Portfolio Construction Based on Mean Variance Portfolio Theory

Haokai Yin[1, *]

[1]School of International Economics and Trade, Shandong University of Finance and Economics, Jinan, China

*Corresponding author: 20190554136@mail.sdufe.edu.cn

Abstract. Asset allocation is considered an important task and is highly regarded, people are eager to earn attractive profits through the correct allocation of investment assets. In this paper, an optimal portfolio of five stocks under different conditions is constructed to calculate and predict the expected return of this portfolio. When constructing a portfolio with equal weights for different underlying symbols, the Sharpe ratio is 1.14, which means that the portfolio return is higher than the volatility risk. Based on the evaluations, Tesla is more volatile, its yield has risen a lot recently. The efficient frontier is obtained by means of Monte-Carlo concepts with random weights arrangement. The results of two different cases with different optimal conditions are presented. In the end, the conclusion is that choose the portfolio with the largest Sharpe ratio, The risk premium per unit of risk is the highest, and it's worth it. Get a possible optimal investment portfolio by analyzing data, so that people can have a deeper understanding of asset allocation theory. Overall, these results offer new insights on portfolio construction and provide guidelines for construction of portfolio.

Keywords: Asset allocation; mean variance portfolio; Monte Carlo simulations.

1. Introduction

In recent years, various types of underlying assets are appeared around the global financial market including stock security, bonds (country, city or corporation), commodities (futures, gold, oil), options and cryptocurrency (e.g., Bitcoin) etc. [1-3]. However, different underlying assets show different features of price fluctuations and some of them have strong correlations and lead-lag effects [4-6], while some of the assets are irrelevant to each other and some even has negative correlations. In order to satisfy the hedging risks motivations of investors, the separating investment strategy is one of the most applied measures [7-9]. However, a critical issue for separating is how to determine the weight of different products and symbols.

To address the issue, the concept of portfolio is proposed with a specific weight arrangement following mathematical models, where various scholars have carried out study to give hints for choosing the weights [10-13]. In retrospect, in 1952, Markowitz first mentioned the efficient portfolio and efficient frontier of mean-variance model in his paper. In 1956, Markowitz introduced quadratic utility function with risk preference to transform nonlinear optimization into quadratic optimization problem [10]. With regarding to the Markowitz proposed the efficient portfolio in 1952, the portfolio with the least risk under a given level of return or the portfolio with the highest return under a given level of risk. These efficient portfolios constitute the efficient frontier [10]. This model will be used in this study for investigation. Nevertheless, scholars recognize that to achieve stable and well performance portfolio, the prediction of price for the portfolio is one of the important tasks [14].

In this paper, portfolio optimization is done using mean variance portfolio theory and Monte Carlo simulations, and then the expected return on this portfolio. Finally, one can get the conclusion that choose the portfolio with the largest Sharpe ratio, The risk premium per unit of risk is the highest, and it's worth it. This study conducts research through an example to strengthen the understanding of asset allocation theory and strengthen its own theoretical advantages. Besides, one can learn how to rationally allocate assets. The reminder of the study is divided into five parts: introduction, Data & Method, Results & Discussion, Limitations & Future outlooks, and Conclusion. Introduction introduces the background. Data & Method introduces the data, model construction and the whole process of operation. Results & Discussion introduces the results of the analysis and suggestions for

investors. Limitations & Future outlooks introduces the limitations of this research and based on the analysis. The prospect of this study for the future, the Conclusion section summarizes the results of this paper and the significance of this study.

2. Data & Method

2.1 Data

Five stocks are chosen. The five stocks selected are Apple, Amazon, Google, Microsoft and Tesla. The ticker symbols of these companies are AAPL, AMZN, GOOG, MSFT and TSLA respectively. The adjust closing stock prices of the companies from November 1, 2019 to October 28, 2022 is chosen as the data source for the model. Here are the details about the company. Apple Inc. is a multinational technology company. Amazon specializes in e-commerce, digital streaming, and artificial intelligence. Google is a subsidiary of Alphabet engaged in Internet-related work, like cloud computing. Microsoft is a multinational computer technology company in the United States. Tesla is the largest electric vehicles corporation around the world.

