The Impact of Factors in Capital Assets Pricing Model and Fama-French Models

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Abstract. Despite the fact that other researchers have looked into hundreds of potential factors that could affect equity returns, Fama and French initially proposed three and have since allowed for five, with the sporadic appearance of a sixth factor. By using regression which is a statistical technique that is used to isolate and quantify the significance of a variable. It works as a test to ascertain whether a stock's average returns may be influenced by factors like leverage or sector performance. This study uses R programming language and the “lm” function to research the impacts of factors in CAPM and different factors of Fama-French Models. To conclude, more factors included in the Fama-French model are not better than those with fewer factors, and there is a multicollinearity problem for different portfolios. This study aims to help other researchers have a brief consideration before using the model.

Keywords: CAPM; Fama-French Models; Impact; Factors.

1. Introduction

The fundamental tenet of investing is that the return and risk of financial assets should be comparable. The return on stocks has attracted attention as a crucial area of financial economics and has long been a central concern of the investing sector. For this aim, there are three primary alternatives: the first is Capital Asset Pricing Model proposed by Sharpe and Lintner in 1964, Over time, plenty of critiques of CAPM have surfaced, and several writers have suggested substitute models as a way to make it better. The intertemporal capital asset pricing model (ICAPM), developed by Merton in 1973 to account for the multi-period component of financial market equilibrium, is one example of these CAPM modification models [1].

The second is the three-component model proposed by Fama and French in 1992. As the three-factor model was insufficient for estimating expected returns because its three factors overlook a significant portion of the variation in average returns related to profitability and investment, the Fama-French five-factor model added two factors. Fama and French developed profitability and investment in 2015. Instead of the normal CAPM, three factors Fama-French Model and 5-factors Fama-French Model, there are also some unique Fama-French Models with different combinations of factors. In 1993, Jegadeesh and Titman discovered that the market-style-size model was improved when a fourth element, momentum, was included. Investing in companies whose prices have grown and sold companies whose prices have previously declined is how momentum is assessed (winners minus losers). Together, the market, style, size, and momentum components make up today's Fama-French 4 Factor Model. From the perspective of this essay, the factors indeed impact the correlation and fitness of the data. There is another 6-factors Fama-French Model with the facto MoM (Momentum). [2].

Instead of typical stocks, the CAPM and Fama-French models may be used to analyse different industries, such as green energy. The yield to maturity of a renewables portfolio is contingent on the returns and risks of the wider market when the CAPM model is applied to green energy companies. Since returns are negatively correlated with risk, returns rise when risk exceeds the risk-free rate. When the market is typically losing money, an investment portfolio of green energy equities with a negative beta yields excess returns. Since these businesses are still in the early stages of their trends and are not part of the broader market, the stocks of wind or electric vehicle firms are probably making excess returns. Green stock investors might seek gains in hazardous green stock startups to increase their [3] investments' cash on hand or access to risk-free credit while reducing their overall portfolio of risky holdings. Certain businesses, like Rivian Automotive, will proliferate while generating value or having a high book-to-market ratio. Other businesses, like Bright Automotive, Coda, and Aptera, might enter the renewable energy industry as glitzy stocks, fail to live up to initial expectations, and leave the market. The market returns (R_m), market risk sensitivities (β), book-to-market ratio, fixed investments, and profitability variances influence green energy stock portfolio's surpluses, among other things. For the international renewables equities portfolio, the period 2010–2019 [4].

This study will focus on the impacts of factors for both CAPM and Fama-French Models[5], using the data from the Kenneth website with US market returns and different portfolios to check the importance of the factor by using the R programming language. Moreover, testing the hypothesis that more is not the best for the Fama-French Model. Also, the questions occurred about multicollinearity.

2. Methodologies

2.1 The Capital Asset Pricing Model

A unique, one-factor portfolio model called the capital asset pricing model(CAPM) demonstrates the relationships between the return on securities and the risks. The estimated coefficient for security is calculated by measuring the values of the reduction in the value of the securities relative to the market value by adding a portion of the lowest possible return, the risk-free interest rate, and the return of the securities that are higher than the market value.

The formula for the CAPM is:

$$R_j = R_f + b_j(R_m - R_f) \tag{1}$$

Where R_j means the expected return on a stock/security; R_f means the risk-free rate; b_j means the beta coefficient; R_m means the market return; $R_m - R_f$ means the excess return on market.

