Theory and Practical Application Based on Game Theory

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Abstract. Game theory refers to the method in which one of the interacting parties obtains the decision-making information of the other party and uses it to influence the other party in turn. Therefore, game theory is also called "game theory". Game theory can be divided into cooperative game and non-cooperative game. In recent decades, the theory of game theory has developed rapidly with the establishment of John Nash and has been applied to many fields such as medical, engineering, military, and business. In this paper, the author starts from the basic definition of game theory and analyze the prisoner's dilemma, concordance fallacy, median voter theorem, Gounod model, Bertrand model, iterative deletion of dominated strategy, etc. through specific cases. The research results show that game theory has been widely used in the economy. For bilateral games, the results of Nash equilibrium are unstable. The research in this paper broadens the theory of game theory and has important practical significance for the application research of game theory.

Keywords: Game theory; Nash Equillibrum; dominated strateg.

1. Introduction

Game theory is now widely used in medicine, for example in the treatment of diffuse cancers, where initial systemic therapy is effective but oncology presents a huge obstacle to a complete cure due to the resistance of malignant cells and their ability to adapt under Darwinian selection [1]. In this model, the treatment strategy of the physician and the resistance strategy of the cancer cells form a game-theoretic game. However, this game is asymmetric; the physician is rational in the game, but the cancer cells can only adapt to the current environment and cannot predict the physician's behaviour; furthermore, there is a clear leader effect in cancer cells, where the leader adapts first and drives the other cells [2]. Therefore, unilateral changes in treatment strategies by physicians cannot significantly and effectively change outcomes [3]. As a result, physicians can adopt more dynamic treatment protocols and continually adjust treatment plans to improve outcomes [4].

In addition, breaking down the cooperation between tumours is an ideal strategy for making improvements in cancer. A comprehensive understanding of cooperation between tumour cells requires the methods and concepts of evolutionary game theory, which has been successfully used in other areas of biology to understand similar problems, but is underutilized in cancer research. Game theory can provide insight into the stability of cooperation between tumour cells and the design of anti-evolutionary treatments that have the potential to disrupt this cooperation [5].

Game theory also provides a rigorous mathematical method to evaluate and predict stakeholder interactions [6, 7]. Research in game theory on construction engineering and management (CEM) Findings show the contribution of game theory in seven CEM applications (conflict resolution; bargaining and negotiation; distribution of cooperative benefits; governance and oversight; evolution of cooperation and trust; construction bidding; and risk allocation) [8-10]. Social network analysis (SNA) also reveals the need to further investigate the interactions between CEM stakeholders within PDS beyond the traditional design-bid-build and public-private partnerships. In addition, games featuring strategic alliances and multi-agent supply chains can more fully reflect actual interactions [11, 12].

2. Theoretical Analysis

Assume that there are two people which both has two strategies, then it will have four outcomes (player 1 and 2 both choose strategy A, player 1 and 2 both choose strategy B, player 1 choose strategy

A and player 2 choose strategy B, player 1 choose strategy B and player 2 choose strategy A). So, four different outcomes correspond four different payoffs for these two people. As a result, player's strategy A strictly dominates his strategy B, if his payoff from A is strictly greater than that from B regardless of what others do.

2.1 Prisoner's Dilemma

The first classical concept in game theory, do not play a strictly dominated strategy. This leads to the first classic case of game theory, the Prisoner's Dilemma. In the Prisoner's Dilemma case, the background is such that two accomplices are brought into two different interrogation rooms and each has two choices, to confess to the crime or deny the charge, corresponding to four outcomes: if both deny the charge, both serve one year in prison (corresponding to gains of (-1, -1)), if one pleads guilty and the other pleads not guilty, the pleader is immediately released and the non-pleader serve 10 years (corresponding to gains of (0, -10) or (-10,0) respectively), and if both plead guilty, both serve 5 years (corresponding to gains of (-5, -5) respectively (in Table 1).