2.2 Model

An optimal portfolio of five stocks under different conditions is constructed to calculate and predict the expected return of this portfolio. When doing investment analysis, the principle of building a portfolio is not to put eggs in the same basket, to diversify and reduce risk, that is, to choose stocks in different industries to construct a portfolio, because different companies are in different industries, so their risk coefficients are different, affected by the systemic risk of market factors will perform differently, such a diversified portfolio is more risk-resistant, more conducive to reducing the risk of the portfolio. Because investors choose a portfolio with large returns under the same risk, and a portfolio with the same return, investors will definitely choose a portfolio with less risk. After discussion, this paper decided to select the stocks of the following five companies.

2.3 Processing

After collection of the data, this study first imports the data to the analysis software. To process the data, this paper first calculates the return ratio of different underlying assets and clean the data to satisfy the following calculations. Then, the basic statistics for the different securities will be calculated including the annual return ratio as well as the annual volatility (variance of the return). Thus, one can derive the Sharpe ratio of the underlying assets directly based on above data. Subsequently, the Monte-Carlo simulations will be carried out with the generation of random matrix for the weights of different underlying assets. After that, the return ratio as well as the variance will be calculated for each dot to form the effective frontier with abundant simulation points.

With regard to the portfolio construction, the optimal process will be given. On this basis, the weights of different securities will be obtained via solving the optimal functions. To mathematically describe the procedures, the two optimal condition corresponds to:

$$Maximum\ Sharpe: Max\ E(r_p)/Var(r_p) \tag{1}$$

$$Minimum\ Variance: Min\ Var(r_p) \tag{2}$$

Here,

$$E(r_p) = \Sigma_i \omega_i E(r_i) \tag{3}$$

$$Var(r_p) = \Sigma_i \omega_i^2 Var(r_i) + \Sigma_{i \neq j} \omega_i \omega_j Cov(r_i, r_j) \tag{4}$$

with

$$\Sigma_i \omega_i = 1 \tag{5}$$

Where the E() represents the expected return, Var() for the variance function, Cov() for covariance function.

Nevertheless, it should be noted that above models have some assumptions. It considers only one investment cycle. Investors only need to make one allocation at the beginning of the period and do

not need to change their decision until the end. In practice, investors usually allocate funds at the beginning of the period, but also need to adjust the decision at any time according to the market situation, in order to achieve better investment results. In addition, it ignores other costs like transaction fees. Transaction costs play a very important role in investment management. Moreover, it supposes the security is infinitely divisible. This is inconsistent with the actual securities market. Besides, markets are considered to be efficient. However, the securities market began in the 1990s, and still belongs to the immature market. Investors are perfectly rational, i.e., one follows the principle of choosing the securities with the least risk for the same expected rate of return, or the securities with the highest expected rate of return for the same investment risk The return on each asset follows a normal distribution. Under the premise that the market is efficient and reasonable, one limits the rate of return and believe that all of them obey normal distribution.

3. Results & Disucssion

3.1 Effective Frontier

To begin with, one needs to have an overall map of the performances (return ratio as well as volatility marked by the standard deviation) combination of different securities with different weights. At this point, the important thing is how a given stock pool finds the balance of risk and return. On this basis, a random selected weights matrixes based on the concepts of Monte-Carlo simulations are presented to show the distributions of the returns and volatility (marked by the annul standard deviation of the portfolio). To ensure full usage of the capital as well as forbidden the leverage, the sum of the weights for all the securities are fixed at 1. The results of the random points are presented in Fig. 1, where the combination of the results depict the clear effective frontier of the selected underlying assets.

