Optimizing Stock Portfolio using Markowitz Model

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Abstract. The stock market is a well-known illustration of an investing option with obvious returns. Its volatility provides a basis for carefully analyzing its prior results in an effort to forecast future market returns. This paper discusses the various models used for selecting investment portfolios and utilizes the Markowitz decision-making model to aid in selecting an optimal investment portfolio based on historical price data and US Treasury 10-year bond rate data. Four stocks were included in the model, with two popular and highly volatile stocks, and two fast-growing stocks. The Markowitz model that maximizes the Sharpe ratio favors the two fast-growing stocks, whereas the same calculations for minimizing risk favor the more volatile stocks, but only lowers risk marginally and reduce returns significantly. The paper further analyzed the accuracy of the Markowitz model in this specific case and provided the Fama French model as a usable alternative that will likely provide alternative suggestions that are more correlated to real-life scenarios.

Keywords: Stock market; Markowitz Model; Sharpe ratio.

1. Introduction

The stock market is a well-known example of an investment opportunity that has clear returns and risks and is updated on a daily basis. Its frequency of change is ground for precisely evaluating its past performances in hopes to predict future market returns. Multiple models have been developed that can be utilized in this instance, such as the Markowitz model [1] and the Fama-French model [2]. These models are widely recognized and utilized in supporting the decision making of investments. Their positive effects cannot be ignored and have proven to be effective in certain cases. The Markowitz model has demonstrated effectiveness, under specific circumstances, in selecting bond investments [3]. For this study, the Markowitz model will be utilized to aid in optimizing an investment portfolio. The risk-minimized portfolio yields much lower returns but only marginally lower costs than the risk-return balanced portfolio, as suggested by the model. However, it is still important to recognize the limitations of the model and alternatives may be better in improving the accuracy of the optimized portfolios.

In this study, we begin with an overview of the stocks that we have chosen to work with, as well as their respective prices history. We will proceed to discuss how the Markowitz model will be applied in this case and determine the optimal investment portfolio using the Markowitz model's output with our data. Additionally, we will interpret the numbers that the model returns and discuss ways of potentially improving the prediction.

2. Data and Company Selection

In this study, we selected two high-return stocks and two high-volatility stocks. Both Occidental Petroleum and ExxonMobil experienced significant growth in 2022 due to the increase in gas prices. On the other hand, Tesla and Gamestop carry very high volatility, are extremely popular, and can be highly profitable if entered at the correct time.

2.1 Tesla Inc

Figure 1. TSLA's stock price history

Tesla inc. is an American multinational corporation that specializes in electric vehicles, energy storage, and solar panel manufacturing based on a sustainable energy economy. It was founded in 2003 by entrepreneur Elon Musk with the goal of reducing the world's dependence on fossil fuels and combating climate change. Tesla designs and manufactures electric cars, solar products, and energy storage systems. The company is known for its innovative technologies and its mission to accelerate the world's transition to sustainable energy. Tesla is an example of an extremely popular and influential stock and has shown major upward trends in prior years, although declining in value drastically towards the end of 2022, as seen in figure 1. Contrary to the poor stock performance, Tesla announced in the latest report that its revenue was 74.8 billion US Dollars in 2022, and its net income was 11.2 billion, over tripled from its income in 2021 [4].

