

# Reasonable Conjecture and Demonstration of Fama-French Model and Its Applications in Australia and US

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**Abstract.** Fama-French three-factors model and Fama-French five-factors model have been considered as significant models for calculating stock returns and estimating potential risk events. However, as Fama and French did in 2015, one reasonable conjecture has been come up with: Is there any extra factors could be added into the Fama-French five factors model? This paper raises a reasonable conjecture about the number of factors in Fama-French model and use the method of data analysis process with R software. Then this paper prove that the Fama-French five-factors model is still the most reliable to estimate the returns of portfolios currently based on the statistical data summaries. Besides, this paper also talks about the application and applicability of Fama-French models in Australia and US with some real cases and data analysis process. The result of this study shows that the applicability of Fama-French models is confirmed in both Australia and US. So the investors in Australia and US could use Fama-French models to estimate their expected return for some specific candidate projects and choose the best one to make investment based on the analysis results.

**Keywords:** reasonable conjecture; Fama-French Model; applications; different countries.

## 1. Introduction

The asset pricing model has experienced continuous amelioration so that it could have better result to estimate the expected stock returns of asset portfolios. Initially, Harry M. Markowitz came up with the idea about portfolio theory. He used Mean Variance Model to find out the best portfolio based on the rate of expected returns and variances of returns [1]. Compared with Markowitz, William Sharpe suggested the idea of market portfolio and made an assumption that asset returns and market portfolio are linearly related. Later on, John Lintner, Jack Treynor and Jan Mossin indicated one advanced model, which is familiar as CAPM at present. Core theory is that there is linear relationship between a single asset portfolio and corresponding potential risks. After a series of practices and verifications, it was demonstrated that beta value of CAPM model could validly make the definition of stock returns rate more clear [2].

Starting from 1970s, with the development of finance and economy around the world, it was verified that CAMP model was not good enough to estimate the expected return. After that, in 1993, Fama and French presented the Fama French three factors model, where two new factors SMB and HML were added into it. This model incredibly increased the accuracy of prediction value of expected asset portfolio return at that time. However, starting from the 21st century, there was clear evidence to prove that Fama-French model with only three basic factors was no longer practical for everywhere. As a result, based on this, Fama and French raised Fama-French five factors model, where another two factors RMW and CMA were added into the Fama French three factors model. The new one is considered as the most appropriate one so far in the world. But it is also possible that Fama-French model is not suitable for every country in everywhere.

Around the Fama-French models, a number of researches have been conducted, they are majorly concentrated on the following perspectives: the evolution process of the Fama- French Models, the impact and differences of the utilization of Fama-French Models before and after the COVID-19 pandemic disease. For example, in the view of evolution process of the Fama- French Models, Chongming Ma (2007), Karp, A. and van Vuuren (2017) studied about it and got the result that Fama-French five factors model is the better one for current economy situation [1,2]. For the application of Fama-French models, Guo, D. & Zhou, P (2021), M. Uddin, A. Chowdhury, K. Anderson, K.

Chaudhuri (2021) and Baker, S. R., Bloom, N., Davis, S. J., Kost, K. J., Sammon, M. C., & Viratyosin, T. (2020) claim that the COVID-19 pandemic disease does not bring any negative influence to the applicability of Fama-French models, this means Fama-French models could still be used when estimating the returns of portfolios.

In the first part of this paper, the main topic is to demonstrate the above conjecture based on some data analysis by R software about stock returns as known. Afterwards, in the second part of this paper, it is prior to focus on some applications of Fama-French Model in different countries.

## 2. Method

### 2.1 Fama-French Three Factors Model

Fama and French found that beta value of CAPM model is not good enough to estimate the expected return of asset portfolio. Based on this, in 1993, Fama and French made an assumption that there was a model which includes three factors: Portfolio of market assets, Market factor and Book to market factor [3]. They are well known as  $R_m - R_f$ , SMB, HML today, where  $R_m$  is the return of the market portfolio,  $R_f$  is the risk-free return, SMB means “Small Minus Big” and HML means “High Minus Low”. The fundamental formula is the following.

$$E(R_i) - R_f = \beta_M(E(R_M) - R_f) + \beta_{SMB}SMB + \beta_{HML}HML \quad (1)$$

where  $R_i$  is the return of a generic asset.

### 2.2 Fama-French Five Factors Model

In 2015, Fama and French introduced one advanced model, Fama-French Five Factors Model. This model grounded on the Fama-French Three Factors Model with two more new factors: Profitability factor and Investment factor were included in it [4]. They are well known as RMW and CMA recently, where RMW means “Robust Minus Weak” and CMA means “Conservative Minus Aggressive”. The fundamental formula is the following.

$$E(R_i) - R_f = \beta_M(E(R_M) - R_f) + \beta_{SMB}SMB + \beta_{HML}HML + \beta_{RMW}RMW + \beta_{CMA}CMA \quad (2)$$

## 3. Reasonable conjecture and demonstration

Similarly as what Fama and French did in the process of research of Fama-French models, it is reasonable and natural to have the following conjecture: whether there is one more relevant factor could be involved into Fama-French Five Factors Model? This part will conduct some data and analysis process.

