

# The Upgrading of CAPM Model: from Fama-French 3 Factors to Multi-factors

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**Abstract.** In general, financial theorem has always been particularly interested in the trade-off for dynamics of associated risk and expected return. This mystery had remained convoluted for decades until William Sharpe and John Lintner developed the first practical framework of the Capital Asset Pricing Models (CAPM). As a matter of fact, CAPM is founded on the idea that asset prices are influenced by inherent, risk free and market risk returns, and diversification of portfolios also benefited the investor for risk reduction. Since this finding, many advancements have been made beyond the initial models of CAPM such as the ICAPM and Fama French models. Thus, this paper presents founding theorems of the Capital Asset Pricing Model, discusses its historical advancements into different equations and demonstrate its potential limitations and future outlooks. To be specific, the principle and adding factors for the Fama French model of three factors, five factors and other quantity factors are discussed with the implementations and applications. Overall, these results provide historical summary as well as new insights for further exploration of assets pricing scenarios based on multi factorial concepts.

**Keywords:** CAPM, Fama French, Limitations, Outlook.

## 1. Introduction

Investigating the possible relationships between the return ratios of underlying assets and other relevant factors remain a key issue for financial pricing and forecasting. In retrospect, the oldest model CAPM can be traced back to 1960s by Sharpe and Lintner, who have been regarded as the foundational building block of modern asset pricing theory [1, 2]. Despite the existence of stocks and option markets since the 17th century with the East India Company, there continued to be a vacancy for a financial instrument that enabled clear testable predictions between risk and return. The ingenuity and genius of CAPM was critical as the mid 20<sup>th</sup> century experienced a stagnation in financial theory regarding decision making and risk reward trade-off. There were few rigorous theories of risk preference in a casual or investment setting prior to the existence of CAPM. One notable study would be the works of Latane's experiment which attempts to capture individual behaviour between hypothetical risky and risk-free payouts [3]. Furthermore, empirical studies of risk and reward using market data was also in its infancy during this period as there lacked sufficient technological power for researchers to collect and process descriptive statistics. More recently, the CAPM model has experienced various testing from a collection of papers to challenge the efficient market foundations as the empirical failures of CAPM contradict its theoretical proposals. Basu found that future returns on high price-earning ratio stocks often exceed CAPM predictions when stocks are separated through price-earning ratios [4]. Bhandari documents that the so called high debt-equity ratios are connected with excessively high returns relative to one's market betas [5].

Perhaps the most influential piece of follow up study that truly complicate modern financial research was conducted by Fama and French in 1992 [6]. It was reasoned that although the two appended factors (i.e., the size and book to market equity) are not overall factors with continuous values, the addition of value and size risk factors adjust for the outperformance tendencies found in previous research to better predict growth and performance. This is then further compounded when Fama and French proposed their enhanced model by adding two additional factors: the introduction of robust minus weak (RMW) and conservative minus aggressive (CMA).

Although full of controversy, CAPM is no doubt the fundamental equation in modern finance. This paper seeks to provide a holistic outlook of the CAPM model from its origination to its current

positions in finance through analysing its performance in initial stages and how improvements were made to better suit the current financial atmosphere against the criticism and scrutiny. The paper will start by examining the foundational CAPM model, introducing its core conceptualisations and experimental data. Then it shall proceed to the Fama French model where one studies and explores the most well known 3 factor and 5 factor variations with experimental statistics and move on the relatively extraneous N factor models. Eventually, this paper will conclude on the discussion of CAPM models limitations and what one can expect in future outlook for such models.

