The Progress of Black-Scholes Model and Black-Scholes-Merton Model

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Abstract. Black-Scholes (BS) model was first proposed in 1973, which has been modified by Robert Merton as the Black-Scholes-Merton (BSM) model subsequently. Contemporarily, these two models have been widely used and praised by financial scholars as well as employees. Plenty of scholars have tried to verify the accuracy of the and expressed their views on the existing defects in above models. Based on the existing literature, this article first introduces and derives the two models step by step and discusses the basic assumptions for these models. Subsequently, the applications of the two models are demonstrated separately. Specifically, the project valuation based on BS model is presented detaily while the applications of BSM model are introduced from four aspects (pricing of intangible assets, risk avoidance, default prediction and employee stock option's pricing). Afterwards, the limitations and gaps of the models (e.g., volatility smile) ascribed to the ideal assumptions are discussed. In order to tackle the issue, improvement and suggestions are proposed including extending the models with different forms. These results offer a guideline for the option pricing, which can be widely applied in investment strategy.

Keywords: Black-Scholes model; Black-Scholes-Merton model; application; limitation; modification.

1. Introduction

Option pricing has always been a vital part of the options trading market. The theoretical study of option pricing can be traced back to 1900, when the French financier Louis Bachelier [1] first proposed a stochastic model for the operation of stock prices using the idea of random wandering in his book Theorie de la Speculation. Subsequently, Black and Scholes [2] derived the Black-Scholes (BS) option pricing formula in 1973, which set off the academic wave of option pricing. In the same year, Robert Merton optimized it and established the Black-Scholes-Merton (BSM) Model [3]. Afterwards, other methods sprang up such as Put-Call Parity, which describes the basic relationship that necessarily exists between the price of the selling and the buying option.

Due to the continuous improvement of these option pricing methods, option pricing has become mature gradually. As BS Model and BSM Model are the most popular models for the option pricing, this perspective will mainly discuss the applications and limitations of the above two models. Specifically, we first demonstrate the basic descriptions and intrinsic relationship between BS and BSM models based on literature review. Besides, the research developments and hot spots for the applications of the models are summarized accordingly to recent studies. Furthermore, the corresponding future trends from different aspects of applications are proposed.

The reminder of this paper is organized as follows: Section 2 introduces the basic descriptions, applications and limitations of BS Model; section 3 demonstrates the modification of BSM model from BS model and its application in 5 areas; section 4 discusses the future development routine for these models; section 5 gives a brief summary eventually.

2. The progress of BS model

2.1 Basic descriptions of BS model

BS model is a dynamic model for the European call option price based on partial differential equation, which follows the key idea to hedge the option by buying and selling so as to eliminate risk. According to Ref. [2], this model has lots of assumptions for the assets and markets. As for the assets, the rate of return on the riskless asset is fixed, which is also known as risk-free interest rate. Besides, the stock does not pay a dividend and the price satisfy a geometric Brownian motion with constant drift and volatility. With regard to the markets, one is able to borrow and lend any amount of cash at the riskless rate as well as buy and sell of the stocks without cost fees. Moreover, the markets have no arbitrage opportunities. Based on above assumptions, one deduces the Black–Scholes equation:

$$\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2}+rs\frac{\partial C}{\partial S}-rC=0 \tag{1}$$

where C is the European call option price, S is the current price of the financial asset, r is the continuously compounded risk-free interest rate and σ is the annualized standard deviation. In terms of assumptions, the boundary conditions follow C(0, t)=0, C(S, t)~S when S is infinite and C(S, T) = max(S-X, 0) where X is the strike price of the option, the Eq. (1) can be solved as:

$$C(S,t)=SN(d_1)-Xe^{-r(T-t)}N(d_2) \tag{2}$$

where N(d) is cumulative probability distribution function of normally distributed variables, d1, d2 are two random variables from the distribution follow:

$$d_1=\frac{\ln\left(\frac{S}{K}\right)+(r+\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \tag{3}$$

$$d_2=d_1-\sigma\sqrt{T-t} \tag{4}$$

2.2 Applications of BS model

So far, BS pricing model are widely used to estimate the project value. Traditional way to evaluate the value of R&D investment projects mainly includes cost analysis method, market comparison method, discounted cash flow method (DCF), and real option method (RO)[4]. Through comparative research, it is found that the real option valuation model has made up for some of the shortcomings of other methods, and can better reflect the uncertainty and flexibility value of the evaluation to the future. Among them, the widely used real option valuation models are mainly the binary tree option pricing model and the closed-form equation method. Thereinto, BS option pricing model is the main content of the closed-form equation method. Lots of studies have been carried out based on BS model for different types of objective, e.g., drug research [5], renewable energy sector [6], information technology project [7], etc.

