Portfolio Optimization for 10 Stocks under Constraints

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Abstract. Portfolio optimization is now playing a key role in the financial sector. The aim of this article is to analyze asset allocation across a number of sectors including technology, financial services, and industrials. Ten representative companies from these industries are selected, and then the performance of the portfolios is predicted using the Markowitz model and the Index model. Also, four constraints quantified by real financial market factors are used to observe the differences between the two models. Finally, the comparison of the two models under the same constraint and the comparison of the same model under different constraints are illustrated. The results show that, firstly, The Markowitz and index models are very effective when applied to portfolios of risky assets when combined with capital market lines and efficient frontiers. Secondly, using the S&P 500 index as an indicator of the global stock market, the index is favorable for those who choose a low-risk portfolio. These findings may help investors with specific risk appetites make their own investment decisions.

Keywords: Optimization; Markowitz model; Index model; constraints.

1. Introduction

Portfolio theory is a theory that has become widely known in the financial world since the American economist Markowitz proposed the Markowitz model, which is an optimal portfolio that maximizes returns and minimizes risk. In order to satisfy the requirement of a low-risk, high-return portfolio, and several models were subsequently proposed based on Markowitz's theory. Several models based on Markowitz's theory have been proposed. Since then, the balance between risk and return has become a popular research area and portfolio studies have flourished everywhere [1].

However, a review of the many research papers on portfolio optimization reveals a major problem. Some papers deal with portfolios of stocks in different specific areas. A larger number of studies deal with portfolios of different stocks in a single region. For example, Wang studied investment strategies for energy futures from an individual investor's perspective based on Markowitz's mean-variance model [2], and Wang selected eight stocks from the AI industry and constructed effective frontiers based on Markowitz portfolio theory, under different degrees of risk aversion they select the optimal portfolio based on the investor's investment utility [3]. And Medeiros states in the article that it has outperformed the VW, EW and Markowitz portfolios, even when transaction costs are taken into account [4]. The parametric approach is very flexible in terms of weighting, transaction costs and incorporating listing and de-listing restrictions. In his paper, Xing also determined the first-order asymptote of the spectral dimension of portfolio loss risk [5]. In addition, Naccarato et al selected the 30 most highly capitalized real stocks from the European financial sector, excluding non-financial sector stocks [6].

The purpose of this paper is to focus on the asset allocation of ten representative companies in five sectors: technology, financial services, industrials, energy, and consumer cycle. The specific research process is as follows. First, ten stocks from technology, financial services, industrials, energy, and consumer cycle sectors are selected based on their closing prices over the past 20 years; second, monthly prices are filtered from the original data; third, using EXCEL, annualized mean returns and annualized standard deviations, single Sharpe index beta model, standard deviation, alpha, residual standard deviation, and correlation among the ten stocks. Fourth, the Markowitz and index models were constructed by adding four types of constraints. Fifth, the Excel graph function was used to analyze the performance of these ten stocks under different constraints of the same model and the performance of different models under the same constraints.

The article is structured as follows. Section 2 presents the data and Section 3 describes the methodology. Section 4 presents the empirical results and Section 5 concludes.

2. Data

Data used in this document are from Yahoo Finance. (https://financeyahoo.com/). This paper selected the following 10 companies for their closing prices in twenty years from November 2001 to November 2021. This paper considered to choose these data as they have a relatively high beta, and they were all performance well during the past 20 years. Also, this paper chooses these companies from different fields in order to diversify the assets and avoid high correlation. Some descriptive statistics of these assets are shown in the Table 1 below.

Table 1. Descriptive statistics of the selected stocks

	SPX	NVDA	CSCO	INTC	GS	USB
AAR	7.542%	32.802%	9.714%	8.905%	10.825%	9.878%
ASD	14.850%	55.774%	30.809%	30.503%	29.572%	23.680%
β	1.000	1.979	1.321	1.188	1.410	0.971
α	0.000%	17.877%	-0.246%	-0.052%	0.190%	2.533%
RSD	0.000%	47.405%	23.762%	24.889%	20.881%	18.780%
	TD CN	ALL	PG	JNJ	CL	
AAR	11.010%	10.080%	9.437%	8.464%	7.105%	
ASD	18.134%	24.884%	14.587%	14.785%	15.350%	
β	0.787	1.056	0.405	0.540	0.454	
α	5.074%	2.113%	6.381%	4.392%	3.677%	
RSD	13.865%	19.317%	13.288%	12.423%	13.787%	

From the data showing above this paper finds that the highest annualized average return is appeared in NVDA while the lowest annualized average return is 7.105% from CL. Comparing their variances, INTC has the largest variance while PG has the lowest. In addition, the highest beta and highest annualized alpha are both in NVDA. And the lowest annualized residual return is shown in JNJ.