The index fund's beta is a risk factor per return on investment and sensitivity to portfolio returns. Thus, beta evaluates a strategy manager's capacity to produce profits that are comparatively larger than those of the market as a whole. Since it is improbable that all riskier investors will build comparable portfolios, some investors may maintain different portfolios from the typical riskier ones and lower their risks by taking out loans or other less hazardous borrowing arrangements [5].

2.2 The Fama-French Model

2.2.1 Three Factors Fama-French Model

The three factors Fama French Model was created in 1992 by Eugene Fama and Kenneth French[6,7]. The observation that small-cap stocks typically outperform large-cap stocks and that value stocks frequently outperform growth stocks impacts it. The two economists developed their three-factor model by extending the Capital Asset Pricing model in light of their findings (CAPM).The Fama-French Three Factor model incorporates size risk, and value risk into the calculation rather than only measuring market risk like the CAPM does. This includes factorsR_m, SMB, and HMLwhich are built from six size/book-to-market portfolios. Rm is the excess return on the market [8], which is calculated by subtracting the one-month Treasury bill rate from the value-weighted return on all NYSE, AMEX, and NASDAQ [9]equities and is used to calculate the risk premium (R_f). SMB (Small minus Big) is the average return on three small portfolios minus the average return on three big portfolios with the formula: [10]

$$SMB_{B/M} = 1/3(SmallValue + SmallNeutral + SmallGrowth) - 1/3(BigValue + BigNeutral + BigGrowth) \tag{2}$$

HML (High minus Low) is the average return on two value portfolios minus the average return on two growth portfolios, the formula for HML is:

$$HML = 1/2(SmallValue + BigValue) - 1/2(SmallGrowth + BigGrowth) \quad (3)$$

The overall three factors Fama-French Model formula :

$$Return = R_f + \beta_1(R_m - R_f) + \beta_2(SMB) + \beta_3(HML) + \alpha \quad (4)$$

Where $\beta_1\beta_2\beta_3$ is the market coefficient for each factor in the model; R_m is the total market return; α is the investment's alpha.

2.2.2 Five Factors Fama-French Model

The 5-factors Fama-French model criteria are derived from the six value-weighted portfolios depending on size and book-to-market, size and operating profitability, and size and investment.

SMB(Small Minus Big) is calculated as the average return on these nine minor stock portfolios divided by the average return on the nine big stock portfolios. The formulas for SMB are:

$$SMB_{B/M} = 1/3(SmallValue + SmallNeutral + SmallGrowth) - 1/3(BigValue + BigNeutral + BigGrowth)$$

$$SMB_{OP} = \frac{1}{3(SmallRobust + SmallNeutral + SmallWeak)} - \frac{1}{3}(BigRobust + BigNeutral + BigWeak) \quad (5)$$

$$SMB_{INV} = 1/3(SmallConservative + SmallNeutral + SmallAggressive) - 1/3(BigConservation + BigNeutral + BigAggressive) \quad (6)$$

$$SMB = 1/3(SMB_{B/M} + SMB_{OP} + SMB_{INV}) \quad (7)$$

HML (High Minus Low) is calculated as two value portfolios' average returns divided by two growth portfolios' average returns, the formula for HML is:

$$HML = 1/2(SmallValue + BigValue) - 1/2(SmallGrowth + BigGrowth)$$

RMW is the average return on the two operational profitability portfolios that are both strong and is subtracted from the average return on the two operating profitability portfolios that are both weak.

$$RMW = 1/2(SmallRobust + BigRobust) - 1/2(SmallWeak + BigWeak) \quad (8)$$

The difference between the average returns on the two conservative investment portfolios and the average returns on the two aggressive investment portfolios is used to compute CMA (Conservative Minus Aggressive) [11,12]

$$CMA = 1/2(SmallConservation + BigConservation) - 1/2(SmallAggressive + BigAggressive) \quad (9)$$

Hence, the regression equation for the Five factors Fama-French Model is:

$$Return = R_f + \beta_1(R_m - Rf) + \beta_2(SMB) + \beta_3(HML) + \beta_4(RMW) + \beta_5(CMA) + \alpha \quad (10)$$

There are two other Fama-French Models which are four factors and six factors.

