weights [6][7]. Hight points out that security portfolios with securities of smaller correlation coefficient between each is exposed to lesser risk [8]. Thereby, the associated standard deviation would be reduced by arithmetically increasing the proportion of uncorrelated securities in the portfolio. A systematic investment strategy evolved based on Hight's research is Index tracking: an investment strategy that tries to gain returns by establishing a portfolio that approximates a well-diversified market-proxy index. Index tracking is in line with CAPM as a passive strategy with the theoretical justification provided by the academia. Historically, mutual funds based on passive investment strategy have grown tremendously since the 1980s[9]. Through diversification, the non-systematic risk of an equity and the loss associated can be reimbursed by profits made from other stocks [10]. Donalee Brown explores the psychological aspect of investment by focusing on individual's psychological attachment and such concept's application onto real life sell-side client relationship maintenance [11]. And lastly, there is voice of the critique such as Yuwono dan Ramdhani, who claims the rate of return obtained from securities using the Markowitz model is no higher than the risk-free bonds [12].

Against the historical backdrop of investment strategies, this paper would also highlight the lingering impacts of the COVID-19 pandemic, including but not limited to hampered economic growth, skyrocketed inflation, and widened wealth gap. The pandemic has presented unprecedented challenges for institutional investors to keep up with the market benchmark return. Hence, it becomes increasingly vital for investors to be aware of the impacts from the external shocks on the portfolio. This paper focuses on Markowitz Model and Index Model to analyze the step-by-step construction of optimal investment portfolios based on different risk tolerance levels. The data set includes a recent 20 years of daily return data (in percentage) from ten chosen stocks, belonging to four equity market sectors categorized by the Yahoo finance, one market index (SPX), and a proxy index for the risk-free asset choice (FEDL01). The collected raw daily returns of each stock would be processed into the monthly data in order to reduce the non-Gaussian effects. This paper focuses on the differences in performance between portfolios constructed based on the two theoretical models. Based on the observed differences, this paper proceeds by analyzing the differences with insights to hypothesize the best weight allocation to each of the ten stocks. To optimize the allocation, investors frequently appeal to Markowitz model and Index model. Therefore, this paper uses these MM and IM optimization inputs to identify the region of permissible asset allocation combinations under the benchmark (non-constraint) and four additional constraints to assist investors navigating through uncertainties.

2. Data

2.1 Data Introduction

This paper investigates a hypothetical portfolio that covers consumer cyclical, technology, financial services, and industrial sector as listed in table 1 based on the corresponding data collected from the past 20 years. In addition, as presented by table 2(1), S&P 500 Index (SPX), being the first one listed on table 1, is an equity index that records the weighted average performance of 500 major listed companies' stock in the United States. Due to the index's broad inclusion of stocks across a multitude of sectors, its non-systematic risk is dispersed. Therefore, SPX is capable to serve as a numerical proxy to the market. Besides SPX, there are also ten chosen stocks as mentioned above, which is listed as the following: Amazon (AMZN), Apple (AAPL), Citrix system (CTXS), J.P Morgan Chase & Co (JPM), Berkshire Hathaway Inc (BRK/A), The Progressive Corporation (PGR), United Parcel Service, Inc (UPS), FedEx Corp (FDX), J.B. Hunt Transport Services, Inc. (JBHT), and Landstar System, Inc (LSTR).

Table 1. Selected Stocks

Ticker	Company Name	Sector
SPX	-	-
AMZN	Amazon.com, Inc.	Consumer Cyclical
AAPL	Apple Inc.	Technology
CTXS	Citrix Systems, Inc.	Technology
JPM	JPMorgan Chase & Co.	Financial Services
BRK/A	Berkshire Hathaway Inc.	Financial Services
PGR	The Progressive Corporation	Financial Services
UPS	United Parcel Service, Inc.	Industrial(s)
FDX	Fedex Corporation	Industrial(s)
JBHT	J.B. Hunt Transport Services, Inc.	Industrial(s)
LSTR	Landstar System, Inc.	Industrial(s)
FEDL01	-	-

Table 2(1) to table 2(5) are the charts demonstrating price trend of SPX 500 and the ten chosen stocks. All stock fell sharply through the outbreak of the COVID-19 since late 2019 to early 2020, with some sectors exhibiting greater fluctuations than others. But within a year, most of the stock prices rebounded to the their pre-pandemic level, reflecting how the security market has strong resilience and responsiveness under unusual external shocks. The following parts of this paper are constructed to analyze shock prices by grouping them into sectors with exception of the SPX 500, which, as mentioned above, indexes the overall stock market.

