Constructing An Optimal Portfolio Based on Markowitz And Nonlinear Programming Methods

Suning Jiao*

Department of business and economics, Monash University, Melbourne, Australia

*Corresponding author: Sjia0051@student.monash.edu

Abstract. Portfolio construction is extremely essential for the trading of stocks construct an optimal portfolio is one of the frequently asked questions in the financial sector as it is directly related to the profits of investors. While some people use financial theory only in their investments and do not combine it with mathematical methods. So, we explore how to construct an optimal portfolio through financial and mathematical models and theories based on the analysis of stock prices of some Chinese companies in the past year. We find that the stock of Xi'an Catering Co., Ltd and Joyvio Group Limited is extremely attractive for us, at 91 percent and 6 percent of our cash invested respectively. The Sharpe ratio is 0.09948, which means the reward of unit risk is 0.09948. So, we need to invest more in stocks with higher expected returns compared to others. Because it could bring more profits for us. Usually, the companies with an increased stock price during a period will satisfy the requirements as they have better development potential. Our study can also be improved by using the Fama-French three factors model and Machine learning.

Keywords: Markowitz; nonlinear programming; optimal portfolio; stocks.

1. Introduction

With the rapid spread of globalization, financial markets in various countries are also developing rapidly. More and more companies and people are also participating in trading financial assets in the financial markets. Therefore, Rationally making correct judgments in financial decisions like buying and selling financial assets is becoming more important compared to before. Because the right trade allows people to earn extra rewards. There are many different methods and theories for pricing and investing and selling financial assets like stocks. For example, the Black–Scholes option pricing formulas Black–Scholes option pricing formulas are used in the pricing of stock[1], and the Capital Asset Pricing Model for predicting the expected return of financial assets[2].

There are many quantitative analysis models widely recognized and used in the financial decisions of many companies like Deppon Securities Co., Ltd and Beijing Crayfish Technology Co., Ltd. However, pure financial theories and formulas are not necessarily the most accurate and effective in practical application. We also need to combine applications of mathematics and theory reasonably as mathematical models can help us observe the law of change, then predict it. But how should the two areas be closely linked? How to transfer real financial cases into mathematical problems, and solve them according to financial theory? These questions are very meaningful for improving financial decision-making.

This study uses relevant theories and models of mathematics and finance to construct the optimal investment portfolio based on the stock price data of five stocks in the Chinese market in the past year. We first collect and analyze the data stock price of five companies, and get the describable data like maximum value, minimum value, the mean, and standard deviation in section 2. In section 3, we use the Markowitz model to calculate the risk and expected. Afterward, we use nonlinear programming to find the optimal portfolio construction with the target of maximizing the Sharpe ratio and confirm it by graphing efficient frontier theory. Finally, we decide to invest almost 91% in Xi'an Catering Co., Ltd, 6% in JOYVIO Group Limited, and 1% in Huayi Brothers Media Group, Shanghai Transportation Co., Ltd, and North China Pharmaceutical Co., Ltd to maximize the Sharpe ratio. In section 3, we discuss both advantages and disadvantages of our approach. The same approach can be applied to a finite number of stock portfolio analyses in the reality.

2. Data

In this study, we collected the daily price of stocks of Xi'an Catering Co., Ltd, Huayi Brother media, Joyvio Group Limited, Shanghai Transportation Co., Ltd, and North China Pharmaceutical Co., Ltd.

Xi'an Catering Co., Ltd price is principally engaged in producing, processing, and selling Xi'an special food. The operating income of Xian catering is ￥0.73billion in 2022, and the net profit is ￥-0.28billbion. The maximum of its stock price is 11.64, and the minimum value is 3.67 which is lower than the mean value (5.06). The daily average return is 0.4478%, and the standard deviation is 1.6. The stock price increased from ￥4.2 (04/01/2022) to ￥11.63 (04/12/2022). The price is shown in Fig 1.