3. Methods

In this paper, correlation matrix and modern portfolio theory are selected to do portfolio management for the selected SPX 500 and the ten representative stocks.

3.1 Pearson Correlation Coefficient

The process of stock price changing is a time series data which contains both the randomness and some regular pattern. When dealing with the SPX500 and stock prices, knowledge of probability theory is applied to obtain the correlation among the eleven assets. Besides, correlation matrix are necessary details for portfolio management. The correlation coefficient is calculated by the following formula,

$$r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}} \tag{1}$$

where $\{x_i, y_i\}$ are the annual returns of the eleven assets.

3.2 Markowitz Model

Over a predetermined length of time, an investor makes investments worth a set sum of money. At the beginning of the time, he purchases some securities, and at the end of the period, he sells them. Then, he must select at the start of the time period which securities to purchase and how to divide his funds [7]. The investor must pick the best portfolio out of all those available at the start of the time. The highest rate of return and the lowest risk of uncertainty are the investor's current two decision-making goals. The ideal goal should be to strike the optimal balance between these two competing demands. The investing concept has a variety of ramifications. When purchasing risky assets like securities, expected profit and risk must both be addressed in the beginning [8]. And a crucial topic that market investors must address is how to gauge risk and return in an investment portfolio and balance these two factors in the asset allocation. The Markowitz theory was developed in this setting in the 1950s and early 1960s. And these are the Markowitz model's presumptions,

$$\sum_i w_i = 1 \tag{2}$$

where w_i is the weight of $asset_i$ in a portfolio. The portfolio return and standard deviation can be calculated as follows.

$$return = \sum_{i=1}^{n} w_i * asset\, return_i \tag{3}$$

where $asset\, return_i$ is the annualized average return of $asset_i's$ monthly return.

$$\sigma(w) = \sqrt{\sum_{i,j} [(w_i * s_i) * C_{ij} * (w_j * s_j)]} \tag{4}$$

where $\sigma(w)$ is the standard deviation of $asset_i$, and C_{ij} denotes the covariance between $asset_i$ and $asset_j$. One of the three traditional ratios that considers both return and risk is the Sharpe ratio, which is another indicator of asset quality. The extra return for each unit of risk is measured. It can be calculated in the manner shown below [9].

$$SR = \frac{r}{\sigma(w_i)} \tag{5}$$

Where r is the return calculated from Markowitz model or Index model, and $\sigma(w_i)$ is the standard deviation of the portfolio [10].

3.3 Index Model

Market risk (systematic risk) and firm-specific risk are the only two components that make up a security's risk. As a result, both systematic and firm-specific risk are included in the performance of the security, with systematic risk being broadly represented by the returns of the major stock market indices. The returns of the major stock market indexes may typically be used to indicate systematic risk; this form of return is known as an index model.

For analytical purposes, the single-index model makes the assumption that a single macro factor, which may be represented by the return on a market index like the S&P 500, drives stock return risk. According to the model's presumptions, the return on any stock can be divided into three categories:

the return on macroeconomic events affecting the market, the return on unexpected microeconomic events affecting only the firm, and the expected residual return on each stock.

Denote the unpredictable component of the macro factor by F and the sensitivity of security i to macroeconomic events by β_i, then the macro component of the return on security i is $M_i = \beta_i F$, and equation (1) takes the following form

$$R_i = E(r_i) + \beta_i F + e_i \quad (6)$$

If investors use a major stock market index, such as the S&P 500, as a common proxy for macroeconomic factors, they obtain an equation similar to a single-factor model. This is a univariate model and a common proxy for the index model discussed in this paper.

4. Results

The correlation coefficient matrix of the eleven assets was obtained by calculating the correlation coefficients. The results are shown in Table 2 below.