Table 2(1): portfolio stock's price over time

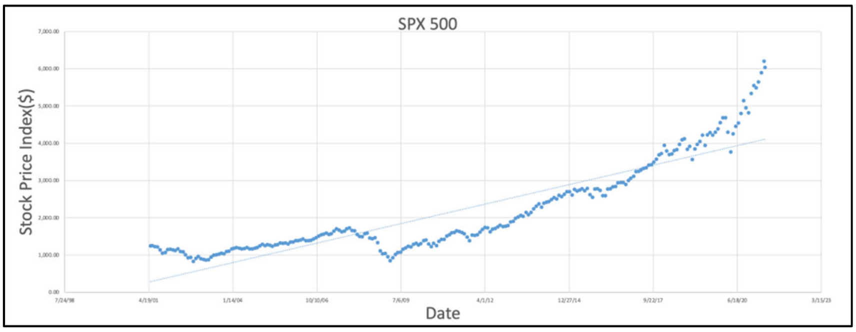
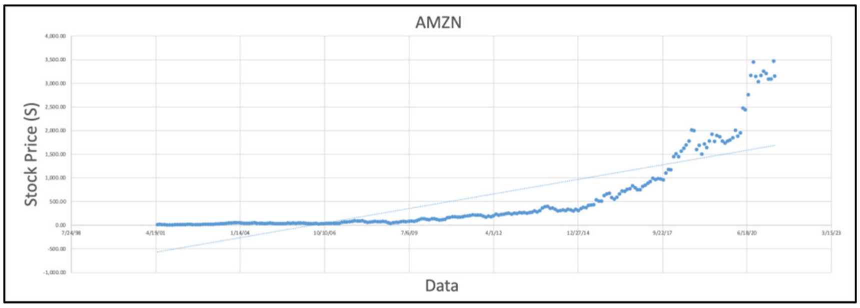


Table 2(2): portfolio stock's price over time (consumer cyclical)



Among the 10 chosen stocks, AMZN represents consumer cyclical industry. Being the only firm from the consumer cyclical sector, Amazon is an American technology company founded in 1994 that specialized in e-commerce, cloud computing, and artificial intelligence. Consumer cyclical industry has always been a popular sector in the equity market to investors especially under the COVID-19 hit, when consumers became increasingly dependent on making online purchases.

Table 2(3): portfolio stocks' price over time (technology)