2.2.3 Four Factors Fama-French Model

For this study, the Four Factors Fama-french Model is the different combination of excess market return,$SMB, HML, RMW, CMA, and MOM$. The MOM factor is called Momentum, and it is equal to the average return on the two portfolios with high past returns minus the average return on the two portfolios with poor prior returns.The formula for MOM is: [13]

$$MOM = 1/2(SmallHigh + BigHigh) - 1/2(SmallLow - BigLow) \tag{11}$$

2.2.4 Six Factors Fama-French Model

The momentum element, first presented by Carhart (1997), completes Fama and French's FF5 and transforms it into the Fama and French Six-Factor Model:

$$Return = \beta_1(R_m - R_f) + \beta_2(SMB) + \beta_3(HML) + \beta_4(RMW) + \beta_5(CMA) + \beta_6(MOM) + \alpha \tag{12}$$

3. The impact of factors in CAPM and Fama-French Models

3.1 US market returns

3.1.1 Data

The data of monthly US market returns of Fama-French three factors model from Kenneth French's website (1926. 07 — 2022.08).

3.1.2 Process

By using the R programming with the function "*lm*" to test the three factors Fama-French Model on the US market.

3.1.3 CAPM model for US market returns

By running the code in R for the CAPM model with the market excess return which is shown by Mkt, it gives the following results in table 1:

Table 1. Results of CAPM

Call: lm(formula y ~ Mkt)				
Residuals:				
Min	1Q	Median	3Q	Max
-1.08183	-0.15370	0.04617	0.23220	0.32891
Coefficients				
	Estimate	Std. Error	Tvalue	Pr(> |t|)
(Intercept)	-0.267195	0.007534	-35.47	<2e-16 ***
Mkt	1.000963	0.001392	719.28	<2e-16 ***
Residual − standard − error: 0.2521 on 1152 degrees of freedom				
MultipleR^2: 0.9978		AdjustedR^2: 0.9978		
F − statistic: 5.174e+05 on 1 and 1152 DF			p − value: < 2.2e-16	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

3.1.4 Three-Factors Fama-French Model applied to US market returns

Table 2 shows the results.

Table 2. Results of Three-Factors Fama-French Model

$Call{:}\ lm(formula = y$ ~ Mkt + SMB + HML)				
$Residuals$:				
Min	$1Q$	$Median$	$3Q$	Max
-1.06863	-0.15245	0.05014	0.22838	0.33528
$Coefficients$				
	$Estimate$	$Std.Error$	$Tvalue$	$Pr(>\|t\|)$
(Intercept)	-0.266832	0.007543	-35.375	<2e-16 ***
Mkt	1.000557	0.001498	668.090	<2e-16 ***
HML	-0.002094	0.002143	-0.977	0.329
SMB	0.003897	0.002466	1.580	0.114
Residual standard error: 0.2519 on 1150 degrees of freedom				
$MultipleR^2$: 0.9978		$AdjustedR^2$: 0.9978		
$F - statistic$: 1.727e+05 on 3 and 1150 DF			$p - value$: < 2.2e-16	

*Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1*

3.2 Discussion about the CAPM and three factors Fama-french Model in the US market

Here is the difference between Multiple R-squared and Adjusted R-squared. R Square is a simple matrix that indicates how much variation is accounted for by the model. In multivariate linear regression, the R square value always rises when more factors are added, regardless of the relevance of the new variables. Calculating R square from only the variables included in the model that has a substantial impact is what modified R square does. Therefore, it is more frequently to consider adjusted R square rather than R square when doing a multivariate linear regression.

Hence, for analysing the differences and suitable for CAPM and Fama-French Model, it is important to analyse the adjusted R-squared number and regression standard error. As is well knowledge, two important goodness-of-fit metrics for regression analysis are the standard error of the regression and R-squared. The percentage of the variation of the dependent variable that the model explains is relative measured by the R-squared. The other one — standard error of regression described how distant the average value in a data set is from a regression line of the data as a logical regression measure to the data set.

As shown in table 1, the useful information in the CAPM model is the residual standard error (0.2521) and adjusted R-squared (0.9978). From table 2 the Residual standard error is 0.2519 and the adjusted R-squared is 0.9978. Compared with those two pieces of information together, we can make a conclusion that the three-factors Fama-French Model is better since the residual standard error is smaller which is 0.2519 based on the same adjusted R^2 (0.9978). And the model is more suitable with more factors included.

Based on the conclusion by the US market returns, this study set a hypothesis on the application of models and the impacts of factors.

