A Newsboy Model for Cross-Selling Products with Demand Impacted by Price

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Abstract. Cross-selling refers to the fact that customers may purchase another product related to them while purchasing one product, or the loss of sales opportunities for another product due to the shortage of one product. Considering the impact of price and cross-selling products on demand, the newsboy problem in the centralized decision-making environment is studied. In the centralized decision-making environment, consider a single retailer, pay attention to the sum of expected returns of all products, and propose the first-order necessary condition for the existence of the maximum value of the return function, which provides the basis for retailers to make decision-making strategies.

Keywords: Cross selling; Price; Order goods; Newsboy Model.

1. Introduction

Cross-selling refers to the fact that customers may purchase another product related to them while purchasing one product, or the loss of sales opportunities for another product due to the shortage of one product. This study takes the cross-selling products whose demand is affected by price as the research object. The product demand is affected by price and other cross-selling products. The product demand is expressed as a function of both. Considering the centralized decision-making environment, the pricing and ordering strategies of such products are studied, so as to provide corresponding decision support for retailers. This study has important theoretical and practical significance.

2. Problem description and model assumptions

2.1 Problem description

Consider that there is only one retailer in the market for sale N products. Each product is perishable, that is, if the product fails to be sold in a single cycle, the residual value is zero. Each product is price sensitive, that is, the demand of the product is affected by the price. There may be a cross-selling relationship between each product, and the sale of one product will promote the sale of related products; Similarly, the shortage of a product will lead to the loss of sales opportunities of related products. Therefore, the demand for products is affected by their own prices and the sales of related products. Based on the above situation, retailers determine the price and order quantity of products to ensure the maximization of the total expected revenue of all products.

2.2 Model assumptions

$i = 1, \dots, N$ correspond to the N products of each vendor. For product i, p_i represents the unit selling price; c_i represents the unit purchase cost; y_i represents the safety stock; Y_i represents the order quantity; D_i represents the demand; π_i represents the profit. $\gamma_{ij}(0 \leqslant \gamma_{ij} \leqslant 1)$ represents the cross-selling coefficient form product i to product j . Impacted by cross-selling, if the number of products i sold increases by 1 for every 100 increases in the number of products j sold, then $\gamma_{ij} = 0.01$. $(a)^+ = \max(0,a)$. $r_x(\cdot) = f_x(\cdot)/(1-F_x(\cdot))$. For random variable x, $r_x(\cdot)$ represents the failure rate; $f_x(\cdot)$ represents the probability density function; $F_x(\cdot)$ represents the cumulative distribution function.

3. Model

3.1 Model construction

For retailers, the deterministic demand affected by price is known, namely $L_i(p_i)$ known, so the retailer decides the price and safety stock of the product to determine the order quantity. Order quantity $Y_i = L_i(p_i) + y_i$.

Product i total demand of $D_i(p_i,y_{-i})$

$$\begin{aligned} &= L_i(p_i) + \varepsilon_i + \sum_{j\neq i}\gamma_{ji}\min\{D_j(p_j),Y_j(p_j)\} \\ &= L_i(p_i) + \varepsilon_i + \sum_{j\neq i}\gamma_{ji}\min\{L_j(p_j)+\varepsilon_j,L_j(p_j)+y_j\} \\ &= L_i(p_i) + \varepsilon_i + \sum_{j\neq i}\gamma_{ji}(L_j(p_j)+\varepsilon_j) - \sum_{j\neq i}\gamma_{ji}(\varepsilon_j-y_j)^+ \end{aligned} \tag{1}$$

Among them, $y_{-i} = y_1, \dots, y_{i-1}, y_{i+1}, \dots, y_n$, $\min\{\varepsilon_j,y_j\} = \varepsilon_j - (\varepsilon_j-y_j)^+ = y_j - (y_j-\varepsilon_j)^+$.