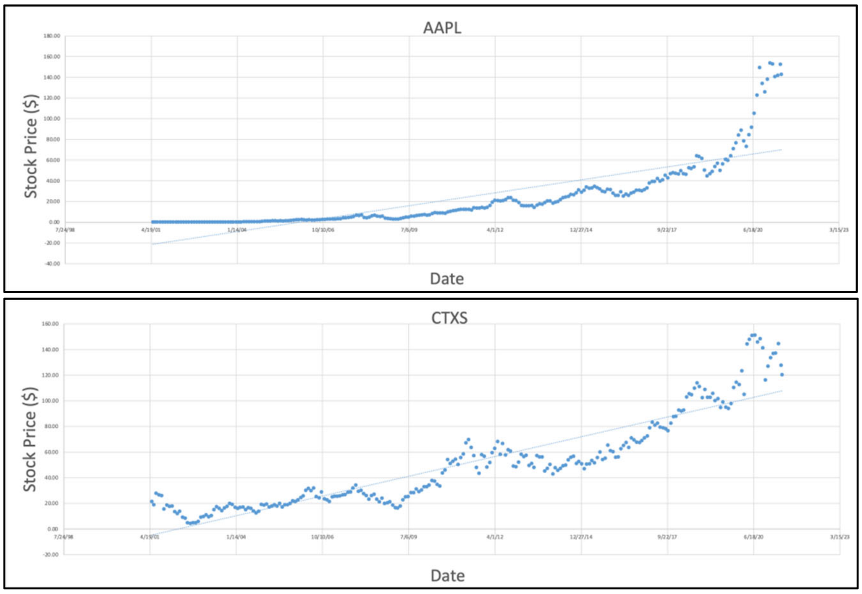
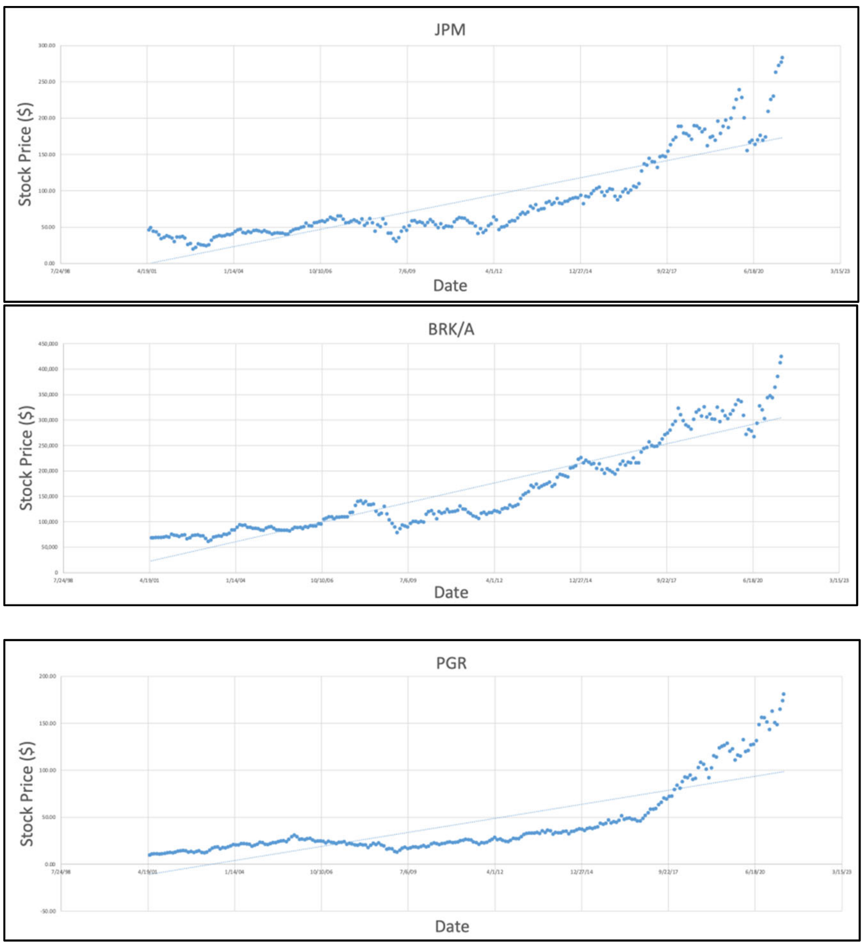


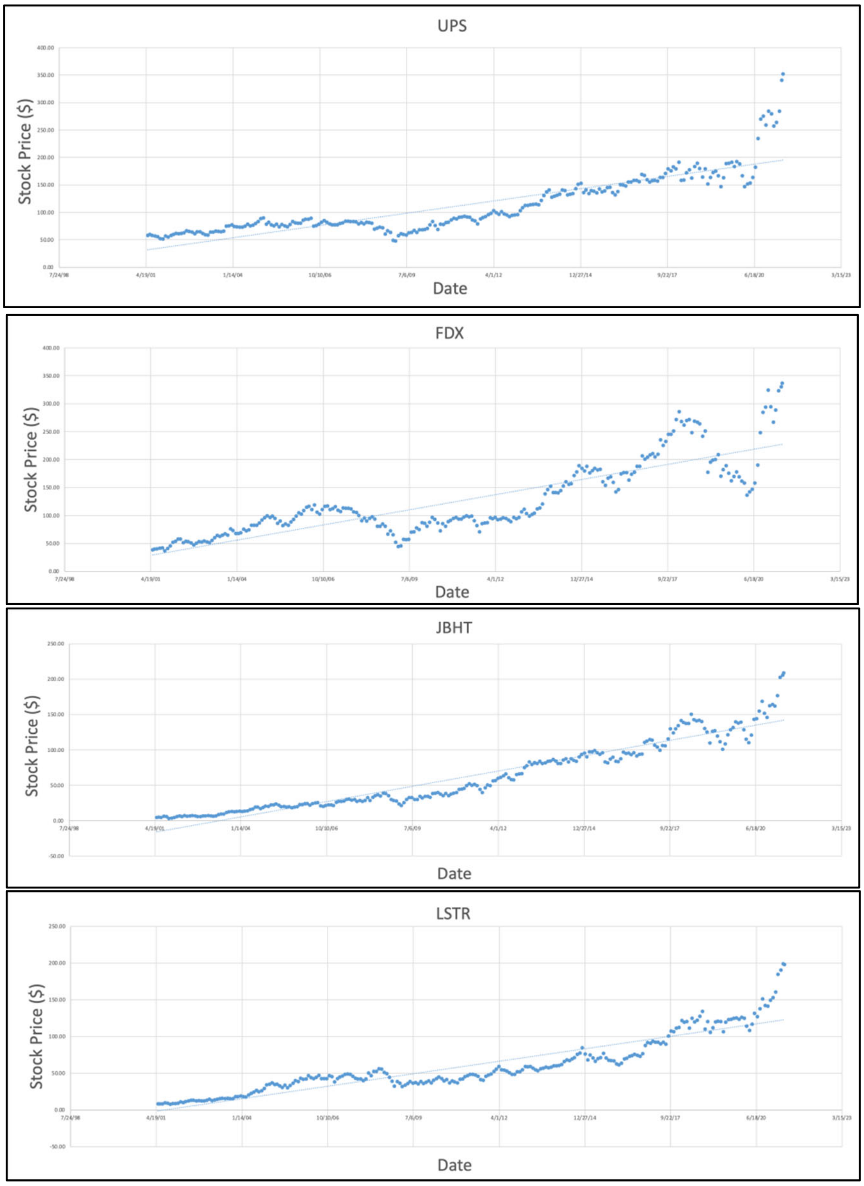
Table 2(4): portfolio stocks' price over time (financial)