Hypothesis: The Fama-French model with more suitable factors has larger adjusted R-squared in the portfolios than other models.

4. Discussion on the impacts of factors in the Fama-French Model

4.1 Data

By using another set of data also from the Kenneth website to test the impact of factors in the different factors Fama-French Models.

4.2 Explain each factor in the Fama-French Model with different combinations and their risks

4.2.1 Process

In this study, by using the difference combinations of those six factors— Mkt, SMB, HML, RMW, CMA and MOM. Therefore, for the six factors Fama-French Model, the formula is the regression equation for these six factors (table 3). See the formula (13).

Table 3. 6 factors combination in portfolio 3

$Call: lm(formula$ =y3 ~ Mkt + SMB + HML + RMW + CMA + MOM)				
$Residuals$:				
Min	$1Q$	$Median$	$3Q$	Max
-1.92675	-0.38358	-0.01724	0.36194	2.85422
$Coefficients$				
	$Estimate$	$Std.Error$	$Tvalue$	$Pr(>\|t\|)$
(Intercept)	0.042534	0.023729	1.793	0.0735
Mkt	1.000190	0.005864	170.577	<2e-16 ***
HML	0.532992	0.011536	46.201	< 2e-16 ***
SMB	0.879050	0.008114	108.336	< 2e-16 ***
RMW	0.060741	0.011441	5.309	1.50e-07 ***
CMA	0.074698	0.016763	4.456	9.78e-06 ***
MOM	-0.002813	0.005608	-0.502	0.6161
$Residualstandarderror$: 0.5795 on 668 degrees of freedom				
$MultipleR^2$: 0.9892		$AdjustedR^2$: 0.9891		
$F-statistic$: 1.021e+04 on 6 and 668 DF			$p-value$: < 2.2e-16	

Note:$Signif.codes$: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

For the Five-factors Fama-French Model, choosing the 5 factors with the formula $6C_5$ = 6, so there are 6 different combinations in the 5-factors Fama-French Model, shown in table 4 and 5.

Table 4. Example of 5-factos combination in portfolio 3

$Call: lm(formula$ =y3 ~ Mkt + SMB + HML + RMW + CMA)				
$Residuals$:				
Min	$1Q$	$Median$	$3Q$	Max
-1.83893	-0.38423	-0.01807	0.35991	2.86213
$Coefficients$				
	$Estimate$	$Std.Error$	$Tvalue$	$Pr(>\|t\|)$
(Intercept)	0.040503	0.023368	1.733	0.0835
Mkt	1.000588	0.005806	172.327	<2e-16 ***
HML	0.534497	0.011133	48.009	< 2e-16 ***
SMB	0.878885	0.008103	108.466	< 2e-16 ***
RMW	0.060095	0.011362	5.289	1.67e-07 ***
CMA	0.073679	0.016630	4.430	1.10e-05 ***
$Residual - standard - error$: 0.5791 on 669 degrees of freedom				
Multiple R^2: 0.9892		$AdjustedR^2$: 0.9891		
$F - statistic$: 1.226e+04 on 5 and 669 DF			$p - value$: < 2.2e-16	

Table 5. Summary Table for five factors models in portfolio 3

$Regressor$	$Model1$	$Model2$	$Model3$	$Model4$	$Model5$	$Model6$
Mkt	1.000588	0.9924027	0.9947485	1.00350	1.11735	
SMB	0.878885	0.8766325	0.8656146	0.90037		1.13432
HML	0.534497	0.5679609	0.5430266		0.60406	0.55703
RMW	0.060095	0.0500603		0.14734	-0.32584	-0.28041
CMA	0.073679		0.0560535	0.60151	-0.04673	-0.77752
MOM		0.0002174	0.0005386	-0.07019	0.02185	-0.13230
Intercept	0.040503	0.0578205	0.0642153	0.07710	0.27647	0.91751
Summary Statistics						
SER	0.5791	0.5876	0.5911	1.186	2.495	3.865
AdjustedR^2	0.9891	0.9888	0.9887	0.9544	0.7981	0.5156
N	675	675	675	675	675	675

For the Four-Factors Fama-French Model, choosing 4 factors and the number of combinations is $6C_4$ = 15. As shown in table 6-8.