2.2 GameStop

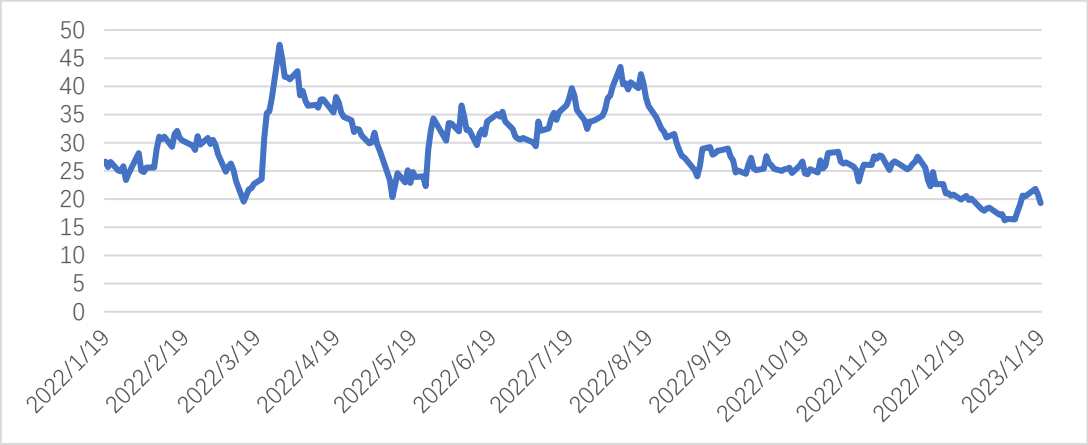

Figure 2. GME's stock price history

GameStop Corp. is an American video game, consumer electronics, and gaming merchandise retailer. The company was founded in 1994 and has its headquarters in Grapevine, Texas. GameStop operates more than 5,000 stores across 14 countries. It is one of the largest video game retailers in the world, offering new and pre-owned video games, consoles, and accessories for various gaming platforms. GameStop was the pioneer of "meme stocks" and gained massive popularity in 2021 as it skyrocketed unreasonably and dropped in a similar fashion. In 2022, this stock continued its tendency of extreme volatility and continues to be the center of focus for many freelance investors. Its

surprisingly high valuation is reflected by its current revenue, which totaled 5.94 billion, whereas it incurred a net loss of 0.52 billion in 2022 [5]. Nevertheless, it is apparent that its stock value has little relevance to GamtStop's actual performance and therefore is a highly interesting case to observe (Figure 2).

2.3 ExxonMobil

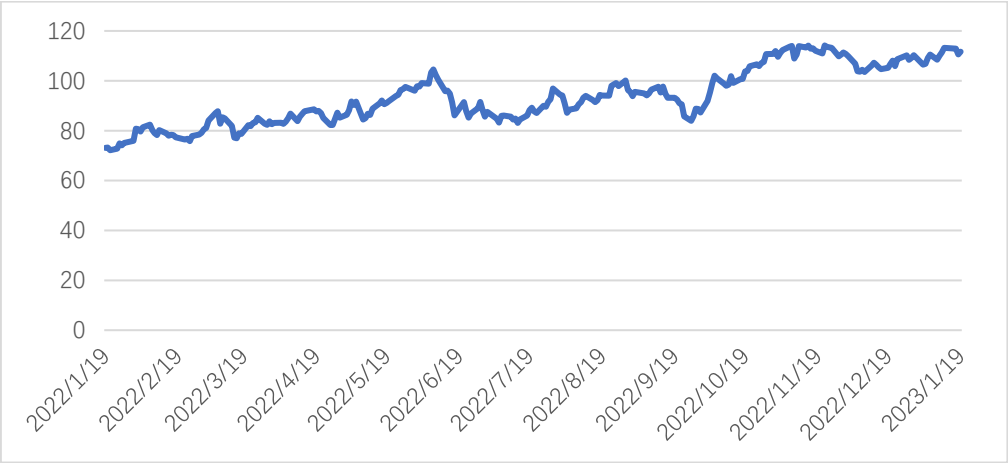

Figure 3. XOM's stock price history

ExxonMobil Corporation is an American multinational oil and gas corporation. It is one of the largest publicly traded oil and gas companies in the world. ExxonMobil was formed in 1999 by the merger of Exxon and Mobil (Figure 3). The company is involved in the exploration, production, transportation, and sale of crude oil, natural gas, and petroleum products. ExxonMobil also has significant refining, marketing, and chemical operations. It has witnessed significant growth in 2022 due to the war in Ukraine which drove international oil prices up drastically. Up to October 2022, ExxonMobil has an annual revenue of 403.2 billion US Dollars and a net income of 56.7 billion [6].