Two stocks are chosen from the technology sector, both of which have potential for growth and solid fundamentals. Because of technology companies' potential, investments to TMT companies are made with expectations to boost the returns of the entire portfolio. In the past 20 years, the development of relevant industries such as data analysis, mechanical software, and chip technology render the associated companies' stock price to soar. Apple. Inc, for example, is the company with the one of the best growth performances in the past 20 years. At the same time, other technology firms such as Citrix system has also shown to be a very good investment option based on its stock price performances. While the two technology companies have shown a general upward trend with their stock prices, it is also clear that the pandemic hit both firms hard. As indicated by sharp plummet of stock price, the external shock's effects on the global supply chain to the technology firms are considerable.

JPM, BRK/A, and PGR are the chosen representatives of the financial sector. The finance sector is also susceptible to external shocks. The chosen stocks clearly demonstrate how COVID-19, as an example of external shock, affects individual's bearish expectation towards the finance institutes. The plummeting of financial stock prices is likely to be related to the decreasing need of FA (financial advisory) services during the pandemic.

Table 2(5): portfolio stocks' price over time (industrial)



Lastly, UPS, FDX, JBHT, and LSTR are the representations of the industrials, particularly the transportation industry. Due to the pandemic, online activities like e-shopping, as described in section introducing Amazon, have greatly boosted the demand on ground-transportation that pushed up the revenue generating capacity of transportation companies. The four chosen stocks are well related to the consumer cyclical sector (AMZN), which would explain the expected influences of COVID-19 as the two are complementary industries.

Table 3: Correlation between the selected stocks and SPX

	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
SPX	1.00	0.48	0.54	0.44	0.70	0.52	0.50	0.57	0.61	0.52	0.50
AMZN	0.48	1.00	0.38	0.22	0.25	0.12	0.20	0.30	0.28	0.31	0.26
AAPL	0.54	0.38	1.00	0.33	0.24	0.17	0.24	0.23	0.33	0.27	0.29
CTXS	0.44	0.22	0.33	1.00	0.32	0.18	0.27	0.26	0.33	0.29	0.25
JPM	0.70	0.25	0.24	0.32	1.00	0.45	0.39	0.36	0.44	0.44	0.37
BRK/A	0.52	0.12	0.17	0.18	0.45	1.00	0.26	0.40	0.38	0.24	0.23
PGR	0.50	0.20	0.24	0.27	0.39	0.26	1.00	0.39	0.36	0.28	0.29
UPS	0.57	0.30	0.23	0.26	0.36	0.40	0.39	1.00	0.67	0.46	0.44
FDX	0.61	0.28	0.33	0.33	0.44	0.38	0.36	0.67	1.00	0.54	0.48
JBHT	0.52	0.31	0.27	0.29	0.44	0.24	0.28	0.46	0.54	1.00	0.59
LSTR	0.50	0.26	0.29	0.25	0.37	0.23	0.29	0.44	0.48	0.59	1.00