Table 2. Correlation matrix

correlations	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
SPX	1.000	0.527	0.637	0.578	0.708	0.609	0.645	0.630	0.412	0.542	0.440
NVDA	0.527	1.000	0.487	0.524	0.343	0.160	0.338	0.157	0.060	0.165	0.069
CSCO	0.637	0.487	1.000	0.614	0.487	0.328	0.410	0.297	0.220	0.239	0.165
INTC	0.578	0.524	0.614	1.000	0.411	0.280	0.412	0.286	0.136	0.325	0.110
GS	0.708	0.343	0.487	0.411	1.000	0.472	0.494	0.417	0.173	0.296	0.203
USB	0.609	0.160	0.328	0.280	0.472	1.000	0.539	0.540	0.336	0.234	0.218
TD CN	0.645	0.338	0.410	0.412	0.494	0.539	1.000	0.417	0.231	0.273	0.212
ALL	0.630	0.157	0.297	0.286	0.417	0.540	0.417	1.000	0.346	0.452	0.407
PG	0.412	0.060	0.220	0.136	0.173	0.336	0.231	0.346	1.000	0.484	0.483
JNJ	0.542	0.165	0.239	0.325	0.296	0.234	0.273	0.452	0.494	1.000	0.527
CL	0.440	0.069	0.165	0.110	0.203	0.218	0.212	0.407	0.483	0.527	1.000

As shown in Table 2, each underlying asset has a high correlation with other companies in the same industry. In addition, 'SPX' has a high correlation with ten other assets. All coefficients are positive, which implies that the assets are not isolated from other assets.

In this paper, certain portfolios are constructed under four constraints and the maximum Sharpe ratio portfolio, minimum variance portfolio, capital allocation line and minimum variance frontier are calculated to find the differences. The detailed results are shown below.

Constraint 1: There is no specific limit, which is a commonly used one in the investment decisions. Its results are shown in Fig. 1 and 2.

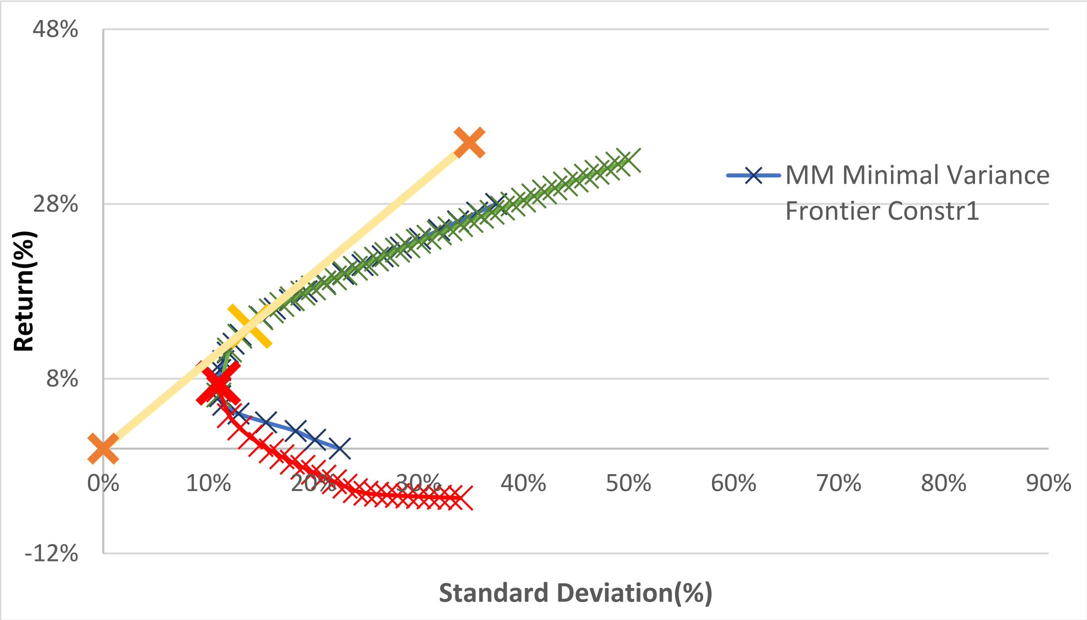

Fig. 1 MM Minimum variance frontier and capital allocation line under constraint 1

