Portfolio Construction Based on Optimization Model and Combination of Various Assets

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Abstract. Since the fifties of the last century, the concept of portfolio management is proposed to enrich the portfolio theory by Markowitz et al., as well as the improvement of portfolio theory by later generations, forming a modern portfolio management theory. This paper uses Markowitz's mean variance portfolio theory to perform a portfolio return maximization analysis on selected assets with assumptions that investors are risk-averse according to mean variance theory. Based on mean variance portfolio theory, as well as Python computational simulation modeling, Monte Carlo simulation is used to find the effective set and effective frontier of the portfolio, and the optimal solution of the portfolio is filtered according to the two constraints of maximum Sharpe ratio and minimum volatility. According to the analysis, the portfolio return of the maximum Sharpe ratio is relatively high, and the maximum Sharpe ratio combination is more worthwhile than the minimum volatility combination. Overall, these results provide enlightenment for further exploration and implementation of portfolio theory.

Keywords: Maximum Sharpe ratio; minimum volatility ratio; portfolio optimization.

1. Introduction

A portfolio is an asset holder's right mix of underlying assets (e.g., funds, stocks, bonds, real estate, and cash). The purpose of an asset portfolio is to maximize the yield on each unit of risk taken by constructing a reasonable asset portfolio, so that every share of risk taken by investors is worth it. In portfolio investing, people always pursuit the higher the return, the better. Due to the risk of investment, the single asset type will concentrate the investment, so the risk cannot be diversified.

Once an unexpected situation occurs, the loss will be large. Therefore, it is necessary to use scientific methods to reduce the risk of the portfolio while maximizing returns. Before 1952, there was a lack of portfolio theory. It was not until Markowitz proposed portfolio theory that the impact of risk diversification on the return of the portfolio was clearly discussed, and the effective set was displayed in a graphical manner, distinguishing the portfolio under different returns and different risks [1]. At this stage, Hicks proposed the separation theorem, and Marschak proposed the ordinal selection theory under uncertainty, while also noting that people prefer high risk and low return. The dividend discount model was proposed by Williams, who believed that the risk could be eliminated by investing enough securities. but Markowitz argued that risk could not be completely eliminated by diversifying investments. Nevertheless, the model remains a standard way to assess the expected value of a security. Leavens demonstrated the benefits of risk diversification on the assumption that risks are independent. However, risks cannot be independent because the correlation between different risk factors can be determined by covariance. Therefore, under uncertain conditions, the basic idea of pursuing risk minimization and benefit maximization is proposed. Markowitz was the first to clarify the way to choose a portfolio, which marked the beginning of modern portfolio theory. The early contributions of Keynes, Marshak, and others enriched only a small part of portfolio theory and ideas, but only indirectly analyzed investment decisions [2].

The Capital Asset Pricing Model (CAPM) was proposed by Sharpe and Lintner, which describes the linear relationship between the necessary return and risk of a stock or portfolio. In a perfectly competitive market, assuming that the mean and variance of allowable returns are only considered, the CAPM model depicts a very intuitive and empirically testable asset return assumption [3]. Ross believes that return on assets is affected by a variety of factors, and the arbitrage pricing model (APT) of the market equilibrium state has been proposed. The capital asset pricing model considers the "beta" to be a numerical indicator of risk. Arbitrage pricing theory states that the riskiness of an asset is related to the sensitivity of four economic variables: (1) Inflation rate, (2) industrial production level, (3) risk premium, and (4) medium slope of interest rate term structure [4]. Fama and French mainly consider market risk factors in the capital asset pricing model, and add the company size factor SMB, SMB refers to the difference between the stock returns of small-cap companies and large-cap stocks, and the price-to-book ratio factor HML, HML is the difference between the stock returns of book-to-market ratio companies and low-ratio companies, based on the above three-factor model was proposed. Contemporarily, with the continuous improvement of mathematical models, the continuous integration of operations research and computer technology, new technologies and methods for analyzing and studying portfolio theory have been put forward, and the powerful data processing capability of computer has also played a key role in financial quantification that cannot be ignored. With the continuous development of portfolio theory and method, some people gradually pay attention to other factors including liquidity, transaction cost and tax.