To better illustrate, new drug research [5] is chosen as an example, which is a dynamic process with a high investment cycle that can be properly evaluate with BS model. After an overall assessment of the future value of the project, the feasibility of the project can be judged. Typically, its further investment depends on the success of the previous stage. The investigation routines of Ref. [5] are divided into two stages: the R&D stage and the commercialization stage. On this basis, the progress of the commercialization stage depends on the success of the R&D stage. In other words, there is only one option before the commercialization stage, and the option structure is relatively simple. Therefore, the literature adopts the BS pricing model to calculate the project value according to the characteristics of pharmaceutical companies, making the calculation result more fit the reality.

2.3 Limitations of BS model

Although BS model are extensive utilized in option pricing, there are some drawbacks and limitations of this model. First of all, BS model assumes that stocks do not pay dividends, which is obviously an unrealistic assumption. Dividend payment will increase the connotative value of put option and decrease the connotative value of call option. Previously, OTC stock options have protective measures for dividend payment, and the effect of dividend payment on option value can be eliminated by adjusting the exercise price of options. However, at present, the contract terms of stock options traded on and off the market are not adjusted with the payment of cash dividends, which makes the dividends paid during the validity period of options an important factor affecting the value of options. Fortunately, this deficiency has been overcome with the amend model of BSM, which will be discussed in Section 3 detailly.

Besides, BS model is not applicable to American option pricing. In addition, the implied volatility derived from the model is always higher than the historical volatility. In fact, the deviation may be ascribed to the unperfect lognormal distribution of the stock price, where the middle is sharper and tail is fatter, i.e., results in pricing bias [8]. Moreover, the risk-free interest rate is assumed to be fixed during the duration of the option, which refers to the interest rate of the national debts in most cases. Nevertheless, these rates are usually fluctuated rather than fixed. Furthermore, the BS pricing model assumes that the future stock price volatility remains constant, but it is affected by the stock price and other factors (e.g., expiration time of the option), which cannot be kept unchanged.

3. The progress of BSM model

3.1 Modification from BS model

As pointed out in Section 2.C, the BS model is unable to tackle with paying dividend stocks. To address the issue, Robert C. Merton modified the formula in the same year the model proposed, which is known as Black-Scholes-Merton (BSM) Model [9]. After considering the dividend effects, the Eq. (2) is extension to:

$$C = Se^{-kt}N(d_1) - Xe^{-r(T-T)}N(d_2) \tag{5}$$

where the k is the annual dividend yield of the underlying stock (assuming that dividends are paid continuously, rather than in discrete installments). Besides, the Eq. (3) is also changed to:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right)+(r-k+\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \tag{6}$$

As same as BS model, the BSM model assumes that the option can only be exercised on its expiration date or maturity date [10], i.e., it can be used only in the European options marke and no-yield American calls. Besides, the BSM model also ignores transaction costs (e.g., commissions and brokerage operations) and assumes a fixed interest rate. Moreover, stock returns are still considered as normally distributed, i.e., means that the volatility of the market is constant over time.

3.2 Application of BSM model

With the amendment of BS model, BSM model are more suitable in different conditions. In this case, we introduce the feasible applications of this model as following.

3.2.1 Pricing of Intangible Assets

As discussed earlier, BSM model considers the dividend effect, i.e., it provides an estimation of assets that accommodate price changes and are subject to the impact of project delays on prices. In reality, it is the first pricing model that can be used for intangible assets [11], e.g., patents, goodwill, brand names or copyrights. In this case, a discounted cash flow model can be applied to forecast future cash flows and estimate the net present value of the asset. Thereby, the BSM model is feasible to calculate the present value of an intangible asset.

3.2.2 Risk Avoidance

The BSM model provides a general evaluation framework for the companies partly financed by borrowing, especially assisting in cases that conventional techniques are invalid to fully reflect the involved risks. If the shareholders are protected by limited liability, they can subscribe to the underlying assets of the company. Based on BSM model, one is able to estimate the value of a company according to the value of its assets and volatility [12]. As shown in Fig.1, the option price predicted by BSM model is in line with other models, which all shows good calibration for the optimal one. For financially strong companies, their time value will be small and the intrinsic value of the firm (i.e., the present value of its assets minus its liabilities) will dominate the value of its equity [13]. In this case, normal risk aversion is expected to apply as the intrinsic value will be equally exposed to both positive and negative movements in the value of the company's assets.

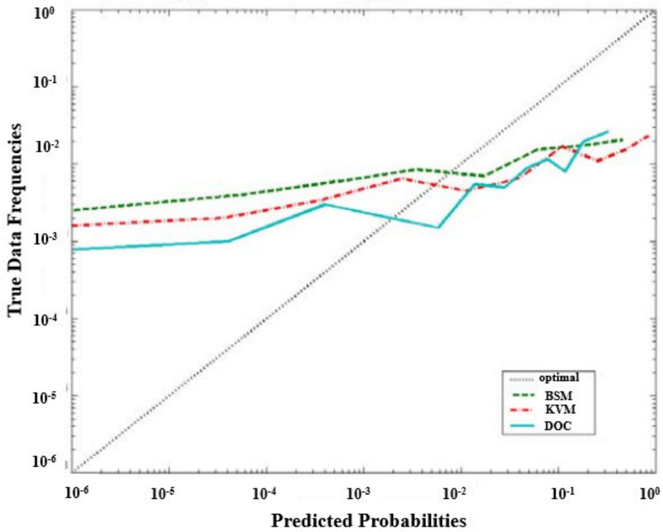

Fig. 1 Prediction calibration statistics for one-year-ahead options based on different models [12].