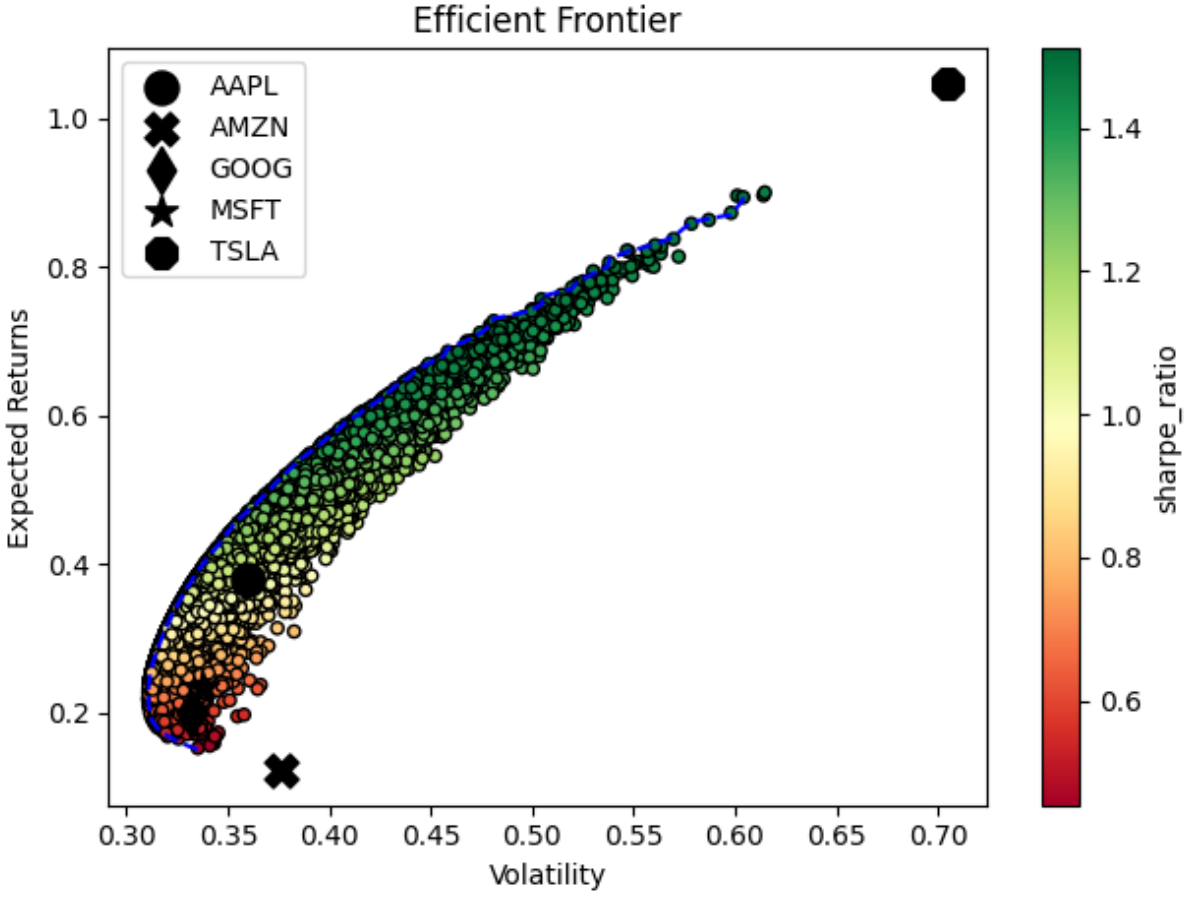

Fig. 1 Effective frontier obtained by Monte-Carlo simulations, the five selected underlying assets performances are marked in the portfolio.

3.2 Performances of Portfolio

For comparison, two kinds of portfolios are found following the different optimal criteria, i.e., one pursuit the largest Sharpe ratio (balance and trade off between returns and volatility) and the variance minimum (the least risks at the cost of high gains). The optimal results compared with Monte-Carlo simulations results are given in Fig. 2. When the Sharpe ratio is the largest, one can get the following results. The return ratio of the whole portfolio is 72.68% while volatility is 48.11% with Sharpe ratio of 151.07%. In addition, the weights for different security symbols are AAPL: 43.44% AMZN: 0.33% GOOG: 1.57% MSFT: 1.67% TSLA: 52.98%, respectively. The portfolio here has a yield of 72.68% and a Sharpe ratio of 151.07%, well above 1, indicating that the return on such a portfolio is much higher than the risk-free rate. It's a good portfolio. In terms of the weight of individual stocks, such a portfolio is mainly composed of Apple and Tesla.

When the variance of the portfolio is the smallest, i.e., the portfolio is the most stable with least risks, one can derive the following data. The performance of the portfolio is that returns ratio equals 21.90%, volatility is 31.08% with Sharpe ratio of 70.47%. In addition, the weights for different security symbols are AAPL: 15.38% AMZN: 20.20% GOOG: 46.42% MSFT: 17.71% TSLA: 0, respectively. From the above data, if the portfolio has the smallest variance, the return of such a portfolio is only 21.90%, and the Sharpe ratio of this portfolio is less than 1, indicating that the operational risk of this portfolio is greater than the rate of return. It should be noted that there is no Tesla in this portfolio, which can be attributed to the extremely large variation of the Tesla.

