Portfolio Analysis of Different Industries under Constraints

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Abstract. Building an effective portfolio is critical for it can decrease investment risk and the impact of fluctuations in the share price of a single firm. This paper selects five example stocks from diverse industries, using the mean variance model and the Index model to estimate the best combination of these stocks, then adds five significant limits to both models to see how components in the real financial market effect the portfolio. The Minimal Variance Frontier is used to compare the impact of five constraints in this paper, and it is discovered that different models have different effects under the same constraints, constrained portfolios have less volatility than unconstrained portfolios, and not allowing short positions reduces risk while also lowering the overall return rate. The findings in this paper are valuable to the industry's research on the best allocation of financial assets and can benefit financial market investors.

Keywords: Portfolio; Markowitz model; Index model

1. Introduction

The Markowitz mean-variance portfolio optimization model is a traditional model used to solve typical portfolio optimization issues [1]. In addition to the typical portfolio theory, the generalized portfolio theory also includes the derived various models based on Markowitz's theory such as Single-Index Model .For today's strong demand of investment, it is necessary to use this model to build an effective and reasonable portfolio, and diversification is a key approach to reduce risk in portfolio optimization, particularly for individual investors. Adding appropriate constraints to different realities will also provide the possibility of better asset allocation and make the portfolio more universal.

Portfolios and diversification is a Worth discussing topic, and there is a lot of research out there. However, after reading a large number of research papers on portfolio optimization, a major problem arises that most articles are aimed at stock selection of portfolios without considering the industry differentiation, which selected all industries or a single industry as a data sample. Li and Wang investigated the influence of portfolio diversification on bank systemic risk by integrating network effects and examining three types of interbank networks: stochastic networks, small-world networks, and scale-free networks [2]. Li, Wu, and Zhang incorporated the multifractal characteristics of capital markets into the investment portfolio research framework, overcoming the shortcomings of previous research results that did not take into account the multifractal fluctuation characteristics of asset prices and the characteristics of multifractal-related characteristics [3]. Rupinder created an optimum portfolio for the Indian computer software sector using the Sharpe index concept [4]. Panja, Campbell, and Kanjilal compare SRI and non-SRI portfolios depending on investors' risk tolerance to arrive at heuristic optimum portfolios [5]. From an integrated risk and return perspective, Stefan, Dennis, and Markus present a model that online retailers (e-tailers) can use to determine the best combination of customer segments within a customer portfolio. Markowitz's Portfolio Selection Theory is used to determine the best composition of customer portfolios [6].

This article focuses on different industries, with different representative stocks selected from the services, technology and financial sectors. The specific research process is as follows. First, this article selects SPX500 and the following 5 companies which are PG, KO, NKE, JNJ and MSFT for their closing prices in twenty years from October 4th 2002 to October 5th 2022.Second, 5,036 pieces of daily data were collected and filtered out as monthly data on the last day of the month. Third, the paper using excel to calculate return, annualized standard deviation, Sharp Ratio, beta, alpha, and correlation between 6 stocks. Fourth, this paper constructs a minimum volatility portfolio and a maximum Sharpe ratio portfolio for selected stocks; fifth, this paper uses the solver table to find the minimum variance boundary and capital allocation line under each constraint.

The structure of this article is as follows. The data used in this paper are displayed in Section 2. The two models used—the Markowitz model and the index model—are described in Section 3; the results are described in Section 4; and the article is summarized in Section 5.

2. Data

The data in this article is derived from Investing.com (https://cn.investing.com). This article selects SPX500 and the following 5 companies which are PG, KO, NKE, JNJ and MSFT for their closing prices in twenty years from October 4th 2002 to October 5th 2022. The reason for selecting these ten stocks is that they are among the most representative films of various industries in the US stock market, and they have been listed for over twenty years.

In this paper, 5,036 pieces of daily data were collected and filtered out as monthly data on the last day of the month. The paper then computed the difference between the adjacent month prices and divided it by the previous day's price to get the return. Following the analysis and calculation of these data, some basic statistics of these assets are shown in Table 1.

Table 1. Descriptive statistics of these selected assets

	SPX	PG	KO	NKE	JNJ	MSFT
Annualized Average Return	8.809%	6.447%	5.269%	16.946%	6.370%	14.727%
Annualized StDev	14.867%	14.871%	16.133%	22.993%	14.579%	22.814%
Sharp Ratio	0.5925	0.4336	0.3266	0.7370	0.4369	0.6455
Min	-16.519%	-11.841%	-17.274%	-21.916%	-13.330%	-16.564%
Max	12.698%	13.142%	14.219%	16.823%	14.421%	24.949%

As the table shows, 'NKE' has the highest average return while 'KO' has the lowest. In terms of standard deviation, 'NKE' has the highest, followed by 'MSFT', and 'JNJ' has the lowest. When their Sharp Ratios are compared, 'NKE' has the highest. Furthermore, 'NKE' has the highest average return, the highest standard deviation, and the highest Sharp ratio, indicating that stocks with higher returns are more risky and volatile.