Film production and shooting is Huayi Brother media's main business. The total revenue for the Huayi brother in 2022 is ￥0.71billion, and the net profit is -￥0.59billion. Both the maximum value and minimum value of its stock price are lower than Xi'an Catering, ￥4.06, ￥2.03 separately. The average value and standard deviation are ￥2.08 and 0.03. Huayi Brother has been hit hard by Covid-19. The stock price dropped from ￥3.94 (i04/01/2022) to ￥2.72 (04/12/2022). This is the exact opposite of the phenomenon of Xi'an Catering. The price is shown in Fig 2.

Joyvio group mainly aims to sell high-quality health foods such as fruits and high protein. The total revenue for Joyvio group is ￥7.59bilion which is pretty higher than the previous two in 2022, and the net profit of Joyvio is ￥0.51billion. Joyvio's stock price got a peak at ￥32.97 which is the highest one among the five stocks. Similarly, the minimum value of it is also higher than others (￥15.36). The stock price of Joyvio is extremely unstable compared to the other four stocks. It experienced many large ups and downs since the beginning of 2022. So, the standard deviation is also very high (5.03). The price is shown in Fig 3.

Shanghai transportation mainly focused on the development of traffic management and manufacturing business in Shanghai. The total revenue in 2022 is ￥5.8billion, and the net profit is-￥0.56billion. The stock price is stable, and the difference between the maximum (￥5.08) value and the minimum value (￥3.49) is small. Therefore, the standard deviation is also at a low level (0.03). The difference between stock prices at the beginning and end of 2022 is not very large. The price is shown in Fig 4.

North China Pharmaceutical mainly engaged in medical care, especially for the research and development and prevention of Covid-19. The net profit is ￥0.12billion although the total revenue in 2022 is ￥15.7billion. The maximum value, minimum value, and standard deviation of its stock price are ￥12.97, ￥5.6, and 1.54, which is close to Xi'an catering's data. Generally, the stock price dropped a lot during 2022.

The five graphs below show the change in the price of these five stocks. The price is shown in Fig 5. Table 1 shows the descriptive statistics of these stock prices.