2.4 Occidental Petroleum

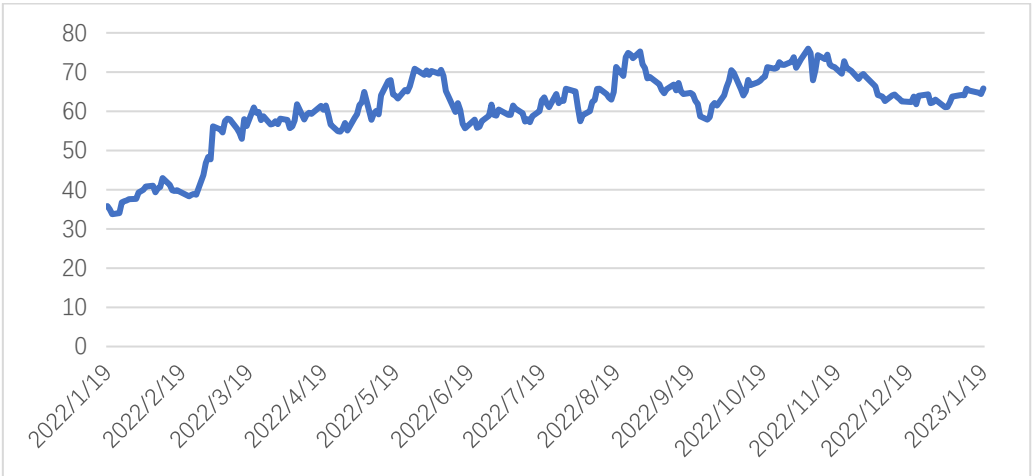

Figure 4. OXY's stock price history

Occidental Petroleum Corporation is an American multinational oil and gas exploration and production company whose stock price doubled in 2022-2023. It is one of the largest oil and gas companies in the world and is involved in the exploration, production, and transportation of oil and natural gas (Figure 4). The company also has significant operations in the petrochemical industry, through its wholly-owned subsidiary OxyChem, which is one of the largest North American

producers of chlorine and caustic soda. Its revenue was 36.32 billion in 2022, and its net income was 12 billion, ending its 2-year streak of net loss in every quarter [7].

3. Method and Outcome

Table 1. An optimized portfolio that maximizes Sharpe ratio

	WEIGHT	ER	W*ER	MIN	MAX
TSLA	0	-75.69%	0	0	1
XON	0.570191663	48.44%	0.276187893	0	1
GME	0	15.04%	0	0	1
OXY	0.429808332	75.03%	0.322483476	0	1

Table 2. Returns of the Optimized Portfolio

SUM OF WEIGHT	0.999999995
ER(P)	0.598671369
RF	0.030393775
PORTFOLIO RISK	0.025013246
SR	22.71906641

For this scenario, we will craft an optimal investment portfolio with the abovementioned stocks given the data of their prices in the past year (2022/01/19-2023/01/19). Utilizing the Markowitz model, we collected historical data on the stock prices of the four companies that we are optimizing and used the historical data to calculate the expected return and risk for each of the four companies. The Sharpe ratio will be an important factor to analyze, as it provides insights into the risk vs. return of a given portfolio [8-9], and one of this study's goals is to maximize it, which gives the best outcome that balances risks and returns. As we can observe, this portfolio yields nearly 60% of expected returns with a risk of 0.025 (Table 1 and Table 2).

Table 3. An optimized portfolio that minimizes the risk

	WEIGHT	ER	W*ER	MIN	MAX
TSLA	0.176182159	-75.69%	-0.133350977	0	1
XON	0.780234106	48.44%	0.377927683	0	1
GME	0.043583739	15.04%	0.006553868	0	1
OXY	0	75.03%	0	0	1

Table 4. Returns of the Optimized Portfolio

SUM OF WEIGHT	1.000000005
ER(P)	0.251130574
RF	0.030393775
PORTFOLIO RISK	0.019776858
SR	11.16136841

Some other risk-avert investors, they may be more inclined to lower the risk of this portfolio as much as possible (Table 3). However, this may not yield an optimal solution as the opportunity cost of giving up returns for lower risk is quite high in this portfolio. Contrary to the prior trial, this one only yields an expected 25% return, while lowering risk only marginally compared to the previous setup (Table 4).