Table 6. Summary Table for four factors model in portfolio 3

Regressor	Model1	Model2	Model3	Model4	Model 5
Mkt	0.992363	0.989388	0.99009	1.12434	
SMB	0.876643	0.865571	0.86788		
HML	0.567880	0.569087			0.65572
RMW	0.050100			-0.23803	-0.85019
CMA			0.57975	0.54891	-1.07308
MOM		0.002454	-0.06500	-0.05408	-0.11961
Intercept	0.057996	0.073220	0.13277	0.32222	1.36692
Summary Statistics					
SER	0.5871	0.5956	1.221	2.756	5.006
AdjustedR^2	0.9888	0.9885	0.9517	0.7537	0.1875
N	675	675	675	675	675

Table 7. Summary Table for four factors model in portfolio 3

Regressor	Model 6	Model 7	Model 8	Model 9	Model 10
Mkt		1.01440	1.11442		0.89004
SMB	1.13432	0.89756		1.13127	0.88509
HML	0.55703		0.59245	0.62959	
RMW	-0.28041	0.13660	-0.32138	-0.31786	0.07242
CMA	-0.77752	0.61403	-0.03898	-0.84262	
MOM	-0.13230				-0.09103
Intercept	0.91751	0.02534	0.29262	0.83686	0.34018
.Summary Statistics					
SER	3.865	1.22	2.495	3.899	1.614
$\boldsymbol{AdjustedR^2}$	0.5156	0.9517	0.7982	0.5071	0.9155
N	675	675	675	675	675

Table 8. Summary Table for four factors model in portfolio 3

Regressor	Model 11	Model 12	Model 13	Model 14	Model 15
Mkt	1.12244		1.160325		
SMB		1.18464		1.20491	1.15748
HML	0.58221	0.15970	0.550886	0.50987	
RMW	-0.31979	-0.18873			-0.19107
CMA			0.053031	-0.71081	-0.22981
MOM	0.01999	-0.17843	0.004175	-0.15161	-0.20317
Intercept	0.26727	0.82072	0.168879	0.83694	0.95667
Summary Statistics					
SER	2.494	4.013	2.582	3.902	4.011
$AdjustedR^2$	0.7983	0.4779	0.7838	0.5062	0.4784
N	675	675	675	675	675

For the three-factors model, it is combined by three factors picked from those 6- factors. Therefore, the total number of combinations is $6C_3$ = 20. As shown in table 9-11:

Table 9. Summary Table for three factors in portfolio 3

Regressor	Model1	Model2	Model3	Model4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10
Mkt	0.988910	0.90119	1.11894		0.88538	1.155246		1.15604		
SMB	0.865591	0.88101		1.18626	0.86910		1.23054		1.20554	
HML	0.568179		0.57480	0.21429		0.575540	0.15065			
RMW		0.05636	-0.31658	-0.23011						-0.75839
CMA								0.58432	-0.21557	-0.43312
MOM					-0.08805	0.005987	-0.18907	-0.06231	-0.21250	-0.20304
Intercept	0.075358	0.27978	0.28367	0.69741	0.36329	0.177394	0.77067	0.23870	0.89793	1.42403
Summary Statistics										
SER	0.5953	1.656	2.494	4.075	1.619	2.581	4.028	2.798	4.026	5.162
$AdjustedR^2$	0.9885	0.911	0.7984	0.4615	0.915	0.784	0.474	0.7461	0.4744	0.1361
N	675	675	675	675	675	675	675	675	675	675

Table 10. Summary Table for three factors in portfolio 3

$Regressor$	Model 11	Model 12	Model 13	Model 14	Model 15	Model 16	Model 17	Model 18	Model 19	Model 20
Mkt	1.00115	1.13246			1.01878		1.15965			
SMB	0.86747							1.21237	1.15743	1.17756
HML			0.72110	0.10410		0.51335	0.54878	0.58731		
RMW		-0.2454	-0.88268	-0.75699	-0.30053				-0.23369	-0.17447
CMA	0.59289	0.55869	-1.13123			-0.90539	0.05427	-0.77649	-0.21951	
MOM				-0.18359	-0.07337	-0.18391				-0.20055
Intercept	0.08079	0.28172	1.29289	1.25902	0.55885	1.18433	0.17228	0.73009	0.83194	0.88257
Summary Statistics										
SER	1.249	2.763	5.025	5.226	2.93	5.318	2.58	3.948	4.096	4.034
$AdjustedR^2$	0.9494	0.7525	0.1812	0.1145	0.7216	0.0829	0.7841	0.4946	0.4558	0.4724
N	675	675	675	675	675	675	675	675	675	675