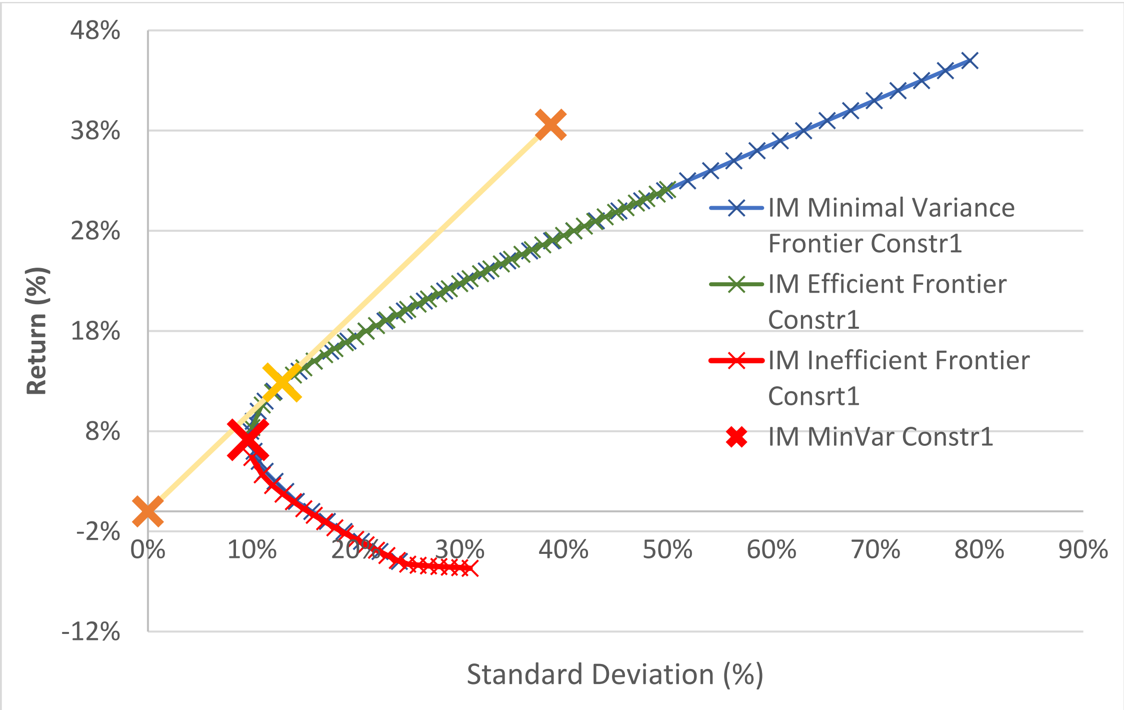

Fig. 2 IM Minimum variance frontier and capital allocation line under constraint 1

Constraint 2: constraint 2 is a usual constraint for an investment manager, which refers to the condition that investor could not borrow money. Its results are shown in Fig. 3 and 4.