In formula (4.1), $L_i(p_i)$ indicates the deterministic demand affected by the price; ε_i indicates the demand affected by random factors; $\sum_{j\neq i}\gamma_{ji}\min\{D_j(p_j),Y_j(p_j)\}$ indicates the demand affected by cross-selling. For the convenience of research, the uncertain part of the total demand is defined as $D_i^s(p_{-i},y_{-i})$.

$$D_i^s(p_{-i},y_{-i}) = \varepsilon_i + \sum_{j\neq i}\gamma_{ji}(L_j(p_j)+\varepsilon_j) - \sum_{j\neq i}\gamma_{ji}(\varepsilon_j-y_j)^+ \tag{2}$$

Expected revenue of retailers $\pi(\vec{p},\vec{y})$

$$\begin{aligned} &= E\sum_{i=1}^{n}[p_i\min\{D_i(p_i,y_{-i}),Y_i\}-c_iY_i] \\ &= E\sum_{i=1}^{n}[\pi_i^d(p_i)-c_iy_i+p_i\min\{D_i^s(p_{-i},y_{-i}),y_i\}] \end{aligned} \tag{3}$$

Among them, $\pi_i^d(p_i) = (p_i-c_i)L_i(p_i)$, $c_i \leqslant p_i \leqslant p_i^{max}$, $0 \leqslant y_i \leqslant y_i^{max}$, p_i^{max} and y_i^{max} are large enough.

In formula (3), $p_iE[\min\{D_i(p_i,y_{-i}),Y_i\}]$ indicates that retailers sell products i total income obtained, c_iY_i indicates the product i total cost of. $(p_i-c_i)L_i(p_i)$ indicates the product i the benefits of deterministic demand, $p_iE[\min\{D_i^s(p_{-i},y_{-i}),y_i\}] - c_iy_i$ indicates the product i benefits of uncertain demand. $E\sum_{i=1}^{n}$ represents the sum of expected benefits of all products. Therefore, the decision goal of retailers is to determine the price of each product p_i and safety stock y_i make expected benefits $\pi(\vec{p},\vec{y})$ maximize. To facilitate research i the benefits of deterministic demand are defined as $\pi_i^d(p_i)$.

$$\pi_i^d(p_i) = (p_i-c_i)L_i(p_i) \tag{4}$$

3.2 Model analysis

Based on the above situation, retailers determine the price and order quantity of products to ensure the maximization of the total expected revenue of all products. In the centralized decision-making environment, the first-order necessary condition for the optimal solution of the retailer's expected return function is:

$$\frac{d\pi_i^d(p_i)}{dp_i} + E[\min\{D_i^s(p_{-i},y_{-i}),y_i\}] + \sum_{j\neq i}\gamma_{ij}p_j\frac{dL_i(p_i)}{dp_i}\Pr\left(D_j^s\left(p_{-j},y_{-j}\right)<y_j\right) = 0 \quad (5)$$

$$-c_i + \sum_{j\neq i}\gamma_{ij}p_j\Pr\left(y_i<\varepsilon_i,D_j^s\left(p_{-j},y_{-j}\right)<y_j\right) + p_i\Pr(D_i^s(p_{-i},y_{-i})>y_i) = 0 \quad (6)$$

Find the formula (3) separately p_i and y_i let the first partial derivative of be zero, then we can get formula (5) and formula (6).

4. Numerical analysis

Based on specific numerical examples, this section simulates the newsboy model of cross-selling products in the centralized decision-making environment. Consider the existence of a single retailer in the market, selling product A and product B. There are cross-selling between the two products, and both are price-sensitive perishable products. In order to facilitate the comparison with the decision-making strategy of products in the decentralized decision-making environment, the parameters in this section are the same as those in the previous section. It is assumed that the random demand of product A and product B obey normal distribution. The relevant parameters of product A are as follows: $L_1(p_1) = 1000 - 2p_1$, $\varepsilon_1 \sim N(50,1)$, $c_1 = 150$, $\gamma_{12} = 0.1$ (γ_{12} indicates the cross-selling coefficient of product A to product B); The relevant parameters of product B are as follows: $L_2(p_2) = 2000 - 3p_1$, $\varepsilon_2 \sim N(100,1)$, $c_2 = 200$, $\gamma_{21} = 0.1$.