Table 4: Excerpts from Basic Data

	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
Annualized Average Return	7.54%	33.80%	34.02%	15.64%	11.86%	9.00%	15.41%	9.85%	12.95%	22.52%	17.39%
Annualized St.Dev	14.85%	41.41%	34.45%	41.51%	29.01%	16.24%	21.06%	21.44%	26.69%	30.65%	23.92%
beta	1.00	1.35	1.26	1.22	1.36	0.57	0.71	0.83	1.10	1.08	0.80
alpha	0.00%	23.60%	24.54%	6.43%	1.60%	4.69%	10.04%	3.59%	4.63%	14.41%	11.37%
Annualized Residual St.Dev	0.00%	36.22%	28.96%	37.35%	20.82%	13.84%	18.21%	17.54%	21.07%	26.16%	20.79%

Table 3 demonstrates that the stocks' correlations in combination are generally low. Small correlation coefficients mean that the numerical strength of beta and association between individual stocks are not high, which suggests the portfolio is well diversified in terms of security inter-dependence. Diversified portfolio has the advantage that the individual stock's performance does not influence the overall preciseness of the portfolio's results. Table 4 presents the needed calculated metrics of the equities: annualized average return, annualized standard deviation, beta, alpha, and annualized residual standard deviation. The calculated metrics would serve as the basis for the subsequent mathematical analysis.

Mathematical formulas

The raw data is recorded as daily stock prices for the past twenty years. To reduce the amount of calculation needed and the non-Gaussian effects, the filter algorithm in Excel is used to systematically sample one stock price data per month. For simplicity, the last day of each month is chosen, thereby, converting the daily stock price data into monthly observations. The monthly returns of each stock R1 is calculated by using P1, the stock price at present, to divide P0, the stock price of corresponding stock from the previous month, then minus one. The equation is given below:

$$R_i = \frac{P_1}{P_0} - 1 \tag{1}$$

The expected returns from the hypothetical portfolio constructed is given as:

$$\text{Return}\ (R_i) = \sum_{i=0}^{n} W_i R_i \tag{2}$$

Return (Ri) = wSPX*pSPX + wAAPL*pAAPL + wCTXS*pCTXS + wJPM*pJPM
+ wBRKA*pBRKA + wPGR*pPGR + wUPS*pUPS + wFDX*pFDX + wJBHT*pJBHT
+ wLSTR*pLSTR (3)

Formula 3 is the expansion form of return formula where w stands for the weights allocated to each of the stock, and p stands for the market price of the corresponding stock. As noted, the sum of WiRi would generate the expected return of this hypothetical portfolio.

2.1.2 Models

This paper will be using MM and IM to optimize portfolio. The notable strength of using the Index Model is its consideration of the stocks' price that fluctuates with respect to the direction of the market index with consideration of the risk-free assets (notes and bonds). On the other hand, the advantage of the Markowitz Model is its degree of flexibility that allows investors to specify the allocation weights while incorporating one's level of risk toleration: either aiming for lowest associated risk or choosing maximum returns with risks that they are able to bear. Markowitz Model is designed to maximize a given portfolio's overall return through a three-parts method. First, the model determines the minimal variance frontier, namely the opportunity set. Second, the model takes the opportunity set and identify the optimal combination of risky assets, which is when the magnitude of the gradient of capital allocation line (CAL) is in tangent condition to minimal variance frontier. Third, the model chooses the desirable combination with incorporation of the bonds and notes. The formulas associated with the Markowitz Models are:

$$\min\sigma^2(\mathrm{r_p}) = \sum\sum \mathrm{x_i x_j Cov(r_i - r_j)} \tag{4}$$

$$r_p = \sum x_i r_i \tag{5}$$

On the other hand, the second of this paper's interest: the Index Model optimizes portfolio with consideration of both the risk and the returns. Index Model has the advantage through simplification of the covariance coefficient, thereby narrowing the margin of error when estimating the premium of security risk. However, the Index Model is a theoretical model built on the assumption that residuals of returns are unrelated in algorithm; or in other words, stocks are correlated only because of the general macro-economy. The formulas associated with the full Index Models are:

$$\gamma_{\mathrm{it}} - \gamma_{\mathrm{f}} = \alpha_{\mathrm{i}} + \beta(\gamma_{\mathrm{mt}} - \gamma_{\mathrm{f}}) + \epsilon_{\mathrm{i}} \tag{6}$$

$$\gamma_{\mathrm{it}} = \mathrm{E}(\gamma_{\mathrm{i}}) + \mathrm{m_i} + \mathrm{e_i} \tag{7}$$