$$|w_i| \leq 1, for\ \forall i \quad (7)$$

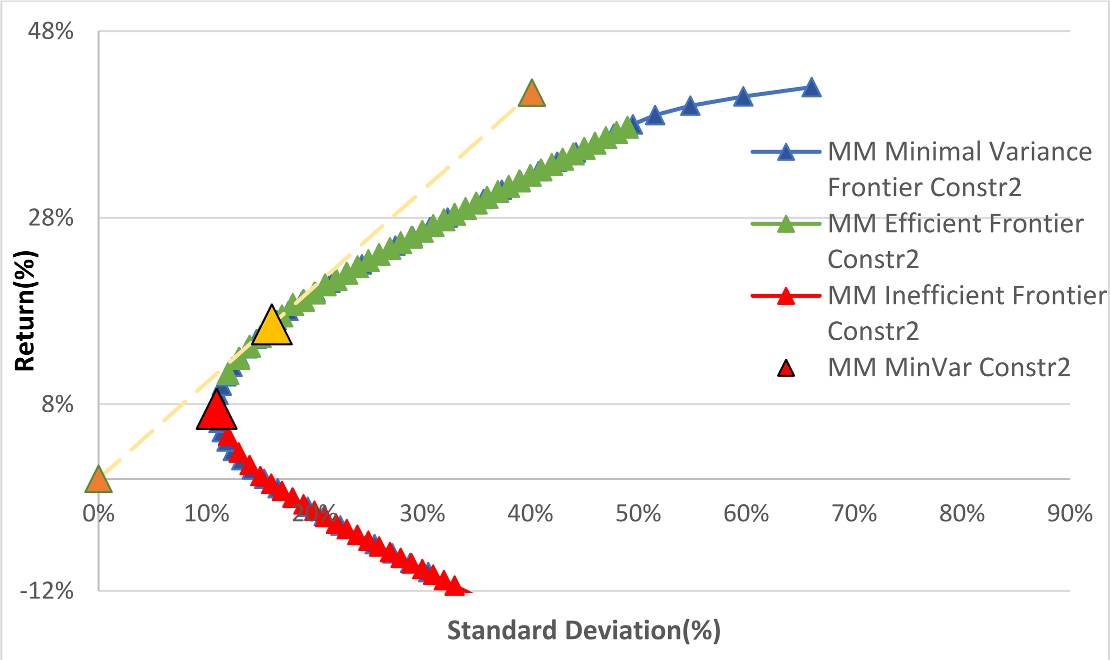

Fig. 3 MM Minimum variance frontier and capital allocation line under constraint 2

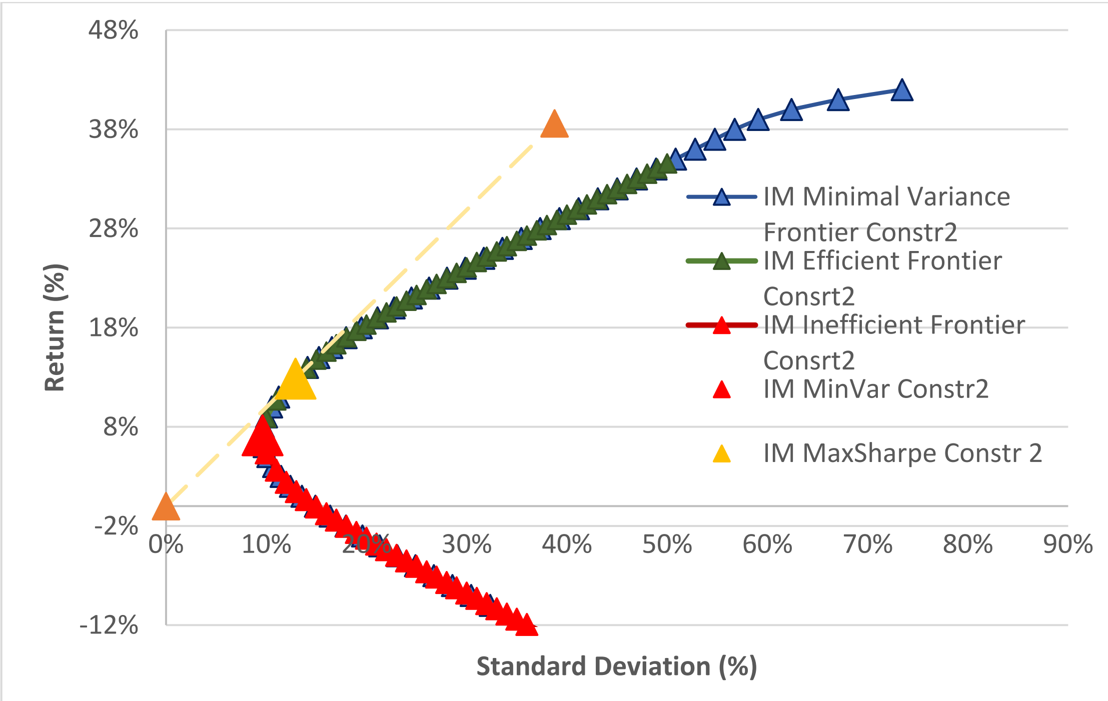

Fig. 4 IM Minimum variance frontier and capital allocation line under constraint 2

Constraint 3: There is no specific limit, which is a commonly used one in the investment decisions. Its results are shown in Fig. 5 and 6.