Fig 1. Xi'an Catering Co., Ltd price

Fig 2. Huayi Brothers Media Group price

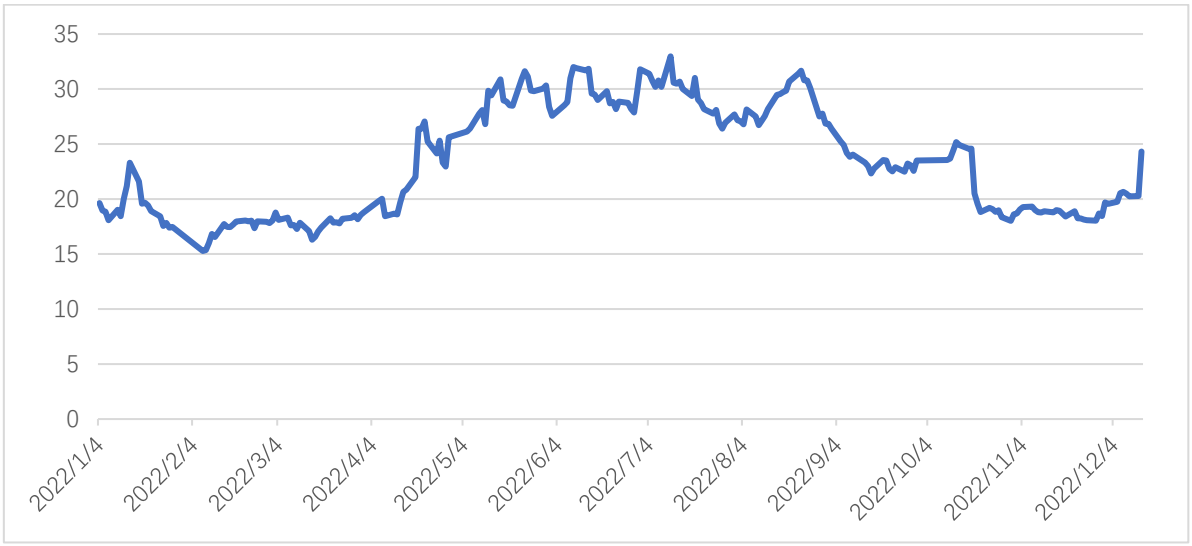

Fig 3. Joyvio Group Limited price

Fig 4. Shanghai Transportation Co., Ltd price

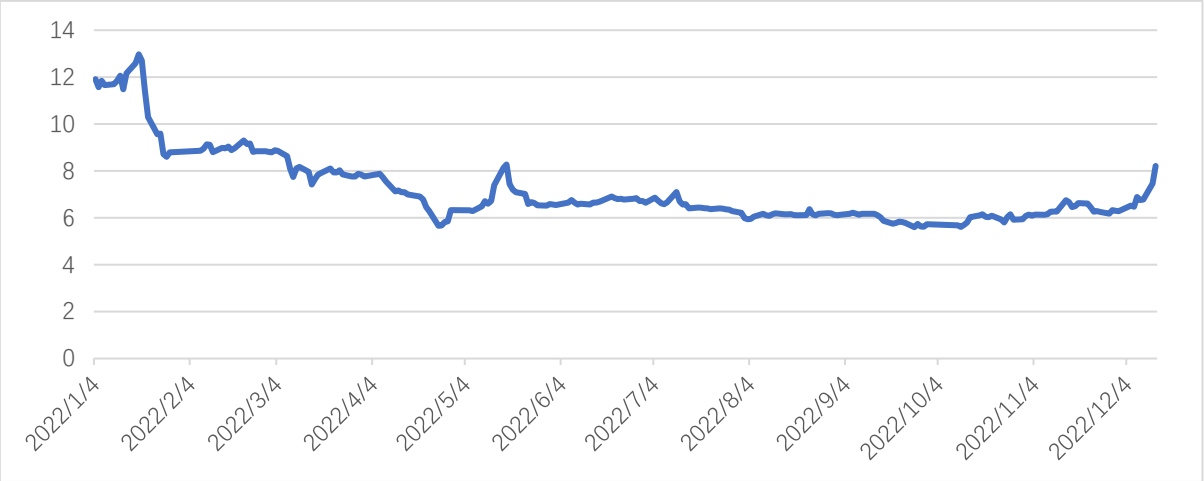

Fig 5. North China Pharmaceutical Co., Ltd price

Table 1. Describable data comparing

	Mean	Std	Min	Max
Xi'an Catering Co., Ltd	5.055721	0.105836	3.74	11.63
Huayi Brothers Media Group	2.80393	0.032258	2.03	4.03
Joyvio Group Limited	23.42629	0.332649	15.28	32.97
Shanghai Transportation Co., Ltd	4.153231	0.022510	3.49	5.08
North China Pharmaceutical Co., Ltd	7.166376	0.102256	5.60	12.97

3. Optimal portfolio construction

3.1 Methods

In this study, we use nonlinear programming in excel to find the optimal portfolio management for these five stocks we analyzed before according to the Efficient Frontier and Markowitz model.

Nonlinear programming means finding a finite number of variables to decide the objective function. Then, by setting constraints to make the objective function meet our requirements [3].