This paper uses genetic algorithm to optimize the retailer's optimal decision-making strategy in the centralized decision-making environment. The specific optimization steps are as follows.

Step 1: construct the expected return function of retailers according to the specific parameters of the example $E\pi(p_1,p_2,y_1,y_2)$.

Step 2: Find the maximum expected revenue of retailers in the feasible region through genetic algorithm.

Step 3: Through continuous iteration, filter out the largest value of all the maximum expected returns and find the corresponding $(p_{1c}^*\ p_{2c}^*,y_{1c}^*\ y_{2c}^*)$, $(p_{1c}^*\ p_{2c}^*,y_{1c}^*\ y_{2c}^*)$ is the optimal solution.

Step 4: Set the optimal solution $(p_{1c}^*\ p_{2c}^*,y_{1c}^*\ y_{2c}^*)$ the expected return function of retailers $E\pi(p_1,p_2,y_1,y_2)$ maximum expected revenue of retailers $E\pi_m(p_{1c}^*,p_{2c}^*,y_{1c}^*,y_{2c}^*)$.

Step 5: Through sensitivity analysis, compare the impact of different values of parameters on retailers' decision-making strategies.

Follow the above steps to solve the example in this section, and the results are shown in Table 1.

Table 1. The optimal solution of the example

Product	p_c^*	y_c^*	Y_c^*	$E\pi_m$
A	345.3	126.9	436.3	281639.4
B	446.4	136.4	792.2	

5. Summary

In this chapter, we consider the centralized decision-making environment, combine the impact of price on demand and the impact of cross-selling effect on demand, and study the newsboy model of cross-selling products whose demand is affected by price. Firstly, a centralized decision-making newsboy model with single cycle and multiple products is constructed; Then, the first order necessary conditions of optimal pricing and optimal order quantity are derived; Finally, the theoretical results of this chapter are verified by an example.

References

[1] Aaron Knott, Andrew Hayes, Scott A. Next-product-to-buy models for cross-selling applications [J]. Journal of Interactive Marketing, 2002, 16 (3): 59 - 75.

[2] Jarrar Yasar F, Neely Andy. Cross-selling in the financial sector: Customer profitability is key [J]. Journal of Targeting, Measurement and Analysis for Marketing, 2002, 10 (3): 282 - 296.

[3] Kamakura W A, Wedel M, Rosa F D, et al. Cross-selling through database marketing: a mixed data factor analyzer for data augmentation and prediction [J]. International Journal of Research in Marketing, 2003, 20 (1): 45 - 65.

[4] Zuo-Jun Max Shen, Xuanming Su. Customer Behavior Modeling in Revenue Management and Auctions: A Review and New Research Opportunities [J]. Production and Operations Management, 2007, 16 (6): 713 - 728.

[5] Kamakura W A. Cross-Selling: Offering the Right Product to the Right Customer at the Right Time [J]. Social Science Electronic Publishing, 2014, 6 (3): 41 - 58.

[6] Margaret Anne Emmelhainz C, Bruce Kavan. Using Information as a Basis for Segmentation and Relationship Marketing: A Longitudinal Case Study of a Leading Financial Services Firm [J]. Journal of Market-Focused Management, 1999, 4 (2): 161 - 177.

[7] Paas Leonard, Kuijlen Ton. Acquisition pattern analyses for recognizing cross-sell opportunities in the financial services sector [J]. Journal of Targeting, Measurement and Analysis for Marketing, 2001, 9 (3): 230 - 240.

[8] Su, Barbara. Banking practices and borrowing firms' financial reporting quality: evidence from bank cross-selling [J]. Review of Accounting Studies, 2021.