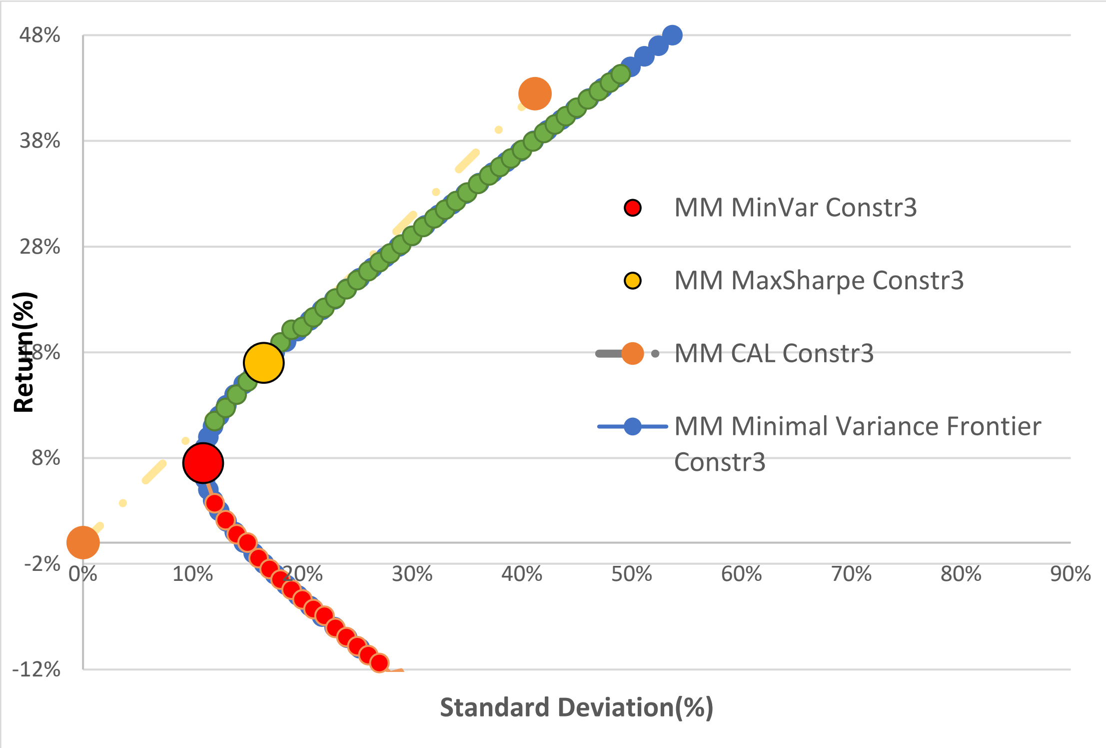

Fig. 5 MM Minimum variance frontier and capital allocation line under constraint 3