3. Method

The significance of building a stock portfolio is to find an optimal portfolio structure based on investors' own risk appetite and expectations of returns [7]. The portfolio was built using the Mean-Variance Model and the Index Model.

3.1 Mean-Variance Model

The earliest method for resolving portfolio issues is the mean variance (standard deviation) model, and investment diversification is its cornerstone [8]. In this model, investors utilize mean-variance analysis to decide which investments to make, which means a rational investor should satisfy two assumptions: (1) the return is not satisfied, and (2) risk aversion [9].

According to Markowitz's portfolio theory, investors should constantly aim to maximize returns at the lowest feasible risk and minimize it as much as possible. Therefore, the precondition of Mean-Variance Model is as follows:

$$\sum_{i=1}^{n} Weight_i = 1 \tag{1}$$

where $\mathrm{Weight_i}$ is the weight of $\mathrm{asset_i}$ in a portfolio,and the Standard Deviation can be calculated as below:

$$portfolio\ StDev = \sqrt{\sum_{i=1}^{n} Weight_i^2 StDev_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} Weight_i Weight_j StDev_i\, StDev_j\, Cov(i,j)} \quad (2)$$

3.2 Index Model

The Single Index Model was proposed by William Sharp. The price of most stocks increases when the market price increases, and vice versa when the market price decreases. This demonstrates that different securities respond to market fluctuations in a similar way, and the yield of securities should be linked. The degree of correlation may be determined by comparing a security's return to the return of the stock market index. Sharp suggests that a stock index that accurately reflects the state of the stock market as a whole should take the role of the one-factor model's macro affecting elements [10]. The single-index model for stocks is:

$$r_i = a_i + b_i N + \varepsilon_i \quad (3)$$

Where RI represents the yield of the ith security; N stands for stock market stock price index; AI represents the market-independent portion of securities earnings; BI stands for how sensitive a securities yield is to the yield of a stock index.

4. Results

The correlation function is used in this study to find the optimal model. All of the correlations are non-zero, indicating that all of the assets are inter-correlated. With the exception of SPX, the correlation between any two stocks is not larger than 0.5. As a result, there is less connection between distinct shares and mutual funds. Additionally, all correlations are non-zero, showing that all assets are inter-correlated (See Table 2).

Table 2. Correlation coefficient between ten stocks

correlations	SPX	PG	KO	NKE	JNJ	MSFT
SPX	1.00	0.42	0.49	0.57	0.53	0.64
PG	0.42	1.00	0.50	0.31	0.50	0.22
KO	0.49	0.50	1.00	0.31	0.48	0.31
NKE	0.57	0.31	0.31	1.00	0.27	0.39
JNJ	0.53	0.50	0.48	0.27	1.00	0.35
MSFT	0.64	0.22	0.31	0.39	0.35	1.00

Constraints:

1. This extra optimization constraint is intended to emulate FINRA (https://www.finra.org/rules-guidance/key-topics/margin-accounts) regulation T, which permits broker-dealers to let their clients to hold positions that are financed to a minimum of 50% by their account equity:

$$\sum_{i=1}^{11} |w_i| \le 2 \quad (4)$$

2. This optimization restriction simulates some arbitrary "box" weight limitations that the client could provide:

$$|w_i| \leq 1 \text{ , for } \forall i \tag{5}$$

3. A "free" problem, with no further optimization restrictions, to show how the efficient frontier and the area of allowed portfolios in general appear in the absence of constraints.

4. This additional optimization constraint is intended to mimic the typical limitations that exist in the U.S. mutual fund industry: U.S. open-end mutual funds are not allowed to have any short positions, see Section 12(a)(3)(http://www.law.cornell.edu/uscode/text/15/80a-12) of the Investment Company Act of 1940 for details:

$$\mathrm{w_i} \geq 0 \text{ , for } \forall \mathrm{I} \tag{6}$$

5. Lastly, investors want to determine whether adding the broad index to our portfolio has a favorable or unfavorable impact. To that end, investors would like to take into consideration an additional optimization constraint:

$$w_1 = 0 \tag{7}$$

Based on all the constraints, this paper calculates the asset weights and the results are shown in the following Table 3 and 4, respectively.