## 2. CAPM Model

To derive and understand the equation measuring the CAPM, one must first recognise the critical assumptions made under this theory. Firstly, investors are assumed to be risk averse. Investment choices are analysed, evaluated and compared solely on expected return and standard deviation between different stock portfolios. Secondly, markets are assumed to be strongly efficient thus information are highly transparent to bring all investors at the same conclusion. Thirdly, capital market assets are infinitely divisible with no transaction or short selling costs. These assumptions, although highly impractical and idealised, are the foundations to derive CAPM. Given prevailing prices, investors are tasked with determining the highest Sharpe Ratio portfolio. The formula of the CAPM model is given as follows

$$E(R) = R_f + B(E(R_m) - R_f) \quad (1)$$

Here,  $E(R)$  is the expected return of certain underlying assets or combinations of portfolios;  $R_f$  is the risk-free rate;  $B$  corresponds to the sensitivity of the market; and  $E(R_m)$  represents for the return of the market. The CAPM model focuses on two important parts to calculate the expected return, overall equity market's risk premium. Another way to think about the risk premium is as a perfectly correlated index.

**Table 1.** Descriptive statistics for monthly returns on the  $q^5$  factors

Variable	Mean	Median	1st percentile	99th percentile	Std dev.	SR	p(1)
Mkt (%)	0.503 (2.806)	0.871	-11.792	11.132	4.481	0.112	0.075
$r_{ME}$ (%)	0.288 (2.346)	0.194	-6.442	7.681	3.064	-	0.032
$r_{I/A}$ (%)	0.374 (4.968)	0.320	-4.097	5.195	1.879	-	0.096
$r_{ROE}$ (%)	0.547 (5.450)	0.653	-7.724	7.424	2.508	-	0.100
$r_{Eg}$ (%)	0.834 (11.061)	0.743	-3.939	5.742	1.883	-	0.098

The prominence of CAPM is largely due to its theoretical implications. Firstly, the introduction of beta presents risk measurement of an asset which is undiversifiable. That means the introduction of new stocks cannot diversify the risks away. Secondly, it demonstrates what an asset's expected return is not influenced by. Although a high beta stock often possesses high stand alone risk. As a matter of fact, the stock's expected return does not depend on the growth rate. Thus, this saves the trouble of extensive financial analysis and greatly simplifies the process of understanding each companies profitability as they only need to know parameters such as beta or market risk premium, which are comparatively easily accessible.

**Table 2.** Correlations between the  $q^5$  factors

	$r_{ME}$	$r_{I/A}$	$r_{ROE}$	$r_{Eg}$
Mkt	0.272	-0.382	-0.211	-0.458
$r_{ME}$	-	-0.138	-0.312	-0.369
$r_{I/A}$	-	-	0.036	0.345
$r_{ROE}$	-	-	-	0.504

More recently, there has been further extensive research into CAPM. Hundal, Eskola and Tuan examine the realised return of Finnish companies using CAPM to highlight over/under performances [8]. They have identified that the risk-return relationships are correlated and sample firm returns are documented to be much less volatile and deviating than the market index. Qin tests the consistency

of the q5 with Merton's intertemporal CAPM framework to conclude that it does not suffice to be theoretically applicable for the q5 model [9]. A typical sample of statistics for the return and factors is shown in Table. 1 and the correlations between the factors are given in Table. 2. Using time series tests, they show descriptive statistics of the market risk free, size, investment, ROE and expected growth factors. They find that the investment factor shows a significant ability for forecasting decline which aligns with theories. The analysis results that describe cross sectional average returns demonstrates an inability to foresee long term investment opportunities with high win ratios.

### 3. FF3

Beyond the CAPM model is the Fama French 3 Model, which is proposed by Nobel Laureates Eugene Fama and Kenneth French in 1992. In a 1988 paper, Fama and French found that growth stocks lag behind value stocks and large cap stocks are lower than small cap stocks in stock returns [10]. Hence, portfolios with an abundance of large cap or growth stocks would be higher than the CAPM result. The importance of Fama French and what the CAPM failed to facilitate was that it demonstrated investors must be able to account for periodic volatility and underperformance.