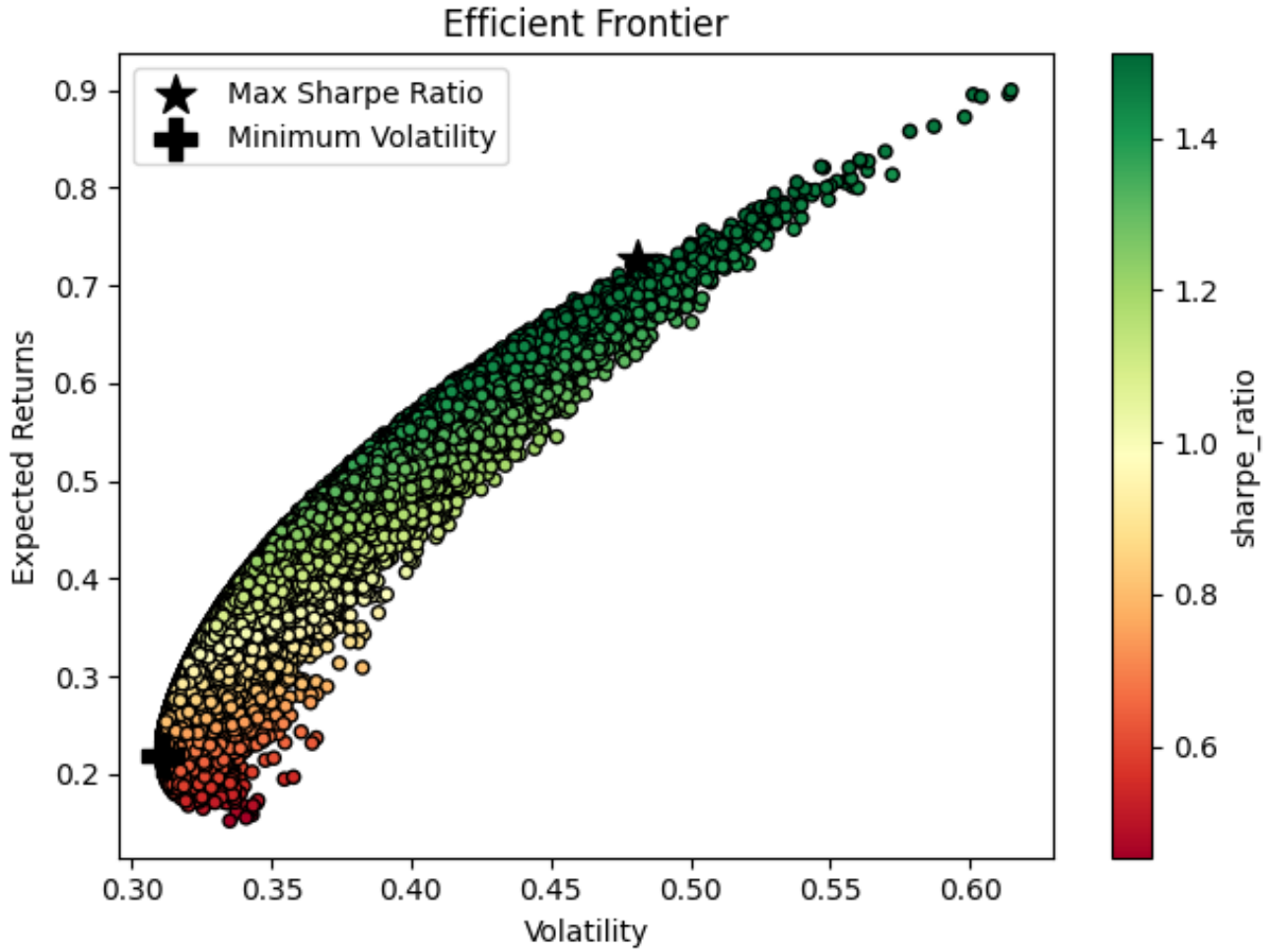

Fig. 2 Performances for different optimal conditions combing with the effective frontier.

3.3 Suggestions

Comparing the portfolio under the above two investment conditions, the decision is that in the combination of these five stocks, Tesla's income fluctuates greatly, when the stability of the portfolio

return is pursued, the return of the portfolio that can be obtained is 30% of the yield of the portfolio with the largest Sharpe ratio, but the volatility of the portfolio is only reduced from the original 48% to the current 31%, and the Sharpe ratio becomes half of the original, so in this analysis, we can get the conclusion that choose the portfolio with the largest Sharpe ratio, The risk premium per unit of risk is the highest, and it's worth it.