4.2.2 Checking Process

By using the *"step"* function in R to run the most suitable model, the results are shown in table 11 -12:

Table 11. Checking table for portfolio 3-process 1

Start: AIC = -729.66				
Y3~ Mkt + SMB + HML + RMW + CMA + MOM				
	Df	**Sum of Sq**	**RSS**	**AIC**
MOM	1	0.1	224.4	-731.40
<none>			224.3	-729.66
CMA	1	6.7	231.0	-711.89
RMW	1	9.5	233.8	-703.76
HML	1	716.7	941.0	236.29
SMB	1	3941.0	4165.3	1240.38
Mkt	1	9770.1	9994.4	1831.17

Table 12. Checking table for portfolio 3-process 2

Start: AIC = -731.4				
Y3~ Mkt + SMB + HML + RMW + CMA				
	Df	**Sum of Sq**	**RSS**	**AIC**
<none>			224.4	-731.40
CMA	1	6.6	231.0	-713.88
RMW	1	9.4	233.8	-705.75
HML	1	773.1	997.5	273.59
SMB	1	3946.0	4170.4	1239.21
Mkt	1	9960.4	10184.8	1841.91

4.2.3 Explanation and Discussion

For portfolio 3, the R-squared of the 6-factor model (Mkt, SMB, HML, RMW, CMA, MOM) by the console is 0.9891 as shown in Table3, which is the same as the 5-factor model (Mkt, SMB, HML, RMW, CMA) as shown in Table 5 Model 1. Therefore, the assumption for portfolio 3 is that the 5-factors model is the most suitable one, and the factor MOM does not become relevant to each other.

Firstly, the Mkt (excess return on the market) is the most critical factor as the R-squared of the 5-factors model (SMB, HML, RMW, CMA, MOM) is 0.5156 as shown in Table 5 model 6, which is the lowest among other combinations, and this indicates that the Mkt is the most influential factor to the model for portfolio 3.

By using the same method and comparing each R-squared with 0.9891, the sequence of the importance of the factors from high to low is Mkt > SMB > HML > RMW > CMA > MOM from Table 5.

Secondly, the R-squared of all the combinations in the 4-factors model and 3-factor model are all lower than 0.9891, which concluded that the 5-factor model with 0.9891 is the best. Moreover, to check the importance of factors, keep three factors the same and check the other one by comparing which also provides the same result. For example, the combination in model 6 in Table 7 (Mkt, SMB, HML, CMA, except RMW and MOM) has adjusted R-squared 0.9887, which is more significant than that in Table 6 Model 2 (Mkt, SMB, HML, MOM, except RMW and CMA, 0.9885), this comparison concludes that CMA is more relative than MOM.

Furthermore, use the step function to analyse portfolio three and check the assumptions, which shows that the MOM is useless and the importance is Mkt > SMB > HML > RMW > CMA with their AIC from high to low.

Finally, the conclusion of portfolio 3 is that the 5-factors model with the combination of Mkt, SMB, HML, RMW, and CMA is the most suitable, and the factors that influence the returns on this portfolio are that Mkt > SMB > HML > RMW > CMA > MOM from high to low.

4.2.4 Same methodology and process for Portfolio 4

The results of Portfolio is shown in table 13-19.

Table 13. results of Portfolio 4

Regressor	Model
Mkt	0.995815
SMB	-0.107385
HML	-0.269704
RMW	0.160031
CMA	-0.021850
MOM	-0.005140
Intercept	0.077479
Summary Statistics	
SER	0.6124
Adjusted R-squared	0.9819
N	675

Table 14. 5-Factors Model for portfolio 4

$Regressor$	$Model1$	$Model2$	$Model3$	$Model4$	$Model5$	$Model6$
Mkt	0.996543	0.998093	0.981478	0.994141	0.981503	
SMB	-0.107687	-0.106678	-0.142783	-0.118172		0.14677
HML	-0.266954	-0.279932	-0.243267		-0.278385	-0.24577
RMW	0.158850	0.163155		0.116210	0.207255	-0.17962
CMA	-0.023713		-0.070972	-0.288425	-0.007015	-0.87034
MOM		-0.006027	0.003690	0.028955	-0.008153	-0.13406
Intercept	0.073766	0.073008	0.134600	0.059987	0.048902	0.94863
Summary Statistics						
SER	0.6123	0.6127	0.6875	0.8055	0.68	3.854
$Adjusted R^2$	0.9819	0.9819	0.9772	0.9687	0.9777	0.2833
N	675	675	675	675	675	675