3.2.3 Default Prediction

Bharath and Shumway thought that the BSM model allows for default forecasting and risk-sharing management techniques to reduce risk due to supplier insolvency and to estimate the insurance premiums that banks can use to pay [14]. Therefore, BSM model would be applied to many banks offering supply chain finance solutions, which may include insurance services to further mitigate trade risks (e.g., supplier default). Valverde chose a sample of companies, which was selected from the New York Stock Exchange and historical stock price data was collected from the CRSP database (Center for Research in Security Prices) to calculate probabilistic BSM models to analyze bankruptcies of a sample of suppliers from different industries [15]. He found that twelve libraries of companies of different sizes were created and a VBA program for Excel was developed to calculate the probability of bankruptcy tables for companies belonging to different libraries. These results verified the function of BSM model for estimating the number of policies sold needed to minimize risk and to predict with a high degree of certainty the losses due to supplier insolvency. Financial institutions can use the expected losses from the risk pool in order to price insurance contracts for hedging companies against supplier default risk.

3.2.4 Employee Stock Option's Pricing

Employee Stock Option (ESOs) is a contract between a listed company and an employee that grants the employee the option to purchase a certain number of the company's common shares in the future at the price agreed upon at the time of signing the contract [16]. In other words, the employee can buy the company's stock at a preferential price in the future. This is a common form of incentive for employees in public companies.

As a matter of fact, the cost of their ESOs for financial reporting purposes can be calculated through the BSM model by adjusting the time to maturity [16]. John D. Finnerty modified the BSM model in closed form to explicitly account for the special features of ESOs so as to clarify the impact of forfeiture vesting rates and early exercise rates on ESO costs [17]. He used historical exercise and forfeiture data for 10 firms consisting of 127 separate ESO grants and 1.31 billion ESOs to calibrate the model. According to the results illustrated in Fig. 2, the values obtained from the modified BSM model are consistent with those from the more computationally intensive trinomial lattice and μ lattice models.

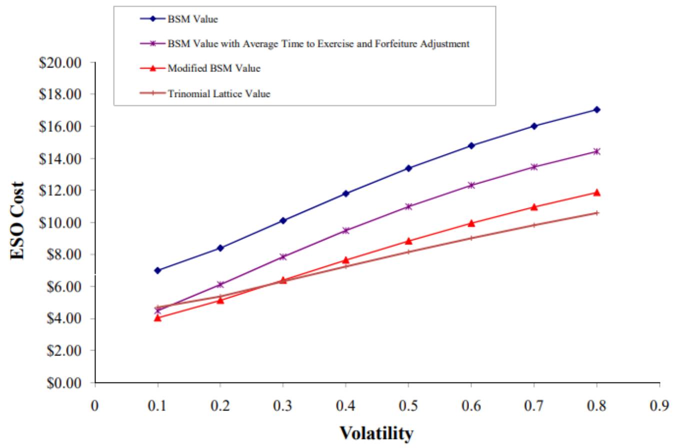

Fig. 2 ESO cost as a function of volatility for different models [17].

4. Discussion

Although there are plenty of applications for the BS and BSM model, the shortages and limitations of the models cannot be ignored during processing due to the ideal assumptions. “volatility smile” is one of the well-known issues faced most of analysis cases [18]. Specifically, it refers to the distribution of yields on financial assets is more characterized by spikes and fat tails, which is verified by a large number of empirical studies [19]. On this occasion, the volatilities of companies are fluctuated instead of fixed, which is inconformity with the models’ assumptions. Thus, the prediction from BS and BSM models might underestimates the price of deep real and deep imaginary options correspondingly [20]. In addition to “volatility smile”, there are other issues occurred in different cases, e.g., valuing bond options [21, 22], interest-rate curve [23, 24], Short stock rate [25], etc.

In order to address above issues and other limitations caused by assumptions of the models, lots of extensions have been proposed, e.g., these extensions include models for American options [26]. Despite the proposals from large amounts of extension models, the applicability and operability for real assets are remain unknown according to previous literature [21, 22]. In the future, more extensions with high accuracy and simple operations should be investigated and various empirical studies should be carried out in order to verify the practicability for all the extension models.

5. Conclusion

In summary, two pricing models, which are BS Model and BSM Model, are introduced and reviewed detailly. Specifically, we demonstrate the basic descriptions and assumptions as well as explored the applications and limitations of the above two models. Besides, the connection between the two models is also inferred. Moreover, the usage limitations on account of the models’ assumptions and future research hotspot (extension the models) are also discussed. According to the analysis, a deeper understanding of the theoretical structure of option pricing is presented based on BS and BSM models. These results shed light for pricing and risk evaluation for investors and researchers as well as offer suggestions for pricing theory developments.

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