The set of all efficient portfolios with the highest expected return at a given level of risk is called the efficient frontier [4]. Capital market line plays a vital role in our portfolio analysis, especially for mean-variance portfolio theory. It can be graphed as the visualization of the determination of the

optimal portfolio [5]. It can be also described as the linear relationship between the risk-free rate and optimization. In the mathematical equation, it could be shown as flow [6].

$$CML = E(R_p) = r_f + \{E(R_m) - r_f\}\sigma_P/\sigma_m \quad (1)$$

$E(R_p)$: the expected return of the portfolio lying in the capital line
E(R_m): the expected return of markets
r_f: risk-free rate
σ_p: risk of the portfolio
σ_m: risk of markets

Markowitz's model is also called the mean-variance model of portfolio investment. The precondition of all investors is risk aversion [7]. The mean-variance model is aim to maximize the expected return of the portfolio and minimize the risk of it by calculating the standard deviation or variance [8]. The excepted return of a portfolio is the sum of the weighted expected return of each stock. The risk of a portfolio is calculated by the weighted matrix and the variance-covariance matrix. In the mathematical equation, these could be shown as flow [7]

$$E(R_P) = \sum_i W_i\, E(R_i) \quad (2)$$

R_p: the return of the portfolio
W_i **:** the weighted of each stock
R_i: the return of each stock

$$\sigma_p^2 = \sum_i W_i^2\, \sigma_i^2 + \sum_i \sum_{j \neq i} W_i W_j \sigma_i \sigma_j\, \rho_{ij} \quad (3)$$

σ_p: portfolio standard deviation
W_i: weighted of each $stock_i$
σ_i: standard deviation of $stock_i$
W_j: weighted of $stock_j$
σ_j: standard deviation of $stock_j$
ρ_{ij}: correlation between $stock_i$ and $stock_j$

3.2 Constructionn

In this case, assuming we have a ¥1million to invest among these five stocks, and the main purpose is carrying out reasonable investment management of this ¥1million to maximize the return.

At first, we collected the daily stock price of Xi'an catering, Huayi Brothers Group, Joyvio Group, Shanghai Transportation, North China Pharmaceutical, and China Treasury from January 4, 2022, to December 13, 2022. Because a large amount of data makes our results more accurate. Then we calculated the daily expected return of each stock, and find the mean return of them. After that, we use the variance function in excel to create the variance-covariance matrix of these five stocks. So, we could find the variance and the covariance we need directly. In this study, we use the Sharpe ratio as our objective function. Because the sharpe ratio is considered the most common metric of the performance of financial assets like stocks. Sharpe ratio is defined as the difference between the expected return of the portfolio and risk rate divided by the standard deviation of the portfolio [9]. In the mathematical equation, it can be shown as flow [10].

$$SR = \frac{\mu - R_f}{\sigma} \quad (4)$$

$\boldsymbol{\mu}$: mean of return
$\boldsymbol{\sigma}$: standard deviation of return
R_f: risk-free rate

Table 2. Variance-covariance matrix

	Xi'an Catering Co., Ltd return	Huayi Brothers Media Group return	JOYVIO Group Limited return	Shanghai Transportation Co., Ltd return	North China Pharmaceutical Co., Ltd return
Xi'an Catering Co., Ltd return	0.001927201	0.000523599	0.000243528	0.000331197	0.000410843
Huayi Brothers Media Group return	0.000523599	0.000789999	0.000297093	0.000303535	0.000287664
JOYVIO Group Limited return	0.000243528	0.000297093	0.001675691	0.000222127	0.000306948
Shanghai Transportation Co., Ltd return	0.000331197	0.000303535	0.000222127	0.000526928	0.000250756
North China Pharmaceutical Co., Ltd return	0.000410843	0.000287664	0.000306948	0.000250756	0.000880112

As we want to know how to allocate our ￥1million, we assume that the weighted of these five stocks are variables. The weights of Xi'an catering, Huayi Brothers Group, Joyvio Group, Shanghai Transportation, and North China Pharmaceutical are X, Y, Z, P, and Q separately. Since the expected return of single stock has been calculated. The total weighted return of this portfolio is

$$0.004478X-0.001746Y+0.000937Z+0.0002P-0.001632Q \quad (5)$$

and the subjective function is equal to

$$\frac{0.004478X-0.001746Y+0.000937Z+0.0002P-0.001632Q-0.000053}{risk\ of\ the\ portfolio} \quad (6)$$

In addition to maximizing the target equation, we also set a few constraints to make our decision model close to reality (Table 2).