First of all, the five factors in Fama-French Five Factors Model will be kept but here a new candidate factor, MOM, which means “Momentum Factor”, will be added. Then two asset portfolios are collected, the main idea of this study is to find out the relevance of the new candidate factor, MOM. R studio is used to analyze the data.

**Table 1.** data analysis of 6 factors

Regressor	Model
Mkt	1.000190
SMB	0.879050
HML	0.532992
RMW	0.060741
CMA	0.074698
MOM	-0.002813
Intercept	0.042534
Summary Statistics	
Adjusted R-Squared	0.5795
SER	0.9891
N	675

**Table 2.** data analysis of 5 factors

Regressor	Model
Mkt	1.000588
SMB	0.878885
HML	0.534497
RMW	0.060095
CMA	0.073679
MOM	
Intercept	0.040503
Summary Statistics	
Adjusted R-Squared	0.5791
SER	0.9891
N	675

From these two figures, the data analysis shows that for this portfolio, the R-squared of 6-factors model (Mkt, SMB, HML, RMW, CMA, MOM) by the console is 0.9891 which is the same as the 5-factors model (Mkt, SMB, HML, RMW, CMA). Therefore, the assumption for this portfolio is that the 5-factors model is the most suitable one and the factor MOM is not relative. The reason for this statement is that the model is always the prior choice with smaller number of factors. In this case, since the R-squared is 0.9891 regardless of the MOM factor, it is natural to kick out the MOM factor.

In order to get the more accurate result for this portfolio, the function “step” is used to let R studio automatically analyze by AIC and get the most relevant model. Here is the final result.

**Table 3.** Automatic analysis result from R studio

Start: AIC=-729.66				
y ~ Mkt + SMB + HML + RMW + CMA + MOM				
	Df	Sum of Sq	RSS	AIC
MOM	1	0.1	224.4	-731.40
<none>			224.3	-729.66
CMA	1	6.7	231.0	-711.89
RMW	1	9.5	233.8	-703.76
HML	1	716.7	941.0	236.29
SMB	1	3941.0	4165.3	1240.38
Mkt	1	9770.1	9994.4	1831.17
Step: AIC=-731.4				
y ~ Mkt+SMB+HML+RMW+CMA				
	Df	Sum of Sq	RSS	AIC
<none>			224.4	-731.40
CMA	1	6.6	231.0	-713.88
RMW	1	9.4	233.8	-705.75
HML	1	773.1	997.5	273.59
SMB	1	3946.0	4170.4	1239.21
Mkt	1	9960.4	10184.8	1841.91

To avoid the accidental circumstance, the data analysis of another portfolio is necessary.

**Table 4.** data analysis of 6 factors

Regressor	Model
Mkt	0.995815
SMB	-0.107385
HML	-0.269704
RMW	0.160031
CMA	-0.021850
MOM	-0.005140
Intercept	0.077479
Summary Statistics	
Adjusted R-Squared	0.9819
SER	0.6124
N	675

**Table 5.** data analysis of 5 factors

Regressor	Model
Mkt	0.996543
SMB	-0.107687
HML	-0.266954
RMW	0.158850
CMA	-0.023713
MOM	
Intercept	0.073766
Summary Statistics	
Adjusted R-Squared	0.9819
SER	0.6123
N	675

From these two figures, the data analysis shows that for this portfolio, the R-squared of 6-factors model (Mkt, SMB, HML, RMW, CMA, MOM) by the console is 0.9819 which is the same as the 5-factors model (Mkt, SMB, HML, RMW, CMA). Therefore, the assumption for this portfolio is that the 5-factors model is the most suitable one and the factor MOM is not relative. The reason for this statement is that the model is always the prior choice with smaller number of factors. In this case, since the R-squared is 0.9819 regardless of the MOM factor, it is natural to kick out the MOM factor.

In order to get the more accurate result for this portfolio, the function “step” is used to let R studio automatically analyze by AIC and get the most relevant model. Here is the final result.