In contrary to CAPM, the Fama French model breaks down market pricing differently. They find that systematic prices are often attached to risk assessments and that the combination of market, sizing and value factors most adequately explain stock performance and pricing. Generally speaking, investor returns convey a firm's cost of capital which can be most accurately estimated by the price of their securities. Therefore, distressed and undersized firms experiencing operational challenges must incur a payment premium for capital when borrowing or issuing as negative signals in areas of prospects, performances, earnings or management are all detrimental to potential investors. Thus to compensate for this risk and induce investor interest, they are forced to bring down their stock prices for these risks. Fama French found that the most adequate measurement for this phenomenon was the ratio of between the market price and the adjusted book value as stocks with high BTM are value stocks and vice versa. Furthermore, depending on how each investor weights their portfolio relative to the total market weighted portfolio, there will be a different premium for including stocks that are smaller or larger than the market, and/or a preference towards value or growth then the market. Therefore, the formula becomes

$$R = R_f + B1(Mkt - R_f) + B2(SMB) + B3(HML) + \alpha \quad (2)$$

which is the sum of risk-free return, the market, size and value premium's and the impact of extraneous random errors. Blanco tests the expected stock return of a portfolio selection according to CAPM and FF3 in the American NSYE market with a database range starting at July 1926 to January 2006 [11]. The results indicate that FF3 is better than CAPM when it comes to measuring and explaining the expected returns of a portfolio. However, the author shows that the results largely fluctuate with how investors form their investment portfolios with different size and value firms. Jan, Iqbal and Aamir investigate return differences, accuracy and reliability of CAPM and FF3 models using time series descriptive data set from January 2014 to December 2018 to analyse yield spread estimates for 50 listed companies from the list of KSE 100 [12]. It is found that the additional factors of FF3 model provides more significant results than single factor models. In predicting stock performances.

### 4. FF5

Since the introduction of the FF3, the framework to optimise stock return expectations have undergone many alterations and evolutions. In 2014, Fama and French updated their model adding two more factors in an attempt to further predict asset returns. The Fama French five factor model derives its theoretical implications from the dividend discount model which brings the foundational understanding that future dividends determine current stock value. From this understanding, Fama

French derived two new factors, investment and profitability. the equation of the five factor model time series regression is given as

$$R_{it} - R_{ft} = a_i + b_i(R_{Mt} - R_{ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it} \quad (3)$$

Here,  $R_{ft}$  is the specific month  $t$  stock return of the portfolio,  $R_{ft}$  is the risk free investment rate,  $R_m - R_f$  is the difference between the average risk and the risk free return, commonly known as the market risk premium,  $SMB$  is the size effect which calculates difference small minus large stocks,  $HML$  is the value effect,  $RMW$  is the value between the most and least profitable firms,  $CMA$  equals firms that invest conservatively minus aggressively.

Despite the addition of two more factors, Fama French 5 has encountered several problems of its own. Firstly, the 5-factor model continues to analyse the CAPM relation between return and market beta, implying market beta is positively and linearly related to expected return. Fama and French attempts to validate this issue and argue the their 5-factor model has great explanatory power for the low beta anomaly. However, this conclusion is at best deduction as evidence for direct correlation is still absent. Secondly, the 5-factor model falls short of providing an explanation to the existing momentum premium. Due to the momentum factor's importance and wide application, most studies choose to report 3/5-factor alphas, but also 4/6- momentum augmented factor alphas. However, no models including those that account for standard momentum factors are able to explain the related idiosyncratic momentum effects [13, 14]. Thirdly, the two new factors may not be robust. Linnainmaa and Roberts also discussed robustness concerns as their study finds little empirical evidence for the two factors before 1963 and between other asset classes [15].

## 5. FFN

Aside from the most notable derivative of Fama French 5 from the original Fama French 3 model, financial economists worldwide have sought to create different variations of this Fama French foundation, which is generally known as the Fama French N group. The reason for such variations is simple. Although the initial model provided a holistic basis to estimate stock returns, it's over generality from solely American stocks undermines its precision thus renders it unapplicable universally. When examining different markets, effective variables are often different, thus to improve the models' accuracy additional factors must be included/excluded for better performance measurement.