2. Literature Review

Markowitz's mean variance theory is based on investors' preference for portfolios with large returns and less risky portfolios under the same returns, and finds the optimal solution under different constraints through econometric models. However, levy uses prospect theory to optimize and find a common valid set that fits the combination of prospect theory and mean variance. Cumulative prospect theory and MV valid sets almost coincide. These two theories seem to have serious contradictions when looking at the theory and publicity, but the use of MV diversification algorithm to construct the effective set of foreground theory is almost overlapping, which means that a bridge is built between mean variance theory and prospect theory [5].

Ralph and Markus propose a powerful computational algorithm to determine the effective edge (non-dominant alternative) of the portfolio selection problem in financing. They call it a computational algorithm programmed in Java multi-parameter quadratic programming (MPQ). It has many advantages over previous calculation methods: it can be used in a wide range of applications, is characterized by too short calculation times, and the number of valid alternatives can be determined in a fraction of the traditional calculation time. In these problems, the valid set is no longer a boundary, but can be described as a sphere that is not perfectly rounded [6]. Portfolio selection issues, including transaction costs and limits on risk exposure, are added by Miguel to describe a method that can find suboptimal solutions that produce suboptimal solutions, as well as upper limits on optimal solutions. The convex optimization method can effectively deal with line easy cost, benefit variance boundary, and different shortage probability boundary [7]. The portfolio that caused problems with standard MVOs was discovered by Pedersen and proposed a simpler approach, enhanced portfolio optimization. It was applied to industry momentum and time series momentum across equities and global asset classes, and Pedersen identified significant alpha that outperformed market, 1/N portfolio, and standard asset pricing factors. It unifies existing methods and allows for enhanced portfolios [8].

Fahmy expanded the classical mean variance (MV) portfolio theory by allowing the time dimension of post transaction. This extension validates MV theory by proving the uniqueness of utility representation with good behavior. Its postponement is also consistent with the observed short-term returns and long-term momentum reversal. This paper adds a time dimension on the theoretical basis of the MV framework, so that the construction of the portfolio can be regarded as an activity, including the return of risky assets and the duration of the portfolio (i.e., the time of the investor's best trading strategy) [9]. After applying the mean-variance method to the average compound return in the fixed-two-asset case, the set of effective portfolios in any one period is inversely proportional to the horizon N, and converges to a single effective series with attractive long-term attributes [10].

3. Theory and Hypothesis

The assumptions of the mean-variance model are as follows:

The basis for investors to make investment choices each time is the probability distribution of securities returns within a certain holding time.

The variance or standard deviation of the expected rate of return of securities is an indicator used by investors to evaluate investment risks

Investors choose solely on the basis of the risk and yield of each security.

Investors prefer portfolios with more returns under the same risk and portfolios with less risk under the same returns

Assuming that the type of assets that can be invested in the whole market is n, and the return rate of each asset respectively are r_1, r_2, r_3, ..., r_n, the proportion of each asset in the portfolio are w_1, w_2, w_3, ..., w_n, the return of the portfolio is r_p, Thus, the expected return on the portfolio formula can be shown:

$$E(r_p) = \sum_{i=1}^{n} \omega_i E(r_i) \tag{1}$$

where $\sum_{i=1}^{n} \omega_i = 1$. The variance formula is:

$$Var(r_p) = \sum_{i=1}^{n} \omega_i^2 Var(r_i) + \sum_{i \neq j} \omega_i \omega_j Cov(r_i, r_j) \tag{2}$$