**Table 6.** Automatic analysis result from R studio

Start: AIC=-654.99				
y ~ Mkt + SMB + HML + RMW + CMA + MOM				
	Df	Sum of Sq	RSS	AIC
MOM	1	0.3	250.8	-656.23
<none>			250.5	-655.46
CMA	1	0.6	251.1	-654.99
SMB	1	58.8	309.4	-514.66
RMW	1	65.7	316.2	-499.80
HML	1	183.5	434.1	-286.03
Mkt	1	9684.8	9935.4	1827.17
Step: AIC=-656.23				
y ~ Mkt+SMB+HML+RMW+CMA				
	Df	Sum of Sq	RSS	AIC
<none>			251.5	-656.40
CMA	1	0.7	250.8	-656.23
SMB	1	59.2	310.1	-515.11
RMW	1	65.6	316.4	-501.49
HML	1	192.8	443.7	-273.26
Mkt	1	9880.0	10130.9	1838.32

#### 4. Applications of Fama-French Model in Different Countries

In the following parts, the main topic will focus on some real examples of Fama-French three factors model and Fama-French five factors model in Australia and US, and their corresponding applicability under specific background.

##### 4.1 Australia

The data used for this study are mostly concluded from some previous studies [5-8]. These studies tried to make application of Fama French three-factors model in Australia based on local stock background. They are presented in table 7, where most statistical summaries are involved there.

Table 7 shows that the percentage of risk premium of the factor HML changes from 6.0 to 14.6, with all of them larger than 0. In addition, the proportion of risk premium of the factor SMB floats around 0, where -6.5 is the smallest one and 17.2 is the largest one. Based on these statistical summaries, presence of linear relationship of two factors HML and SMB in Fama-French three factors model could be confirmed. This also means that the Fama-French model with three factors is useful in Australia during corresponding research period.

**Table 7.** Statistical summaries of previous studies in Australia

Studies	Period	Risk Premium		Parameter Analyse		
		HML(%)	SMB(%)	Insignificant Intercepts	Coefficients of HML	Coefficients of SMB
Fama and French (1998)	1975-1995	12.3	None	None	None	None
Halliwel (1999)	1980-1991	14.6	6.0	23/25	6/25	18/25
Faff (2004)	1996-1999	6.0	-6.5	19/24	14/24	18/24
Gharghori (2007)	1996-2004	10.4	17.2	24/27	20/27	14/27
Kassimatis (2008)	1993-2005	12.6	11.5	11/25	20/25	11/25

However, in 21st century, with the development of global economy and the discovery of Fama-French five factors model, it is natural to have a doubt that whether the Fama-French model with three factors is excellent enough to estimate everything in the world. Duc Hong Vo (2015) claimed that the Fama-French five factors model and Fama-French four-factors model that excludes HML are semblable on all characters. So in Fama-French five factors model, HML seems to be redundant and HML is not priced as it is contained in the Fama-French five-factors model [9]. In fact, nowadays, the argument about the comparison of the significance between Fama-French model with three factors and Fama French five-factors model has not stopped yet. It could be expected that this argument will last for a period. Although there is not quite obvious evidence to prove the superiority Fama-French five factors model, learners and researchers should consider both of these two models when they do research or predict some financial data.

## 4.2 America

In this part, the study will focus on the impact in the US due to the COVID-19 pandemic disease and explore the changes of Fama-French Model in the US.

During the COVID-19 pandemic, the financial market experienced serious decline or exhibited varying degrees of volatility all around the world. At the very first period, most people think the speed of economy decline in the world could not be extremely fast since they know what happened in China in 2003 relating to the SARS pandemic disease. The stock market in China even increased by more than 10 percent during the SARS pandemic in 2003 [10]. However, the rate of economy decrease is much more prolonged and disappointing than the initial expectation. This trend is mainly represented by continuous volatility increase in US markets, compared with other previous pandemic period. In addition, there are incredible 18 market jumps in US market from 24th February to 24th March (totally 22 office days), this is the first time that such situation happens in the history of US [11].

Under this specific situation, the study concentrates on the difference between some statistical data and the corresponding data before and after the COVID-19 pandemic. All the data are from a website which is created by French where some helpful data are available for experiments and test conduction. Here a 50 portfolios data set which includes the mean return of 50 different portfolios from 1970 to 2020. The reason for this is to make the accidental errors and mistakes as less as possible. The following are the data analysis report tables based on Fama-French three factors model and Fama-French five factors model.

**Table 8.** effectiveness of SMB and HML at 5% level

Effectiveness of SMB and HML at 5% level	Corresponding rate			
	SMB before COVID-19	SMB after COVID-19	HML before COVID-19	HML after COVID-19
	0.8626	0.9278	0.7492	0.8395

This table above suggests that the relevance of two factors HML and SMB are higher after COVID-19 pandemic than those before the COVID-19 pandemic. This means the Fama-French three factors model is more applicable after the COVID-19 pandemic when it is used to estimate the excess return of portfolios. In order to find out significance of Fama-French five factors model, a similar data analysis procedure is made and the summary is below.