Table 1. Prisoner's Dilemma

	Deny	Confess
Deny	(-1,-1)	(-10,0)
Confess	(0,-10)	(-5,-5)

For instance, for the first co-offender, his optimal strategy should be to confess to the crime if the other denies it; if the other one confesses to it, his optimal strategy is likewise to confess to it. Thus, under this analysis, both offenders will confess to the crime, resulting in a sentence of five years each. In Table 1, the best results for them is both not to confess, but they end up with a relatively bad outcomes.

So it follows the second basic idea of game theory: Rational choice can lead to bad outcomes. There are many realistic cases which is given by this idea. Firms who are competing in prices, both firms have an incentive to undercut the other firm. If both firms behave that way, prices will get driven down towards marginal cost and industry profits will suffer, then the price war happens.

Consider another case, as shown in Table 2:

Table 2. Best response

	A	B
A	(0,0)	(-1,-1)
B	(-3,3)	(1,1)

Based on different choice of second person, there is no best response for first person. But from the second person's point of view, no matter what choice the first person makes, the second person's superior strategy is always A. Therefore the first person guesses that the other person will rationally choose the strategy A that is advantageous for him or her, and in the case that the second person chooses A, the first person should choose A.

Therefore, sometimes when one player cannot determine his own superior strategy, considering the problem from the opponent's point of view and determine optimal decision by the opponent's superior strategy is relatively wise. Therefore, the third basic idea of game theory is to put yourself in other's shoes and try to figure out what they will do.

In a game of chance, the following factors are necessary: players, strategies, payoffs. In the broader game situation, players often do not have strictly dominant strategies, but there are weakly dominated strategies.

Table 3. Weakly dominated strategies

	E	H
E	(1,1)	(1,1)
H	(0,2)	(2,0)

For example, in Table 3, it can see that for the offense, when the defender chooses strategy E, the offense gains the same amount regardless of which strategy it chooses, but when the defender chooses strategy H, the offense chooses H as the dominant strategy. Therefore, for the offense, strategy E is weakly dominated by H. This is a strategy that provides the same or worse results than the alternative strategy.

In the previous cases of game theory were studied games between two people, in a wider range of situations the game theoretical ideas are still present. For example, in the game of guessing numbers, each person in a classroom chooses a number between 1 and 100, averages the numbers written by all, and the winner is the person who chooses the number closest to two-thirds of the average.

This game is a good example of applying the weakly dominated strategy, all people choose among 100 numbers. When thinking about the problem from someone else's point of view, the author finds that the better strategy for all people is not to choose a number greater than 66. Therefore, for most people, the number they choose will be below 66, so the extreme case is reassumed and 44 becomes the optimal answer if all people choose 66. Noting that 44 is the optimal answer assumes that the majority of people will choose 66, but someone who chooses 66 assumes that everyone will choose 100, the question arises as to the perception of the surrounding players, i.e., whether the surrounding players are as stupid as the player assumes.

There is a difference in the better strategy between a classroom that has studied game theory and a classroom that has not studied game theory. In this game, the process of continually eliminating the worse strategy is called iterative elimination. Since no one will choose the number 100, everyone will choose a number below 66, and since no one will choose a number above 66, everyone will choose a number below 44; at this point, if everyone in a high-level game theory classroom knows how to iterate, everyone knows that their opponent is cunning enough, and thus the number will keep getting closer to 1. In fact, in the Yale game theory classroom, the optimal answer to the first guessing game was already around 17, but when everyone played the same game a second time after learning the iterative rejection method, the optimal number was already in the middle of 2 or 3.

Thus, it can be found that in a game theory it is very important to clearly know and understand the opponent. Depending on whether the opponent is a small child with no knowledge of numbers at all or a game theory master, it is obvious that completely different conclusions will be reached. This is the beauty of game theory, in calculating numbers or other optimal strategies is that each person has to consider the impact that other people bring to this game and what decision is optimal given the different decisions of other people.