Table 15. 4-factors Model for portfolio 4

Regressor	**Model1**	**Model2**	**Model3**	**Model4**	**Model 5**
Mkt	0.999190	0.988266	0.983565	0.978281	
SMB	-0.106965	-0.142728	-0.143797		
HML	-0.277698	-0.276263			-0.23300
RMW	0.162067			0.166789	-0.25335
CMA			-0.305581	-0.281521	-0.90858
MOM		0.001264	0.033050	0.026840	-0.13242
Intercept	0.068136	0.123199	0.103890	0.027816	1.00678
Summary Statistics					
SER	0.6127	0.6938	0.8373	0.8686	3.873
$Adjusted R^2$	0.9819	0.9768	0.9662	0.9636	0.2761
N	675	675	675	675	675

Table 16. 4-factors Model for portfolio 4

Regressor	$Model6$	$Model7$	$Model8$	$Model9$	$Model10$
Mkt	0.980872	0.989642	0.982595		1.048544
SMB	-0.142754	-0.117012		0.14368	-0.110847
HML	-0.245124		-0.274055	-0.17225	
RMW		0.120643	0.205591	-0.21758	0.152137
CMA	-0.069881	-0.293590	-0.009909	-0.93630	
MOM					0.038947
Intercept	0.137606	0.081340	0.042875	0.86690	-0.066162
Summary Statistics					
SER	0.6872	0.8137	0.6803	3.889	0.9612
$AdjustedR^2$	0.9772	0.968	0.9777	0.2703	0.9554
N	675	675	675	675	675

Table 17. 4-fators Model for portfolio 4

Regressor	Model 11	Model 12	Model 13	Model 14	Model 15
Mkt	0.982268		0.954166		
SMB		0.20310		0.19199	0.13655
HML	-0.281666	-0.69053	-0.244563	-0.27598	
RMW	0.208163	-0.07701			-0.21904
CMA			-0.070473	-0.82760	-1.11199
MOM	-0.008432	-0.18570	0.003090	-0.14643	-0.10279
Intercept	0.047520	0.84028	0.117336	0.89702	0.93135
Summary Statistics					
SER	0.6796	4.039	0.8024	3.867	3.88
$AdjustedR^2$	0.9777	0.2128	0.9689	0.2782	0.2734
N	675	675	675	675	675

Table 18. 3-factors Model for portfolio 4

Regressor	Model1	Model2	Model3	Model4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10
Mkt	0.988020	1.043774	0.983745		1.038756	0.9609169		0.956069		
SMB	-0.142718	-0.109100		0.20478	-0.144442		0.22182		0.1916	
HML	-0.276730		-0.278542	-0.63372		-0.277327	-0.69423			
RMW		0.159007	0.206809	-0.12007						-0.28597
CMA								-0.306339	-1.0957	-1.13598
MOM					0.045201	0.0006815	-0.19004	0.032603	-0.1135	-0.10277
Intercept	0.124299	-0.040319	0.040600	0.71195	-0.017617	0.1060209	0.81986	0.086338	0.8640	0.98649
Summary Statistics										
SER	0.6933	0.974	0.6799	4.106	1.008	0.8076	4.039	0.9349	3.902	3.896
$Adjusted R^2$	0.9768	0.9542	0.9777	0.1862	0.951	0.9685	0.2128	0.9578	0.2651	0.2673
N	675	675	675	675	675	675	675	675	675	675

Table 19. 3-factors Model for portfolio 4

Regressor	$Model11$	$Model12$	$Model13$	$Model14$	$Model15$	$Model16$	$Model17$	$Model18$	$Model19$	$Model20$
Mkt	0.977940	0.974252			1.032422		0.953664			
SMB	-0.143588							0.19919	0.13652	0.23371
HML			-0.16063	-0.70006		-0.27543	-0.246119	-0.20119		
RMW		0.170441	-0.28931	-0.17443	0.198844				-0.24061	-0.13871
CMA	-0.312260	-0.286375	-0.97296			-0.85861	-0.069560	-0.89104	-1.10678	
MOM				-0.18658	0.036735	-0.15158				-0.09007
Intercept	0.130318	0.047916	0.92482	0.91542	-0.093548	0.95237	0.119856	0.79382	0.86825	0.57282
Summary Statistics										
SER	0.8478	0.875	3.906	4.076	1.008	3.907	0.8019	3.91	3.901	4.46
$Adjusted R^2$	0.9653	0.963	0.2635	0.198	0.9509	0.2632	0.969	0.2622	0.2656	0.04001
N	675	675	675	675	675	675	675	675	675	675