Table 3. Portfolio performance of Markowitz model

MM (Constr1)	SPX	PG	KO	NKE	JNJ	MSFT
MinVar	23.780%	29.057%	15.232%	2.741%	27.081%	2.109%
MaxSharpe	-15.819%	22.721%	-10.026%	47.190%	19.189%	36.745%
MM (Constr1)	Return	StDev	Sharpe			
MinVar	7.217%	11.771%	61.312%			
MaxSharpe	14.119%	16.464%	85.759%			
MM (Constr2)	SPX	QCOM	AKAM	ORCL	MSFT	CVX
MinVar	30.954%	-2.194%	-1.050%	5.335%	-0.376%	-8.700%
MaxSharpe	-15.819%	22.721%	-10.026%	47.190%	19.189%	36.745%
MM (Constr2)	Return	StDev	Sharpe			
MinVar	7.232%	12.279%	58.893%			
MaxSharpe	14.119%	16.464%	85.759%			
MM (Constr3)	SPX	QCOM	AKAM	ORCL	MSFT	CVX
MinVar	23.780%	29.057%	15.232%	2.741%	27.081%	2.109%
MaxSharpe	-15.819%	22.721%	-10.026%	47.190%	19.189%	36.745%
MM (Constr3)	Return	StDev	Sharpe			
MinVar	7.217%	11.771%	61.312%			
MaxSharpe	14.119%	16.464%	85.759%			
MM (Constr4)	SPX	QCOM	AKAM	ORCL	MSFT	CVX
MinVar	23.780%	29.057%	15.232%	2.741%	27.081%	2.109%
MaxSharpe	0.000%	16.768%	0.000%	41.255%	11.721%	30.256%
MM (Constr4)	Return	StDev	Sharpe			
MinVar	7.217%	11.771%	61.312%			
MaxSharpe	13.220%	15.547%	85.031%			
MM (Constr5)	SPX	QCOM	AKAM	ORCL	MSFT	CVX
MinVar	0.000%	31.611%	19.112%	7.497%	33.288%	8.492%
MaxSharpe	0.000%	21.253%	-11.917%	43.231%	15.456%	31.976%
MM (Constr5)	Return	StDev	Sharpe			
MinVar	7.632%	11.972%	63.750%			
MaxSharpe	13.708%	16.045%	85.434%			

Table 4. Portfolio performance of Index model

IM (Constr1):	SPX	PG	KO	NKE	JNJ	MSFT
MinVar	2.317%	35.365%	25.088%	3.850%	32.071%	1.310%
MaxSharpe	-2.505%	23.492%	3.283%	35.534%	16.103%	24.093%
IM (Constr1):	Return	StDev	Sharpe			
MinVar	6.640%	10.884%	61.007%			
MaxSharpe	12.008%	14.637%	82.041%			
IM (Constr2):	SPX	QCOM	AKAM	ORCL	MSFT	CVX
MinVar	2.317%	35.365%	25.088%	3.850%	32.071%	1.310%
MaxSharpe	-2.505%	23.492%	3.283%	35.534%	16.103%	24.093%
IM (Constr2):	Return	StDev	Sharpe			
MinVar	6.640%	10.884%	61.007%			
MaxSharpe	12.008%	14.637%	82.041%			
IM (Constr3):	SPX	QCOM	AKAM	ORCL	MSFT	CVX
MinVar	2.317%	35.365%	25.088%	3.850%	32.071%	1.310%
MaxSharpe	-2.505%	23.492%	3.283%	35.534%	16.103%	24.093%
IM (Constr3):	Return	StDev	Sharpe			
MinVar	6.640%	10.884%	61.007%			
MaxSharpe	12.008%	14.637%	82.041%			
IM (Constr4):	SPX	QCOM	AKAM	ORCL	MSFT	CVX
MinVar	2.317%	35.365%	25.088%	3.850%	32.071%	1.310%
MaxSharpe	0.000%	23.024%	2.794%	35.088%	15.500%	23.594%
IM (Constr4):	Return	StDev	Sharpe			
MinVar	6.640%	10.884%	61.007%			
MaxSharpe	11.985%	14.614%	82.012%			
IM (Constr5):	SPX	QCOM	AKAM	ORCL	MSFT	CVX
MinVar	0.000%	35.811%	25.565%	4.229%	32.647%	1.747%
MaxSharpe	0.000%	23.024%	2.794%	35.088%	15.500%	23.594%
IM (Constr5):	Return	StDev	Sharpe			
MinVar	6.655%	10.890%	61.114%			
MaxSharpe	11.985%	14.614%	82.012%			

Through the table 3 and 4, investors can build portfolios with greater anticipated returns and lower standard deviations with the help of diversification.

5. Conclusions

This paper chooses five sample stocks from various industries, using the Markowitz model and the Index model to determine the best combination of these stocks, and adds five important limitations to both models to see how elements in the real financial market impact the portfolio. The impact of five constraints is compared in this paper using the Minimal Variance Frontier, and it is discovered that different models have different effects under the same constraints, constrained portfolios have less volatility than unconstrained portfolios, and not allowing short positions reduces risk while also lowering the overall return rate.

However, deficiencies also exist. Because there are fewer industries in which stocks are chosen, which is not in step with reality, this may result in some limits that have minimal influence on the outcomes.

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