$\mathrm{r_{it}}$ is return to stock i in period t
$\mathrm{r_f}$ is the risk − free rate (e.g. FEDL01)
$\mathrm{r_{mt}}$ is the return to the market portfolio in period t
α_{i} is the stock′s alpha or abnormal return
ϵ_{it} is the residual (random) return

2.2 Model Constraints

Benchmark Constraint:

Optimization benchmark is the condition without any additional optimization constraints. Benchmark is needed to illustrate the area of permissible portfolios and the efficient frontier with zero constraint.

Constraint 1:

Optimization constraint 1 is designed to simulate some arbitrary "box" constraints on weights, which may be provided by the client:

$$|w_i| \le 1, \forall \mathrm{i} \tag{8}$$

Constraint 2:

Optimization constraint 2 is designed to simulate the Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity:

$$\sum_{i=1}^{11} |w_i| \le 2 \tag{9}$$

Constraint 3:

This additional optimization constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions, for details see the Investment Company Act of 1940, Section 12(a)(3):

$$w_i \geq 0, \forall i \tag{10}$$

Constraint 4:

Lastly, we would like to see if the inclusion of the broad index into our portfolio has positive or negative effect, for that we would like to consider an additional optimization constraint:

$$w_i = 0 \tag{11}$$

Using the processed data, this paper uses Excel spreadsheet to calculate the permissible region of portfolio combinations for each of the given constraints (Benchmark + 4 additional). In addition, the opportunity set and the efficient frontier together would form the variance frontier as the other vital component of modeling. This research uses excess returns as the starting point of analysis. The results are listed in the tables as following:

3. Results

Table 5: Portfolio weights (no constraint)

Ticker	Minimum Variance Portfolio		Maximum Sharpe Portfolio	
	Markowitz Model	Index Model	Markowitz Model	Index Model
SPX	0.722	0.658	-0.411	-0.473
AMZN	-0.023	-0.041	0.178	0.178
AAPL	-0.038	-0.047	0.384	0.305
CTXS	-0.010	-0.024	0.000	0.005
JPM	-0.185	-0.129	-0.046	-0.027
BRK/A	0.362	0.345	0.231	0.224
PGR	0.139	0.134	0.357	0.315
UPS	0.034	0.086	0.076	0.012
FDX	-0.103	-0.036	-0.042	0.001
JBHT	-0.005	-0.017	0.184	0.192
LSTR	0.108	0.072	0.090	0.267

The first set of results is obtained with none of the extraneous limitations. It is the benchmark results with the purpose to be referred later. Table 5 demonstrates the estimated weights assigned under the given target as minimum variance (risk) and maximum Sharpe (return) respectfully. Between the two, the allocation based on maximum Sharpe is unobtainable in real life because the hypothetical allocation imposes no limit on the stocks' position: the investor may increase the weight endlessly without constraints on the long positions while cut the allocation on the short positions as long as the sum of the total weights equals to 1. The estimated results of the Markowitz Model create a portfolio that generates the minimum variance as low as 12.24%. On the other hand, the maximum Sharpe portfolio reveals a better approximation of real investors as it is more balanced between risk and return. The calculated Sharpe ratio is 1.5385, meaning that per every one percent more risk the investor bears, 1.5385% of return would be compensated. Maximum Sharpe portfolio bears a much higher risk in terms of standard deviation, specifically, 2.5 times higher than that of the minimum variance portfolio. Yet, with higher risk comes higher gain, the max Sharpe portfolio forecasts an excess return of 49.61%, 7 times higher that of minimum variance's 7.15%.

To scrutinize the results for more details, one can notice that the allocation reveals the rationale behind the model. The S&P index stock, the most diversified stock among the chosen ones, has the smallest standard deviation of 14.85%. Therefore, in order to lower the overall risk of the entire portfolio as much as possible, the allocation assigns the heaviest weight on SPX, 73.24% of the portfolio, till the point where standard deviation of SPX matches the other stocks' standard deviation. However, once the investor renounces the objective of minimizing risk for maximizing returns, SPX is shorted heavily to purchase other securities with higher potential expected returns.