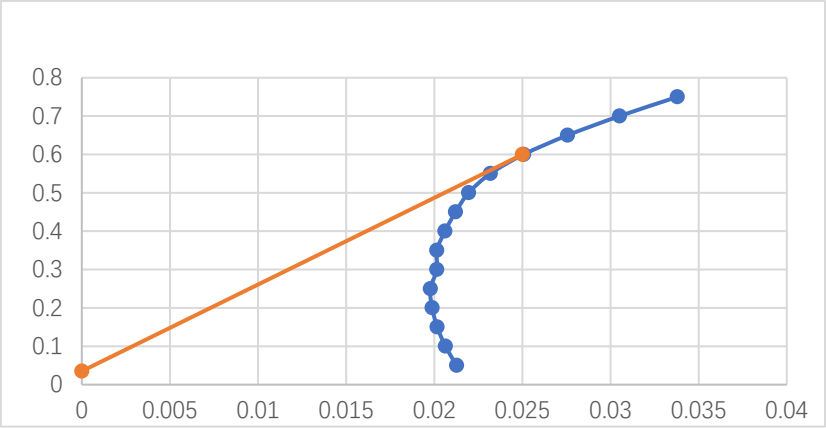

Figure 5. Efficient Frontier

The efficient frontier graph would aid in interpreting returns and risk. In this scenario, the efficient frontier comes quite friendly to risk-averse investors as its risk only starts increasing significantly after the expected return reaches around 55%, but for risk-seeking investors, it could come as a disappointment as the maximum potential returns caps at 75%, and investors will suffer more increase in risks than in returns after the Sharpe-ratio maximized point at 59% returns and 2.5% risk. However, this optimization has its limitations and may be inaccurate in predicting the actual returns of these stocks, and the specific limitations will be discussed (Figure 5).

4. Discussion

In this specific case, it is important to note that all four stocks chosen carry high volatility, especially Tesla and Gamestop, and the parameters that the Markowitz model uses may not be sufficient enough to calculate the true expected returns of these stocks. With only past prices as a Moreover, as the stock prices of oil-related companies vary in a close relationship (need citation) to international oil prices, companies that performed well in 2022 like ExxonMobil and Occidental Petroleum might not carry their outstanding performance to the following years as oil prices are determined by many factors and can be volatile in the instance of unexpected events, such as the outbreak of COVID-19 causing oil prices to drop drastically, while the war in Ukraine drove up oil prices significantly, and crude oil prices are found to be closely linked to oil companies' stock prices [8]. These high levels of uncaptured uncertainty, on top of many well-developed alternatives to the Markowitz model, could indicate potential issues with the accuracy of the Markowitz model, thus it's essential to consider these limitations and combine the Markowitz model with other methods in selecting an investment portfolio.

Table 5. Fama French 3 Factor Model's Regression Output of this study's Portfolio Returns

	Estimate	Std. Error	T value	Pr(>|t|)
(Intercept)	0.03774	0.02300	1.641	0.13947
MKT_RF	1.08100	0.31937	3.385	0.00957**
SMB	0.36325	1.11844	0.325	0.75368
HML	1.63030	0.51206	3.184	0.01292*
	Regression Statistics			
Multiple R	0.7632			
Adjusted R Squared	0.6744			

Using the conditions of the optimal portfolio that maximizes the Sharpe ratio, we get the above Fama French three-factor regression output. In interpreting these data, we can observe that the Fama-French model is fairly helpful in explaining the portfolio's returns as the three factors can help explain 76% of the portfolio's returns. The regression on the model that maximizes the Sharpe ratio

significantly outperforms the risk-free market return, but the P ratio suggests that this might not be statistically significant. Nevertheless [9-10], the Fama French model suggests that the high returns are linked to much higher risks in the market, which could support the model that seeks to minimize risk levels. Moreover, this portfolio focuses on companies with rather smaller market caps, as indicated by its somewhat high SMB factor, but this statement is still statistically insignificant considering its very low significance level. Additionally, the high HML factor suggests that we are primarily concerned with a value fund, as the regression output suggested. In this scenario, the Fama-French trial gave more insights into the portfolio as it adds additional parameters to capture more of the underlying uncertainties of these stocks. The Fama-French model adds size and value, which are not present in the traditional Markowitz model. The size factor captures the returns of small-cap stocks and the value factor captures the returns of low P/E ratio stocks. In this case, Fama French's size factor could indicate giving more weight to Occidental Petroleum, which was largely underlooked in the Markowitz model trial, especially in terms of the lowest-risk example. Moreover, the added value factor could help in re-weighing the proportions of the two oil companies and Gamestop, as these firms have much lower PE ratios compared to Tesla, indicating that Tesla, despite its bad 2022 stock performance, could be a great value stock, although it was essentially ignored in the Markowitz model (Table 5).

5. Conclusion

Overall, with the given portfolio that includes two popular but volatile stocks and two rapidly growing stocks, the Markowitz model suggests that the Sharpe-ratio maximized portfolio should consist of 57% of ExxonMobil stocks and 43% of Occidental Petroleum stocks, whereas the risk-minimized portfolio should include 18% Tesla stocks, 78% ExxonMobil stocks, and 4% Gamestop Stocks. However, the risk-minimized portfolio only lowers risk by a small percentage but lowers return by over 60% compared to that of the sharp-maximized portfolio. Moreover, it is important to recognize that the Fama-French model could generate more accurate outcomes in this case, and should be considered for future scenarios.

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