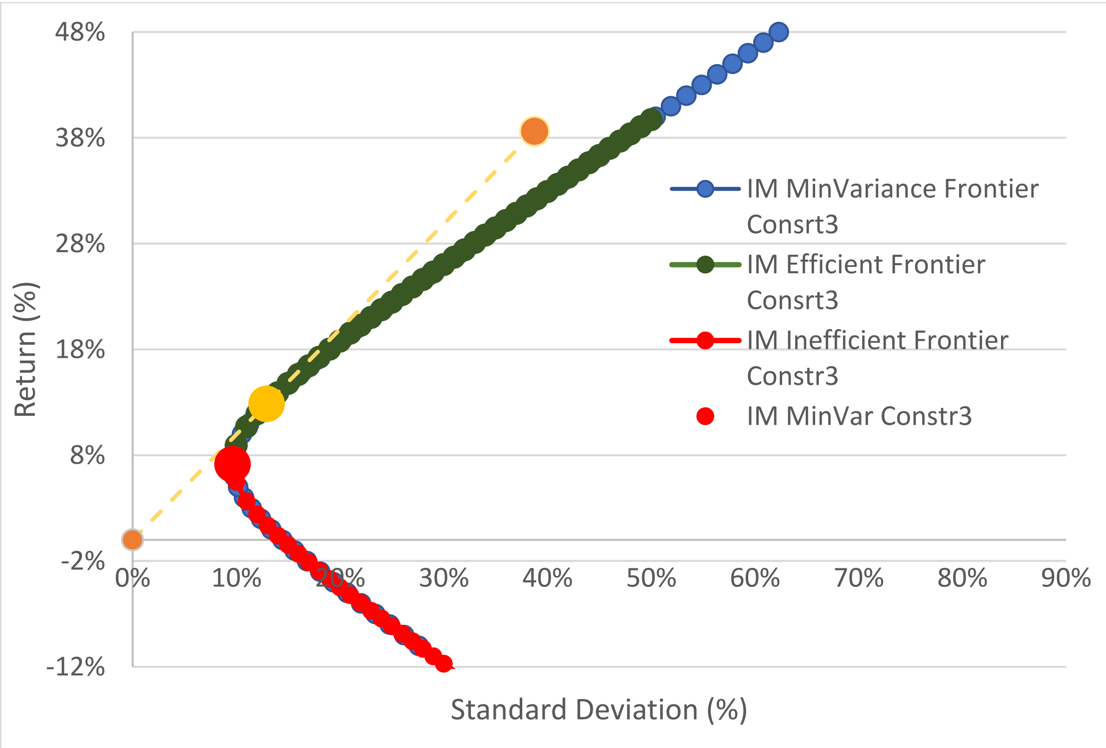

Fig. 6 IM Minimum variance frontier and capital allocation line under constraint 3

As Fig 5 and 6 indicates, the minimum variance frontier of constraint 3 lies in the right of that of constraint 1. Because investors cannot short some bad assets. Besides, an interesting fact may be confirmed that if short selling is not allowed, investing in the SPX to avoid risk is a preferred choice.

Constraint 4: Constraint 4 is that you cannot hold short positions, which is common in U.S. funds. Mutual funds raise money from the public, which has a very low tolerance for risk. Therefore, mutual funds cannot take short positions. The results are shown in Fig. 7 and Fig. 8.

$$w_i \geq 0, for\ \forall i \tag{8}$$

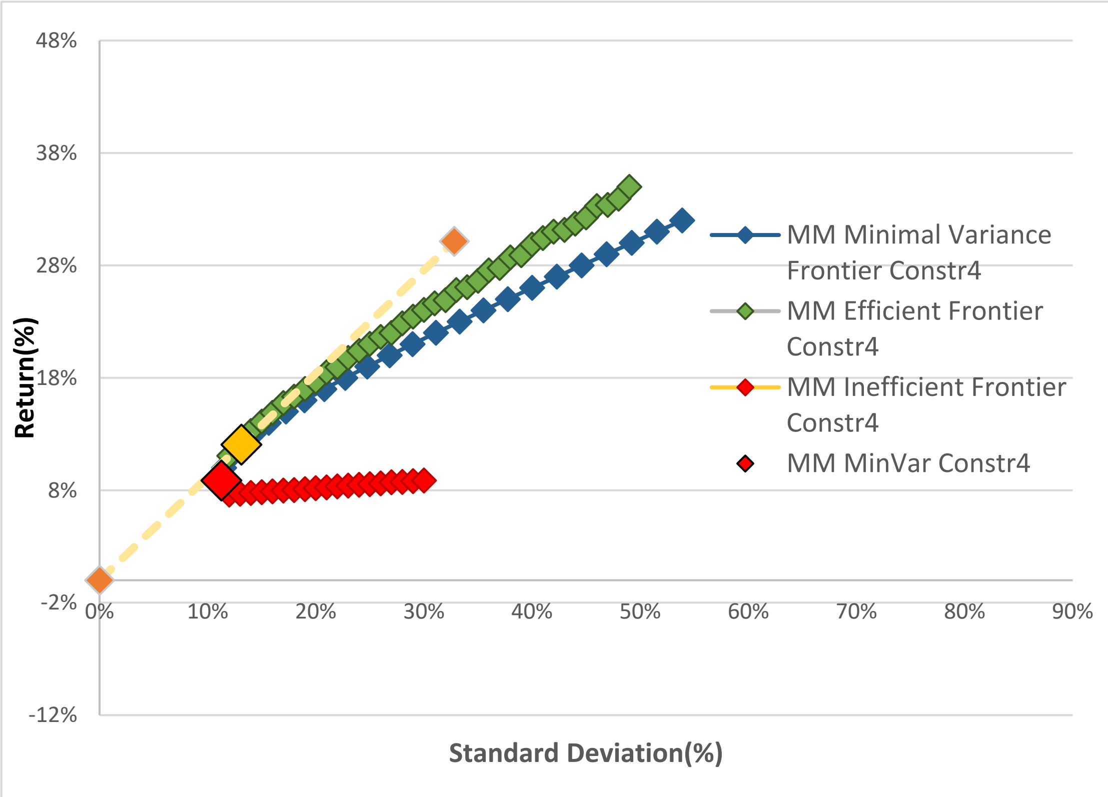

Fig. 7 MM Minimum variance frontier and capital allocation line under constraint 4