4. Data and Method

To avoid the impact of an excessively large sample of daily data on portfolio results, this article collects three-year daily closing prices from the Yahoo finance database for four stocks with a date range from October 1, 2019 to October 31, 2022. The four stocks are Bitcoin (BTC=USD), Gold Dec 22 (GC=F), 10-year T-note Futures (ZN=F), Dow Jones Index (DJI). Next, using the above data, assuming that the proportion of each asset in Portfolio A is equal, that is, 25%, the expected return of this portfolio shown in Table. 1. One can see that the cumulative return is 53.8%, the annual volatility is 18.1%, and the Calmar ratio is 0.3, lower than the Sharpe ratio. A higher Calmar ratio means that the fund returns at unit losses and the better the value for money. So, this combination is also excellent. Skewness is a measure of the degree of asymmetry relative to the mean, a measure of symmetry. By measuring the skewness coefficient, Python can determine the degree and direction of asymmetry in the distribution of the data. The skewness here is -1.05, so it means that this distribution is biased to the left. Kurtosis is a measure of whether data is smooth or steep. The kurtosis of this distribution is 15.64, indicating that the data follow a normal distribution. The kurtosis here is well above 3, indicating that there are some extreme values in the data. When constructing a portfolio with equal weights of each asset, the Sharpe ratio of such a portfolio is 0.62 can be known. The Sharpe ratio here does not exceed 1, indicating that such a portfolio of equally weighted assets does not perform well.

Table 1. Tear Sheet (simple variant)

Start date	2019/10/2
End date	2022/10/31
Annual return	10.10%
Cumulative returns	53.80%
Annual volatility	18.10%
Sharpe ratio	0.62
Calmar ratio	0.3
Stability	0.59
Max drawdown	-33.30%
Omega ratio	1.12
Sortino ratio	0.88
Daily value at risk	-2.20%

The Omega ratio indicator considers the entire distribution of returns and therefore includes information on all higher-order moments. When the critical rate of return is equal to the mean, the Omega ratio is equal to 1. At the same critical rate of return, the larger the Omega ratio, the better, for different investment options. Here the omega ratio is 1.12. The higher Sortino ratio, the better the performance of the portfolio. The difference from Sharp ratio is that Sortino ratio divides the fluctuation of data into good fluctuation and bad fluctuation. It believes that good fluctuation is not a risk, and the positive return of the portfolio meets the needs of investors. Therefore, it only uses the downward standard deviation when calculating the fluctuation. The Sortino ratio at this point is 0.88, which is a very positive sign. Hence, such a symmetrically distributed portfolio is also a portfolio worth investing in.

5. Model and Constraint Condition

In this part, the stock price of each asset is clearly visualized, as shown in Fig. 1, and the second step will use Monte Carlo simulation to simulate a large number of portfolios with randomly allocated weights to build an effective set, in which the paper can find the effective frontier, and find a matching portfolio according to the two different constraints of the maximum Sharpe ratio and the minimum volatility combination, and compare. At this point, it is important to know how a given stock pool (portfolio of securities) finds the balance of risk and return.

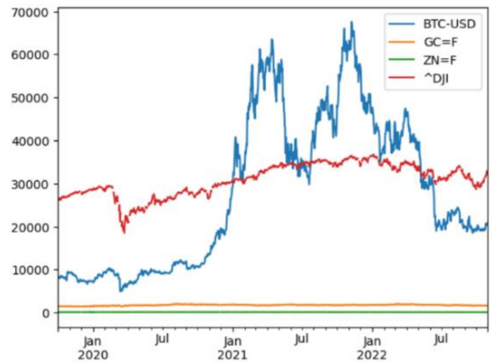

Fig. 1 Stock prices of the considered assets

Fig. 2 exhibits the daily return performance of each asset. Fig. 3 is a portfolio consisting of many randomly weighted assets generated by Monte Carlo simulation, which records the expected returns and volatility of these portfolios. Record important portfolio statistics, including returns, variances, and Sharpe ratios. The optimal solution is obtained by solving the constrained optimal problem. The constraint is that the sum of the weights of each asset in the portfolio is 1. From the Fig. 3, Bitcoin is in the upper right of the effective frontier curve, and it can be known that the volatility of the asset of bitcoin, a digital currency, is very large and will bring high returns, so the higher the risk of the asset, the greater the return. For assets such as 10-year T-note Futures (ZN=F), it is a relatively stable asset, with low volatility and a relatively low yield. The Dow Jones and Gold are two assets in the middle of the risk and return.