After imposing constraint 1, the two sets of result from two models vary slightly with similar emphasis on BRK/A, PGR, UPS, and LSTR. To peek into the details, BRK/A is granted with the largest weight in the portfolio that reaches 0.362, whereas the stock JPM is assigned with the lesser weight in portfolio which is shorted by 0.185. The resulted allocation suggests that BRK/A is the stock that is least affected by risks created from external shocks to the entire portfolio, while JPM is the stock that is most likely to drive up the standard deviation of the entire portfolio under the first constraint. With the additional regulation T requested by FINRA, the calculations based on the two models almost unanimously conclude that the aforementioned four stocks are the more desirable investment allocation choices that should be allocated with more assets.

Table 4. Portfolio weights (constraint 1)

	Minimum Variance Portfolio		Maximum Sharpe Portfolio	
Ticker	Markowitz Model	Index Model	Markowitz Model	Index Model
SPX	0.722	0.658	-1.000	-1.000
AMZN	-0.023	-0.041	0.223	0.218
AAPL	-0.038	-0.047	0.398	0.366
CTXS	-0.010	-0.024	-0.012	0.031
JPM	-0.185	-0.129	-0.005	-0.088
BRK/A	0.362	0.345	0.625	0.333
PGR	0.139	0.134	0.460	0.400
UPS	0.034	0.086	-0.031	0.095
FDX	-0.103	-0.036	-0.106	0.056
JBHT	-0.005	-0.017	0.209	0.250
LSTR	0.108	0.072	0.239	0.338

Table 5. Portfolio weights (constraint 2)

	Minimum Variance Portfolio		Maximum Sharpe Portfolio	
Ticker	Markowitz Model	Index Model	Markowitz Model	Index Model
SPX	0.722	0.658	-2.375	-3.298
AMZN	-0.023	-0.041	0.371	0.430
AAPL	-0.038	-0.047	0.654	0.700
CTXS	-0.010	-0.024	0.004	0.110
JPM	-0.185	-0.129	0.173	0.088
BRK/A	0.362	0.345	0.916	0.585
PGR	0.139	0.134	0.682	0.724
UPS	0.034	0.086	0.013	0.279
FDX	-0.103	-0.036	-0.087	0.249
JBHT	-0.005	-0.017	0.310	0.503
LSTR	0.108	0.072	0.340	0.629

After imposing constraint 2, the two sets of result from two models vary slightly with similar emphasis on BRK/A, PGR, UPS, and LSTR. To peek into the details, BRK/A is granted with the largest weight in the portfolio that reaches 0.362, whereas the stock JPM is assigned with the lesser weight in portfolio which is shorted by 0.185. The resulted allocation suggests that BRK/A is the stock that is least affected by risks created from external shocks to the entire portfolio, while JPM is the stock that is most likely to drive up the standard deviation of the entire portfolio under the second constraint. With the additional constraint of arbitrary 'box' constraints requested by clients and remove the consideration of returns, the calculations based on the two models almost unanimously conclude that BRK/A, PGR, UPS, and LSTR are the more desirable investment choices that should be allocated with more assets.

Outputs illustrated on table 6 are the resulted calculated from the models with the constraint of the US open-ended mutual fund regulation that bans investment of extreme short and long positions.

Under the third constraint, the results from the two models rendered are similar in terms of weight allocations to each of the equity subjects. SPX and BRK/A are allocated with the most weights while the position on the stocks of the technology sector are cleared for the minimum risk portfolio. For the maximum efficiency portfolio, both models indicate to trade weights allocated to SPX for AMZN and AAPL to promote expected returns. The result calculated based on the MM indicates a 1.25 Sharpe ratio, whereas the Sharpe ratio based on the IM suggests 1.27. The other set of result regarding the maximum return portfolio, chooses to allocate assets on the technology sector based stock AAPL to enlarge annualized return in average to generate 34.016% expected portfolio return. Under the constraint 3, the expected returns generated from a portfolio based on IM is considerably higher than that of the Markowitz Model: specifically, Index Models generates a 0.1% lower minimum variance in comparison to that of the Markowitz, nevertheless the Sharpe ratio is also 0.023 higher.