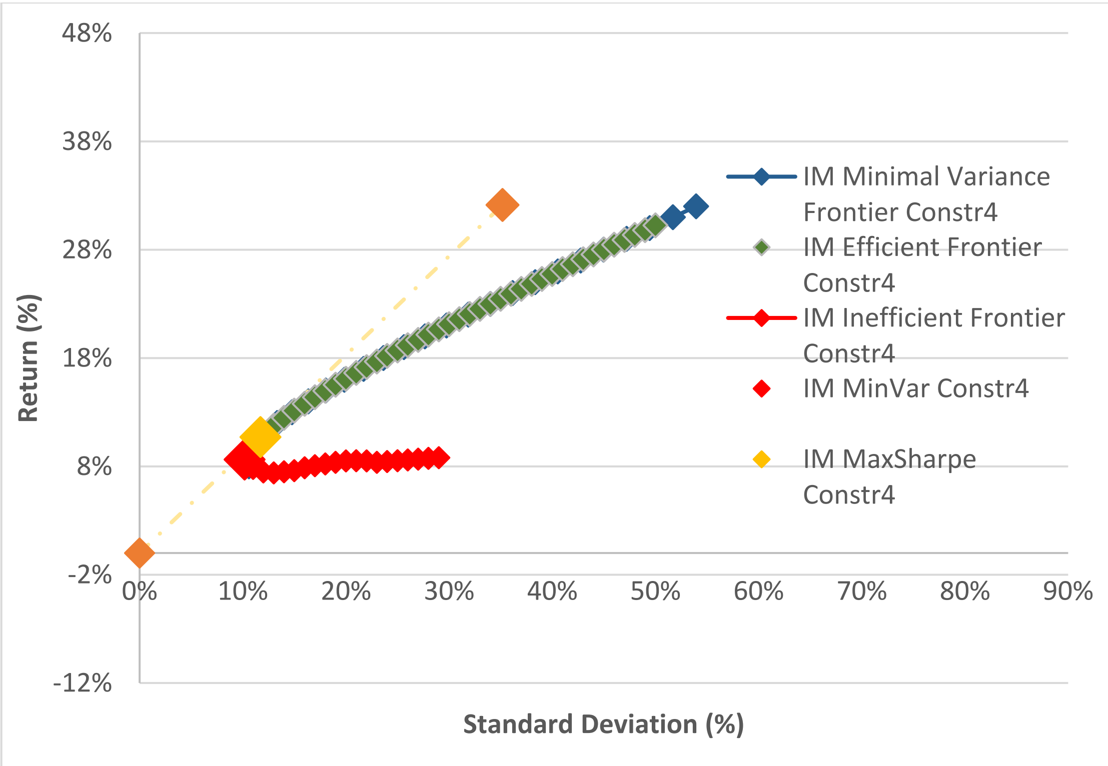

Fig. 8 IM Minimum variance frontier and capital allocation line under constraint 4

5. Conclusions

Today, most portfolio research is based on analysis of general market conditions or specific industries. In addition, few studies focus on legal or industry regulation development constraints. The purpose of our study is to do portfolio analysis of the retail, technology, financial services, and industrial sectors to facilitate potential investors in making investment decisions under different constraints. This paper uses the Markowitz model and Index model to find the optimal portfolio of these securities and identifies the advantages and disadvantages of the process. Also, four constraints of practical significance are added to these two models to observe the impact of factors in real financial markets on the portfolios. Different models under the same constraint and different constraints on the same model are also analyzed. It is found that portfolios under constraints have higher volatility and lower returns than portfolios under unconstrained conditions. Therefore, it is recommended that clients should be made aware of this cost and that non-legal constraints should be carefully considered.

However, there are shortcomings. For example, dynamic correlations are often assumed in empirical finance, and this paper uses constant correlations for calculations; therefore, exploring portfolio management using alternative approaches deserves further research.

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