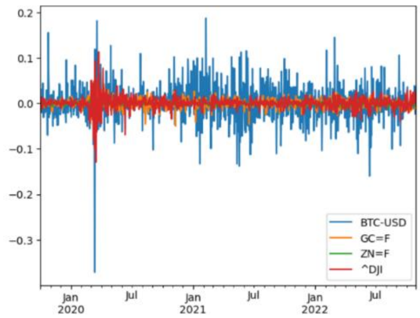

Fig. 2 Daily returns of the considered assets

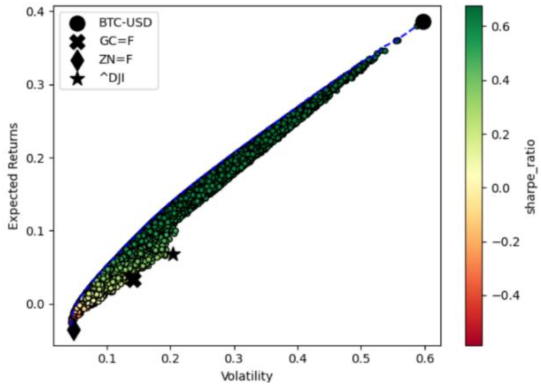

Fig. 3 Efficient Frontier (random weight)

Subsequently, portfolios with the maximum Sharpe ratio and those with the minimum volatility can be found by using Python, respectively. The results of two portfolios performance are summarized in Table. 2. According to the results, when pursuing a portfolio with the maximum Sharpe ratio, that is, when the maximum return per unit of risk is obtained, this portfolio has a return of 17.56% and a volatility of 25.99%. Among them, Bitcoin accounted for 38.08% and gold accounted for 38.56%. Low-risk low-income assets like ZN=F have a weight of only 0.11%, while the Dow Jones index has a weighting of 23.25%. In the portfolio with the minimum volatility, that is, in the more robust portfolio, the return of the portfolio is -2.60%, the Sharpe ratio is -58.56%, and the volatility is 4.44%. Such portfolio investors will never invest. Besides, it can be seen from the table that the proportion of high-risk and high-yield assets such as Bitcoin is almost 0, and the proportion of relatively stable ZN=F assets has reached 90.22%. On this basis, in this random weighting, the minimum volatility portfolio is not ideal.

Table 2. Summary of portfolios.

	Maximum Sharpe Ratio portfolio	Minimum Volatility portfolio
Returns	17.56%	-2.60%
Volatility	25.99%	4.44%
Sharpe Ratio	67.56%	-58.56%
Weight of BTC-USD	38.08%	0.04%
Weight of GC=F	38.56%	2.52%
Weight of ZN=F	0.11%	90.22%
Weight of ^DJI	23.25%	7.22%

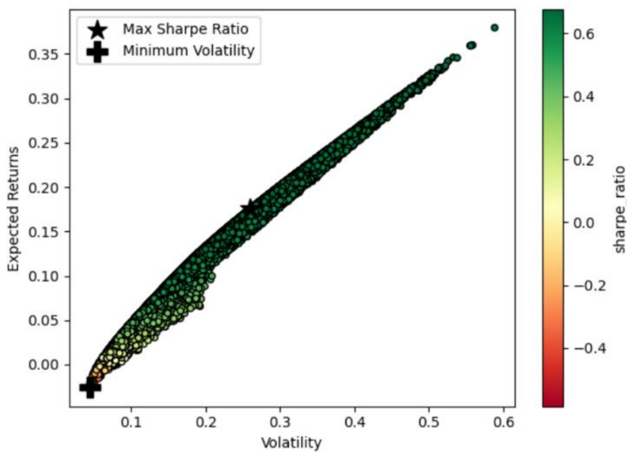

Fig. 4 Efficient Frontier.

Afterwards, this study presents the position of the maximum Sharpe ratio portfolio and the minimum volatility portfolio in the effective boundary in Fig. 4. Following the previous presentation, the position of the portfolio in the effective front under two different constraints can be determined. From the figure can be seen that the maximum Sharpe ratio portfolio is in the middle of the effective frontier, the maximum Sharpe ratio portfolio has a return of 17.56%, and the volatility is 25.99%. The minimum variance portfolio is in the lower part of the effective frontier curve with volatility of 4.44% and the portfolio is negative at -2.60%. Compared to the two portfolios, one prefers the maximum Sharpe ratio portfolio.