All variables cannot be greater than 1, and the sum of them has to be one because they are weighted by different stocks. We also assume that all variables have to be greater than 0.1 as stocks are not allowed to be short-bought or sold in China.

Under these constraints, we find the optimization which is X=0.908119, Y=0.01, Z=0.061881, P=0.01, Q=0.01 by using the nonlinear programming decision model. So, we should invest almost a ￥0.91million in Xi'an catering, ￥0.06million in Joyvio Group, ￥0.01million in Huayi Brothers Group, Shanghai Transportation, and North China Pharmaceutical, and the risk, Sharpe ratio and expected return of this portfolio are 0.0406, 0.0995, and 0.0041 separately.

Finally, we make the risk be some special values like 0.025,0.02,0.03 to get the corresponding rate of return. So, we can draw an efficient frontier with these data. After calculating the equation of a

capital market line. We find that the capital market line is almost tangent to the efficient frontier. This proves once again that our portfolio is optimal (Fig 6).

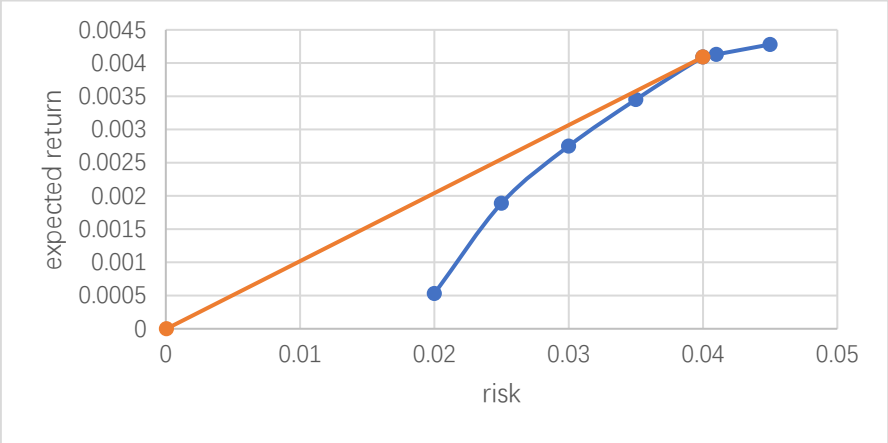

Fig 6. Efficient Frontier

4. Discussion

The primary advantage of our method is the ability to process large amounts of data. The mean return of a stock is been simplified to a single number. So we could analyze a larger amount of stocks. Besides, we just need to increase variables and modify constraints if we want to construct a portfolio with more than five risky assets. The nonlinear function in excel guarantees the accuracy of our method.

However, there are some disadvantages to our study. The calculation of our expected return is not precise. Because we do not take into account the impact factor in some practical situations. While, the Fama-French three-factors model can be used as a potential enhancement method because it considers the value factor, factor, and error term [11]. Besides, the portfolio construction needs to consume part of the human and material resources, especially in the case of insufficient time. Machine learning can solve the question. After a large amount of data detection and learning, artificial intelligence can make fast and accurate judgments about similar situations. So, our study can be developed by machine learning too.

5. Conclusion

In conclusion, we researched how to construct the best investment portfolio to maximize our profits in the case of known stock price information.

According to the Markowitz model and nonlinear programming model, we find that we should invest almost all cash in Xi'an Catering Co., Ltd and Joyvio Group Limited. Because they have the top two highest expected returns than others. It also illustrates that companies with increased price could be the primary goal in our investments, and we need to allocate the majority of cash to this kind of company.

The shortcoming of the current research is inadequate special risk and expected return used in the graphing efficient frontier. So, the efficient frontier is not accurate enough. In the future, we will provide a graph with more points not only lie the efficient frontier but also inside. Moreover, the application of the Fama-French three factors model and Machine learning can make research more accurate.

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