The most notable N factor model would likely be Mark Carhart's four factor model [16]. It was a practical extension of the FF3 model proposed in 1997 which includes an additional cross sectional momentum factor founded by Jegadeesh and Titman that considerably improves the multifactor model's explanatory power [17]. The fourth factor is the Monthly Momentum Factor (MOM), which measures the speed/velocity of price changes in any stock examined. It is calculated through finding the difference between the highest performing firm based on rolling window with shifting one lagging month. The full Carhart model is as follows

$$r = r_f + B_1 \times Mkt + B_2 \times HML + B_3 \times SMB + B_4 \times MOM + e \quad (4)$$

In contrary, there are also examples where adding additional sources of momentum is counter effective. Dirks and Peter found that implementing a FF5 plus momentum factor model in the German market in the analysis is non explanatory. It was concluded that the relevance of the profitability and investment factors are nation and country specific which cannot be utilised internationally [18]. Additionally, Foye, Mramor and Pahor uses a slightly different Fama French 3 model to explain returns of underlying assets in developing/newly developed European countries that joined the EU in 2004 [19]. It was found that the market value factor in the FF3 model was not an adequate variable and performed poorly when used in the context of developing markets. Thus, the researchers replaced the market value of equity factor with cash flow from operating activities to test its explanatory power for stock prices. It was found that replacing the market value of equity factor with cash flow from operating activities yields greatly statistically significant results.

## 6. Limitations & Prospects

CAPM, although astounding in its theoretical importance and foundation, has never been empirically successful since its original introduction. Most empirical test of the CAPM have had trouble explaining the past and future, some even noticed a pattern of divergences. That is because CAPM is built upon several unrealistic assumptions which undermine its authenticity in a practical environment.

Firstly, the CAPM model operates on a single period basis. This means that unlike practical investors who seek maximisation of lifetime consumptions, the model only calculates the optimal wealth a portfolio produces at the end of this period. Investors should not all have homogeneous expectations and follow a path of absolute rationality like a Homo-economicus based on the return and volatility. Moreover, it is unrealistic that the market is not transparent and information is available, a exact risk-free underlying asset also does not exist. CAPM operates on a highly idealised version of the financial market which is nor interfered by any extraneous variables.

Secondly, empirical research finds the relationship between the variables is flatter than predicted by the CAPM. Hence, all CAPM estimates of the cost of equity are blown to disproportionate amounts when it should be much lower. However, since CAPM has inherently empirical failings, it is common for stock portfolios produce abnormal returns, which brings larger than normal returns on paper when in turn the practicality is questionable.

The future of CAPM is clearly still promising. What financial economists must focus on is the addition of effective explanatory variables that better describe the financial atmosphere being practiced today. Finance indicators, technical analysis indicators and industry dispersion indicators are just some of many that could provide better predictions of stock prices. Having said that, when employing additional variables, one must take into account the possibilities of multicollinearity, so as to ensure that stock price predictions are not plagued by multiple overlapping factors. Secondly, when analysing the effectiveness of the model, one could consider the frequency of data and the effectiveness of the market. The current research on CAPM is far too saturated on the major aspects of the equations and in order to make continuous compounding improvements, it is pivotal that emphasis is placed on these minor details.

## 7. Conclusion

In summary, even since its introduction of CAPM has been the fundamental founding stone of modern financial theory. The CAPM was the first that explicitly made relations between portfolio expected returns and market risk expectations. As a matter of fact, CAPM was also the theoretical basis for many newer theories. Whether it was ICAPM or Fama French and its different models all diverge from its internal core understanding of risk reward exchange. Some later models are more effective than others, and despite its convoluted empirical experiences, CAPM has largely shaped the way people view expected return and risk. Investors now learn to allocate their portfolios using diversification and performance measurement and capital budgeting has never been more straight forward. Although CAPM is plagued with errors and empirical inconsistencies, it does not prevent it from exercising its purpose as it continues to be taught at all entry level finance courses around the world for people to set a foundation.

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