Table 6. Portfolio weights (constraint 3)

	Minimum Variance Portfolio		Maximum Sharpe Portfolio	
Ticker	Markowitz Model	Index Model	Markowitz Model	Index Model
SPX	0.385	0.305	0.000	0.000
AMZN	0.000	0.000	0.130	0.155
AAPL	0.000	0.000	0.252	0.271
CTXS	0.000	0.000	0.000	0.000
JPM	0.000	0.000	0.000	0.000
BRK/A	0.385	0.377	0.193	0.033
PGR	0.142	0.146	0.227	0.214
UPS	0.007	0.093	0.000	0.000
FDX	0.000	0.000	0.000	0.000
JBHT	0.000	0.000	0.088	0.140
LSTR	0.081	0.079	0.110	0.188

Table 7. Portfolio weights (constraint 4)

	Minimum Variance Portfolio		Maximum Sharpe Portfolio	
Ticker	Markowitz Model	Index Model	Markowitz Model	Index Model
SPX	0.000	0.000	0.000	0.000
AMZN	0.025	-0.016	0.147	0.186
AAPL	0.042	-0.008	0.267	0.317
CTXS	0.008	-0.001	-0.034	-0.005
JPM	-0.071	-0.050	-0.155	-0.212
BRK/A	0.563	0.480	0.363	0.117
PGR	0.235	0.216	0.320	0.277
UPS	0.114	0.178	-0.122	-0.053
FDX	-0.081	0.035	-0.132	-0.054
JBHT	0.006	0.028	0.178	0.186
LSTR	0.158	0.138	0.168	0.242

Under constraint 4, Markowitz and Index Model generate similar results on weight allocations. Table 7 illustrates the weight predictions when that SPX is not available as an equity investment choice. Besides that, this particular constraint is equal to that under the constraint 3. The result indicates that without having SPX to diversify the entire portfolio, the variance would be higher under the two models: Markowitz generates a associated standard deviation of 13.39% while Index gives out 13.21%. Without SPX, both models allocate the largest portion on the financial sector equity BRK/A, whose standard deviation is the least among the individuals stocks and second largest weight in placed on PGR with the second lowest standard deviation at 21.059%. Without the choice of allocating assets to the index equity, the notable strength of the Markowitz Model to predict risk is less significant in comparison to the benchmark condition as Index Model now allows a portfolio with 0.18% less standard deviation than the Markowitz Model.

4. Conclusion

This paper researches how different constraints affect the strategy of allocating optimal portfolios from 10 chosen stocks, SPX index, and 1-month federal funds rate. Based on the Markowitz Model and Index Model, this paper starts by processing 20-years time span monthly return data. Then, this paper starts to establish each components of the two optimization models: efficient frontiers under 4 constraints by using the excel solver. This paper gathers the results and compared both the calculated portfolios' returns and risks to examine the two financial models under various constraints. Based on the results presented above, a holistic analysis suggests that because both the Markowitz and the Index Model present almost identical resulted allocations, The final result is trustworthy that investors should choose AMZN, AAPL, XTCS, PGR, JBHT, and LSTR as the core of an optimal portfolio positive returns. Under this allocation, both forecasting models indicate strong Sharpe ratio and bearable standard deviations concerning this particular portfolio with or without external shocks. Although the results are very similar, Markowitz model is slightly better when considering the second purpose of minimizing associated risk. In terms of the Index Model, systematic risks in the stock market cannot be alleviated under financial model or diversified with allocations. Unlike the non-systematic risks that exist due to firm specific reasons, systematic risks are associated with the macro-economy. With the two types of risk completely different and one unable to be solved by diversification. Institutional investors should be privy of the risk types behind their stock choices, especially the ones that are heavily weighted. Their knowledge of the associated risk types would help investors to enact appropriate measures in this time frame of uncertainty.

The potential aspects that need further nuance of this paper are related to the innate flaws and theoretical-reality gap behind the two financial models. For example, construction of a Markowitz Model involves a significant number of estimates, which is susceptible to estimation errors that affect the overall results. On the other hand, The Index Model has the assumption that the residuals of return are relatively insignificant. In addition, other assumptions such as the stock market is not correlated with the macro-economy or that not weighing firms of the identical/related sector interact and lead to ripple effects into consideration of the overall results. In the future, it would be worthy of researching suitability of models under different circumstances. While this paper can hardly be compared with the established studies, the conclusion on Index Model and Markowitz Model drawn by this paper aligns in accordance to the former researches that write about the differences between the two. To validate the theory that an efficient portfolio is a portfolio that can provide the greatest expected return with certain risks and provides the greatest risk with certain expected returns, Markowitz Model has clearly achieved it for potential investors to make an earning[13].

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