**Table 9.** The effectiveness of SMB, HML, RMW and CMA at 5% level

Effectiveness of SMB, HML, RMW, CMA at 5% level	Corresponding rate							
	SMB before COVID-19	SMB after COVID-19	HML before COVID-19	HML after COVID-19	RMW before COVID-19	RMW after COVID-19	CMA before COVID-19	CMA after COVID-19
	0.8573	0.9358	0.5086	0.7251	0.3694	0.3957	0.2438	0.4265

Similarly as before, the table 9 indicates that the efficiency of all the factors in the Fama-French model enhance compared with their corresponding ones before the output of COVID-19.

#### 4.3 Discussion on the applicability of Fama-French models in both Australia and US

From the data analysis above, it is prominent that there is enough evidence to identify the usability of Fama French three-factors model and Fama French five-factors model in Australia. In addition, similarly, since the data summaries indicate the effect of Fama-French model stays constant after the outbreak of COVID-19 pandemic, the applicability of Fama-French models in US could be confirmed. In other words, the Fama-French model of both three factors and five factors could be considered to use when investors or some entrepreneurs need to estimate the returns of portfolios and make decisions according to the analysis results.

## 5. Conclusion

Based on the analysis on the “reasonable conjecture of the number of factors in Fama-French models” and “applicability of Fama-French models in both Australia and US”, this paper presents that Fama-French model with five factors is the most appropriate although there may exist some reasonable conjectures about the rationalization of Fama-French five factor model. As a result, Fama-French five factors model is regarded as the first choice to predict the returns for some portfolios under the recent market circumstances. Besides, with the help of some real cases and data analysis process, the applicability of Fama-French models is definitely confirmed in Australia and US, especially after the outbreak period of COVID-19 disease in US. This means Fama-French models are extremely reliable and dependable, which is quite important for further research and economic estimations. This study provides some clear evidence for the applicability of Fama-French models in Australia and US, which is full of application values and quite helpful for future commercial applications. Besides, this study also enriches the research scope of number of factors in Fama-French models on the basis of some previous research.

## References

- [1] Chongming Ma, (2007), On capital asset pricing model and its research progress, Accounting communication: academic edition(3),5. Retrieved on September 21, 2022. Retrieved from: <https://wenku.baidu.com/view/99eb3a1ca8114431b90dd8eb.html?fr=xueshu>
- [2] Karp, A. and van Vuuren, G. (2017) “The Capital Asset Pricing Model And Fama-French Three Factor Model In An Emerging Market Environment”, International Business & Economics Research Journal (IBER), 16(4), pp. 231–256. doi: 10.19030/iber.v16i4.10040. Retrieved on September 21, 2022.
- [3] Liu, H. & Gao, Y.-C. (2019) The impact of corporate lifecycle on Fama–French three-factor model. Physica A. [Online] 513390–398. Retrieved on September 21, 2022.
- [4] Chiah, M. et al. (2016) A Better Model? An Empirical Investigation of the Fama-French Five-factor Model in Australia. International review of finance. [Online] 16 (4), 595–638. Retrieved on September 21, 2022.
- [5] Fama, E. F. & French, K. R. (1998) Value versus Growth: The International Evidence. The Journal of finance (New York). [Online] 53 (6), 1975–1999. Retrieved on September 21, 2022.
- [6] Halliwell, J., Heaney, R. and Sawicki, J. (1999), ‘Size and Book to Market Effects in Australian Share Markets: A Time Series Analysis’, Accounting Research Journal. Retrieved on September 21, 2022.
- [7] Gharghori, P. et al. (2007) Are the Fama-French Factors Proxying Default Risk? Australian journal of management. [Online] 32 (2), 223–250. Retrieved on September 21, 2022.
- [8] Kassimatis, K. (2008) Size, Book to Market and Momentum Effects in the Australian Stock Market. Australian journal of management. [Online] 33 (1), 145–168. Retrieved on September 21, 2022.
- [9] Vo, D. H. (2015) Which Factors Are Priced? An Application of the Fama French Three-Factor Model in Australia. Economic papers (Economic Society of Australia). [Online] 34 (4), 290–301. Retrieved on September 21, 2022.
- [10] Guo, D. & Zhou, P. (2021) Green bonds as hedging assets before and after COVID: A comparative study between the US and China. Energy economics. [Online] 104105696–. Retrieved on September 21, 2022.
- [11] Baker, S. R., Bloom, N., Davis, S. J., Kost, K. J., Sammon, M. C., & Viratyosin, T. (2020). The unprecedented stock market impact of COVID-19 (No. w26945). national Bureau of economic research. Retrieved on September 21, 2022. Retrieved from: <https://scholar.google.com/scholar?q=Baker,%20S.%20R.,%20Bloom,%20N.,%20Davis,%20S.%20J.,%20Kost,%20K.,%20Sammon,%20M.%20%20Viratyosi,%20T.%20.%20The%20Unprecedented%20Stock%20Market%20Impact%20of%20COVID-19.%20Working%20Paper%2026945,%20National%20Bureau%20of%20Economic%20Research>.