6. Limitations & Future outlooks

The mean variance theory assumptions are limited. The assumptions of mean variance theory are very harsh, assuming that investors are risk averse. However, this assumption is inconsistent with reality, investors are divided into risk neutral, risk appetite and risk aversion. Second, the model is based on historical data and does not reflect reality. Finally, this theory ignores transaction costs and taxes, which are also inconsistent with reality. The existence of transaction costs and taxes reduces the actual returns of the portfolio, so investors will want higher returns to compensate for this loss. The mean variance theory also has premise limitations. This theory suggests that the optimal portfolio is one that has the smallest variance at the mean level of a certain rate of return, or the greatest return at a certain level of variance. However, portfolios with large variances can compensate for this deficiency with increased means, and portfolios with small means can also compensate for this deficiency with reduced variance. Secondly, the return of the minimum volatility portfolio calculated by the assets used in this practice is negative, which does not represent the situation of most portfolios, and the time interval selected in this article is the data of the last three years, but because the last three years contain the macro factors of the new crown pneumonia epidemic, the data is not necessarily universally representative, and the data has greater volatility than the ordinary period.

Nowadays, mathematical models have been continuously improved, operations research and computer technology have been continuously integrated, and various new methods for analyzing and studying portfolio theory have been discovered, such as behavioral finance and investment theory, portfolio theory based on the value-at-risk model VaR, dynamic portfolio problems under continuous time conditions, and consideration of liquidity, transaction costs and other factors. Portfolio theory of taxes and various other restrictive factors, etc. In future studies, the impact coefficient of Fama's multi-factor model on each factor on the portfolio can be added. Modern portfolio theory has been developing, intertemporal investment under continuous time conditions and asset pricing with dynamic characteristics, models considering multiple global impact factors and nonlinear relationships, further quantitative research on behavioral financial investment, non-normal distribution research describing investment phenomena under extreme market conditions, and quantitative model research based on big data and complex calculations. When the theoretical assumptions are disagreement with the actual situation, behavioral finance can be used to explain and enrich investors' investment behaviors and choices. These are the directions in which portfolio theory will develop in the future.

7. Conclusion

In sumary, this paper uses the mean variance theory and data processing to maximize the portfolio data of selected assets. To be specific, it analyzes the performance of the portfolio obtained by the portfolio with equal weight allocation weights, the return of the equal weight portfolio is 10.10%, and the sharpe ratio is 62%. Portfolio the paper chose with the maximum sharpe ratio had a yield of 17.56% and volatility of 25.99%. Among them, Bitcoin(BTC=USA) accounted for 38.08% and gold accounted for 38.56%. Low-risk low-income assets like 10-year T-note Futures, Dec-2022 (ZN=F) have a weight of only 0.11%, while the Dow Jones index(^DJI) has a weighting of 23.25%. In the portfolio with the minimum volatility, that is, in the more robust portfolio, the return of the portfolio is -2.60%, the Sharpe ratio is -58.56%, and the volatility is 4.44%. The minimum volatility portfolio calculated in this article will not be liked by investors, because no one will invest in a portfolio with a negative return. In the subsequent random weight Monte Carlo simulation, the effective set of all the portfolios is exhibited that can be composed of the selected four assets. If investors want to improve the return rate of investment portfolio, one should increase the proportion of high-risk and high-yield assets in the investment portfolio. Whereas, risk averse investors want a more stable portfolio, they should increase the proportion of low-risk and low yield assets (e.g., government bonds), which reduces the risk of the portfolio.

This paper is based on Markowitz's portfolio theory of mean variance, but the theory has the hypothetical limitation that not all investors are risk-averse, and the portfolio theory of mean variance does not take transaction costs and tax factors into account. Moreover, the data selected in this paper is simulated based on historical data, and the influence of macro factors such as new crown pneumonia included in the selected data time span is larger than in the general period. In the future empirical research, the Fama multi-factor model can be added to determine the influence coefficient of other influencing factors on the portfolio. In additon, more factors can be added to the modeling process, so that the model simulation is closer to reality, and the behavior of investors can be explained by the theory of behavioral finance, and the shortcomings of the mean variance portfolio theory can be enriched. Overall, these results offer a guideline for portfolio construction based on various assets.

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