2.2 Median Voter Theorm

Since voters tend to vote for candidates who are close to their political leanings, candidates who are located in more neutral leanings will always get more votes when they vote, therefore, candidates crowding close together towards the center, to try and get as many voters who are close to them. In economics, firm's crowding together to try and get shoppers who are close to them.

The application of this voter model in the real world is still very limited. First, in the real world, voters are not evenly distributed, and candidates may have many people, leading to more complex situations. In addition, the problem of voter abstention is not considered in this model. In this model it is assumed that candidates can choose their political positions arbitrarily, however, in real life it is often not feasible to choose political positions arbitrarily, and voters are aware of candidates' past political decisions, so it is not credible that candidates have changed their political positions more dramatically.

In some cases, the probability of the opponent taking each strategy in the game can be obtained from historical data, for example, in the goalkeeper model, L represents the left side, R represents the right side, U represents the top, M represents the middle, and D represents the bottom. The meaning is that the goalkeeper can save to the left and right, and then the attacker can choose to shoot from high, medium and low angles. The benefits are as in Table 4.

Table 4. Expectation of a game

	L	R
U	(5,1)	(0,2)
M	(1,3)	(4,1)
D	(4,2)	(2,3)

Assuming that the probability of a goalkeeper making a save to the left or right is (1/2, 1/2), the expectation of each angle of goal can be obtained. Where the expectation of U, M is 2.5 and the expectation of D is 3, so under this condition, D is the optimal strategy. According to the difference of probability, the image of the function of expected return can be obtained, and from the image change of the expected function we can see that the best response changes from U to D to M as the probability of the goalkeeper saving to the right increases.

3. Nash Equilibrium

Nash Equilibrium is a kind of balance where no player has strict incentive to deviate, which is also called no regrets. The results of Nash equilibrium are shown in Table 5.

Table 5. Nash Equilibrium

	L	C	R
U	(0,4)	(4,0)	(5,3)
M	(4,0)	(0,4)	(5,3)
D	(3,5)	(3,5)	(6,6)

First of all, find the best response of different strategies, in a decision combination of both sides of the decision are each other's best response, the decision combination will reach the Nash equilibrium, in the decision with linear function. The following results can be obtained in this paper: when different decisions corresponding to the intersection of the curve, the intersection point is the Nash equilibrium.

Nash equilibrium is often a decision point that neither side will "regret", from another point of view, assuming that both sides choose the strategy of the Nash equilibrium point, then after this game, in the next identical game, both sides have no tendency to change their strategies.

In some cases, there may be more than one Nash equilibrium point in a game, for example, in the game shown in the Table 6, (1,1) and (0,0) both reach Nash equilibrium, but the better choice here is (1,1). The point (0,0) is also a Nash equilibrium, but we refer to this worse equilibrium as the concordance fallacy.

Table 6. Concordance fallacy

	a	b
a	(0,0)	(3,-1)
b	(-1,3)	(1,1)

Next, the author introduce an investment game to better illustrate the causes and solutions of the concordance fallacy.

In the investment game, the alternative strategies are not to invest or to invest \$10. If you do not invest, the corresponding return is 0. If you invest \$10, then there will be two scenarios, when more than 90 percent of the population invests, each person will make a profit of $5. Once less than 90 percent of the population invests, then not only will there be no profit, but the $10 invested will also be a total loss.

In this investment game for game theory modeling, this model has two Nash equilibrium points, respectively, everyone does not invest, no one loses and no one makes a profit; or all per capita investment, when everyone can make a profit of $5. If everyone does not communicate before the start of this investment game, the probability is that this will lead to a result where some people invest and others do not, resulting in a loss for some people. But if, before the investment game starts, a leader comes forward and suggests that all people invest and go together to get a profit of $5 each. This is when the results change significantly, leading to almost all of the people investing per capita, more than 90 percent, and thus gaining profit.

The bank runs that occurred in the last century were based on this principle. When banks first appeared, people were generally unable to trust them and reached a poor Nash equilibrium, which led to bankruptcy, and as the trust relationship between banks and customers was established, everyone trusted the banks and deposited money with them, thus reaching a better Nash equilibrium.