4. Limitations & Prospects

Nevertheless, it should be noted that this study does has some defects and shortcomings. To be specific, the article only introduces the situation that 5 stocks have equal weights, and the other situations are not detailed, and the explanation is not thorough enough. The quantity of the underlying assets is rather small (compared to all the listed company). Besides, the main challenge for stock price prediction systems is that most of the existing techniques cannot be detected using historical stock data because they are affected by certain factors involving government policy decisions, market sentiment, etc. Therefore, decision-making requires data from different sources, and data preprocessing is a complex task in data mining. Moreover, it should be noted that there are lots of other types of underlying assets can be considered as investments target with larger or smaller volatility and leverage features including the bonds with low volatility and low returns, and futures, options as well as crypto currency with high volatility and high returns). In the future, methods that provide accurate stock market forecasts should be developed. More advanced and state-of-art scenarios (e.g., machine learning approaches) can be implemented to enhance the accuracy. In addition, more types of assets can be chosen to enrich the variety, stability and boost the performances of the constructed portfolio.

5. Conclusion

To sum up, this paper investigates the portfolio construction based on the state-of-art theorem and models. To be specific, an optimal portfolio of five stocks under different conditions is constructed to calculate and predict the expected return of this portfolio. In this paper, portfolio optimization is done using mean variance portfolio theory and Monte Carlo simulations, and then the expected return on this portfolio.

When constructing a portfolio with equal weights of each asset, the Sharpe ratio of such a portfolio is 1.14, which means that the portfolio return is higher than the volatility risk. According to the analysis, Tesla is more volatile, its yield has risen a lot recently, and the other four stocks have similar volatility. The efficient frontier is obtained by means of Monte-Carlo concepts with random weights arrangement. The results of two different cases with different optimal conditions are presented. In the end, there is a conclusion that choosing the portfolio with the largest Sharpe ratio, the risk premium per unit of risk is the highest, and it's worth it. It is a pity that this article discusses less and does not classify the allocation of asset weights more. Moreover, the categories of underlying assets and quantities of the targets are limited, which can be further improved in coming studies (e.g., including other types of underlying assets including bonds, ETF, commodities, futures, or even cryptocurrencies). In the future, the method should be developed to provide the accurate prediction of the stock market via the machine learning scenarios. Besides, this article does point out a possible optimal investment portfolio. Overall, these results offer a guideline for portfolio construction.

References

[1] Petchrompo S, Parlikad A K. A review of asset management literature on multi-asset systems. Reliability Engineering & System Safety, 2019, 181: 181-201.

[2] Garcia P, Leuthold R M. A selected review of agricultural commodity futures and options markets. European review of agricultural economics, 2004, 31(3): 235-272.

[3] Valdeolmillos D, Mezquita Y, González-Briones A, et al. Blockchain technology: a review of the current challenges of cryptocurrency. International Congress on Blockchain and Applications. Springer, Cham, 2019: 153-160.

[4] Zeevi A, Mashal R. Beyond correlation: Extreme co-movements between financial assets. Available at SSRN 317122, 2002.

[5] Frye J. Correlation and asset correlation in the structural portfolio model. The Journal of Credit Risk, 2008, 4(2): 75-96.

[6] Huth N, Abergel F. High frequency lead/lag relationships—empirical facts. Journal of Empirical Finance, 2014, 26: 41-58.

[7] Bernard V, Thomas J, Wahlen J. Accounting - based stock price anomalies: Separating market inefficiencies from risk. Contemporary Accounting Research, 1997, 14(2): 89-136.

[8] Kouwenberg R, Ziemba W T. Incentives and risk taking in hedge funds. Journal of Banking & Finance, 2007, 31(11): 3291-3310.

[9] Stowell D P. Investment banks, hedge funds, and private equity. Academic Press, 2017.

[10] Darst D M. Art of Asset Allocation. McGraw-Hill Companies, 2003.

[11] Markowitz H M. Portfolio theory: as I still see it. Annu. Rev. Financ. Econ., 2010, 2(1): 1-23.

[12] Kazan H, Uludağ K. Credit portfolio selection according to sectors in risky environments: Markowitz practice. Asian Economic and Financial Review, 2014, 4(9): 1208-1219.

[13] Mangram M E. A simplified perspective of the Markowitz portfolio theory. Global journal of business research, 2013, 7(1): 59-70.

[14] Zhang Y, Li X, Guo S. Portfolio selection problems with Markowitz's mean–variance framework: a review of literature. Fuzzy Optimization and Decision Making, 2018, 17(2): 125-158.

[15] Amjady N. Short-term hourly load forecasting using time series modeling with peak load estimation capability. IEEE Transactions on Power Systems, 2001, 16(4): 798-805.