For portfolio 4, the R-squared of 6-factor model ($Mkt, SMB, HML, RMW, CMA, MOM$) by the console is 0.9819 which is the same as the 5-factor model (Mkt, SMB, HML, RMW, CMA) / (Mkt, SMB, HML, RMW, MOM) and 4-factors model (Mkt, SMB, HML, RMW). Therefore, the assumption for portfolio 4 is that the 4-factors model is the most suitable one and the factor MOM and CMA are not relative.

Here is the checking process of the assumption shown above, results are shown in table 20-22.

Table 20. Checking summary for portfolio 4- process 1

Start: AIC = -654.99			
Y4~ Mkt + SMB + HML + RMW + CMA + MOM			
	Df	$SumofSq$	RSS
MOM	1	0.3	250.8
CMA	1	0.6	251.1
<none>			250.5
SMB	1	58.8	309.4
RMW	1	65.7	316.2
HML	1	183.5	434.1
Mkt	1	9684.8	9935.4

Table 21. process 2

Start: AIC = -654.99			
Y4~ Mkt + SMB + HML + RMW + CMA + MOM			
	Df	**Sum of Sq**	**RSS**
CMA	1	0.7	251.5
<none>			250.8
SMB	1	59.2	310.1
RMW	1	65.6	316.4
HML	1	192.8	443.7
Mkt	1	9880.0	10130.9

Table 22. process 3

Start: AIC = -654.99			
Y4~ Mkt + SMB + HML + RMW + CMA + MOM			
	Df	**Sum of Sq**	**RSS**
<none>			251.5
SMB	1	58.7	310.2
RMW	1	71.0	322.5
HML	1	385.1	636.6
Mkt	1	11063.6	11315.1

Firstly, the Mkt (excess return on the market) is the most crucial factor as the R-squared of the 5-factors model (SMB, HML, RMW, CMA, MOM) is 0.2833 in Table 14 Model 6, which is the lowest among other combinations in the scenario of the 5-factors model, this indicates that the Mkt is the most influential factor to the model.

By using the same method, and comparing each R-squared with 0.9891, the sequence of the importance of the factors from high to low is $Mkt > HML > RMW > SMB > (MOMandCMA)$. Secondly, the R-squared of other combinations in the 4-factors model and 3-factors model are lower than 0.9819.

Moreover, to check the importance of factors, keep three factors the same and check the other one by comparing which also provides the same result. For example, the combination in model 2

($Mkt, SMB, RMW, exceptHML, CMAandMOM$) has adjusted R-squared 0.9542, which is smaller than that in Model 3 in Table 18 ($Mkt, HML, RMWexceptforSMB, CMAandCMA,$ 0.9777), this comparison concludes that HML is more relative than SMB.

Furthermore, use the step function to analyse portfolio four and check the assumptions, which shows that the MOM is useless and the importance is $Mkt > HML > RMW > SMB$with their AIC from high to low.

Finally, the conclusion of portfolio 4 is that the 4-factors model with a combination of $Mkt, SMB, HML, andRMW$ is the most suitable, and the factors that influence the returns on this portfolio are $Mkt > HML > RMW > SMB$ from high to low.

These show that the different factors model needs to consider the correlation between each factor when doing the regression and testing whether a multicollinearity problem appears or not. Through testing the t value and R-squared, if there is multicollinearity, it is crucial to exclude those factors. For example, MOM is irrelevant to portfolio 3, CMA and MOM are irrelevant to portfolio 4, and other factors influence the model more.

5. Conclusion

The common conclusion made by this study and tests of US market returns, portfolio three and portfolio 4 is that the market factor is the most influential factor, which is also the only factor used in the CAPM model. It is essential to consider the multicollinearity problem, as it will produce a non-suitable model for the research. A multicollinearity problem will make the whole combination of some particular factors be calculated several times, impacting the extended performance of the original combinations of the factors. Therefore, before using the model to analyse the stock or other areas, it is crucial to consider and solve this problem.

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