Thus the biggest difference between the concordance fallacy and the prisoner's dilemma. The coordination fallacy can be improved by early communication, which can shift the coordination fallacy from a collectively worse Nash equilibrium to a collectively better Nash equilibrium, thus obtaining a collectively better game outcome, whereas the verbal constraint of the prisoner's dilemma is not binding, and early communication does not lead to a change in outcome; the Nash equilibrium is the only one.

So the coordination problem is different from prisoner's delimma. there is a 'scope for leadership'. For example, in a situation where, after communication, both parties are guaranteed to reach a Nash equilibrium in (u,l) or (d,r), the complementary game is called games with strategic complements. The results as shown in Table 7.

Table 7. Games with strategic complements

	l	r
u	(1,1)	(0,0)
d	(0,0)	(1,1)

Game theory can also be applied in daily life such as the sexual war.

Table 8. Sexual war

		he		
		BU	GS	SW
she	BU	(2,1)	(0,0)	(0,-1)
	GS	(0,0)	(1,2)	(0,-1)
	SW	(-1,0)	(-1,0)	(-2,-2)

In Table 8, (BU, BU) and (GS,GS) are two Nash equilibria, which means that in order to reach the Nash equilibrium, one party must make concessions and give up part of its own interests in exchange for relatively better interests.

4. Other Models

4.1 Gounod model

In game theory, there is a model about the change of output and price: the Gounod model. The participants of this game are two companies, respectively, and their corresponding strategies are the quantity of the product produced, while the production of the product bears a certain cost, and the output is linearly related to the total quantity of the product produced by the two companies.

In this model, when two firms produce enough output, the price of the product will be reduced to the cost price. The output that can reduce the market price to the cost price is called perfectly competitive output, meaning that when one firm produces the product at this output, there will not be a second firm in the market that produces the same product. The model illustrates how the output decisions of firms that compete with each other and do not coordinate with each other interact to produce an equilibrium outcome that lies between perfect competition and perfect monopoly. The conclusions of the Gounod model can be easily extended to the case of three or more oligopolistic firms.

4.2 Bertrand model

The Bertrand model also uses manufacturers' output as a means of competition, but the Gounod model is a classic model of output competition, whereas the Bertrand model is a model of price competition. In the Bertrand model, each oligopolist competes by choosing a price; each oligopolist produces the same product and there is no formal or informal collusion between manufacturers. In the Bertrand model, whoever has a low price wins the entire market, and whoever has a high price loses the entire market, so the oligopolists cut each other's prices until the price equals their respective marginal costs. Thus, in the Bertrand model, perfect competition occurs as long as the number of firms is no less than two. This is inconsistent with actual experience. The assumptions of the Bertrand model are therefore problematic because, in reality, there is a limit to the firm's production capacity and it cannot supply the entire market, so prices do not fall to the marginal level. Different firms tend to produce similar but not identical products, so products that are not identical can directly avoid price competition.

5. Conclusion

The research of cluster behavior in complex network environment has become a hot issue in the current academic field. At present, the application theory of game theory in economics is more and more extensive, but the systematic description of the model principle in game theory and the realization process of Nash equilibrium is relatively lacking. The carding and introduction of basic models in game theory is conducive to understanding the impact of game in cluster behavior, and has important and practical significance for expanding the theory and application of game theory. In this paper, the author starts from the basic definition of game theory and analyze the prisoner's dilemma, concordance fallacy, median voter theorem, Gounod model, Bertrand model, iterative deletion of dominated strategy, etc. through specific cases. At the same time, the research results show that game theory has been widely used in the economy for bilateral games, the results of Nash equilibrium are unstable.

There are still some deficiencies in the current research. The research in this paper focuses on the theoretical description, and the lack of data derivation of the model system makes the analysis problem may not be in-depth. Therefore, in the future, when data are available, this paper will use a combination of quantitative and qualitative methods for research, which will enrich the research content of this paper.

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