Optimizing Patrolling Operations with Multiple Traveling Salesmen Problem: A Formulation and Simulation Approach

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Abstract. Integer optimization program is a useful tool for solving realistic problems especially in assigning tasks problems. Traveling salesman (TSP) problem is one of the most classic subjects under integer program. This paper mainly focuses on the multiple traveling salesmen (MTSP) problem, which introduces two ways of formulations for the MTSP problem. The first program is formulated by tracking the traveling direction for each 'salesperson' when they are visiting different locations. On the contrary, the second program does not track the moving direction for salesmen and instead, which only tracks which edge between two locations is assigned to a certain salesperson. The two formulations do share some similarities and have some differences. This study briefly compares the complexity of the two models by counting the number of constraints for two formulations as well. In addition to a simulation test based on the two formulated optimization program, the limitations regarding the two formulations are discussed. According to the analysis, TSP is a classic problem under optimization, hence MTSP is meaningful for companies especially in assigning multiple tasks to workers or machines.

Keywords: Linear optimization; operation research; M-TSP problem; integer program.

1. Introduction

The Traveling Salesperson Problem (TSP) is a well-known problem in the field of integer programming, dating back to 1832 when it was first introduced in a handbook about finding tours through Germany and Switzerland [1]. Although a mathematical model for TSP was not established until 1948 by Flood [2], it has since become a benchmark problem for researchers to solve various optimization problems, such as maximizing profits while selling products to different places [3], assembling rolling schedules in the process of producing steel [4] and optimizing the operation sequence of a chip placement machine [5].

In a TSP problem, the salesperson need to visit each location exactly once and return to the beginning location at the end, with the goal of minimizing the total distance or cost for traveling. However, when multiple salespersons are involved, the problem becomes the Multiple Traveling Salespersons Problem (mTSP), where different salespersons are assigned to visit different locations to ensure that each location is visited exactly once by a certain salesperson. The MTSP model is a derivative of the TSP model and can solve a variety of problems. For instance, the model can help allocate and schedule rescuing units to decrease casualties and lower financial losses during natural disasters [6]. Additionally, it can assist in designing the most cost-efficient offshore wind farm's collector system [7] and establishing a parcel delivery system with different transportation modes [8]. These applications demonstrate the versatility of the mTSP model and its potential in solving real-world problems.

This study aims to introduce a formulation for the mTSP problem, with a focus on patrolling scenarios considering the random movements of officers. To provide a vivid illustration, a simple example generated by random data will be simulated using a computer program, with Gurobi [9] serving as the solver. The rest part of the paper is organized as follows. The Sec. 2 introduces the formulation of the mTSP problem. The Sec. 3 demonstrates the simulation of the problem using random data. The Sec. 4 discusses the limitations and prospects of this study. Finally, Section 5 concludes the paper and provides directions for future research.

The objective of this study is to introduce a formulation for the Multiple Traveling Salesperson Problem (mTSP) and analyze its advantages and disadvantages. To provide a clear illustration of the formulation, a simulation of a simple example will be presented. In contrast to the traditional setup for salesperson problems, this study will employ a different setup that focuses on a patrol considering the random movements of officers.

2. Formulation

2.1 Integer Program for MTSP

Assuming there are m vehicles available for patrolling at campus safety center and there are n total locations that needed to be visited within a patrol, one sets the campus safety center as location 0. In this case, following parameters can be defined:

$$X_{k,i,j} = \begin{cases} 1,\ if\ campus\ safety\ vehicle\ k\ visit\ location\ j\ from\ location\ i \\ 0,\ else \end{cases} \tag{1}$$

$$C_{i,j}: representing\ the\ cost\ for\ vising\ location\ j\ from\ location\ i \tag{2}$$

$$D: working\ capacity\ for\ each\ secruity \tag{3}$$

Here, $0 \le i \le n, 0 \le j \le n, 1 \le k \le m, i \ne j$. Considering above parameters, one gives the objective function to minmize the T with constraints:

$$T \ge \sum_{i,0\le i\le n} \sum_{j,0\le j\le n} C_{i,jk,i,k}\ , \forall k \tag{4}$$

$$\sum_{j,0\le j\le n} \sum_{k} X_{k,i,j} = 1, \forall i \tag{5}$$

$$\sum_{i,0\le i\le n} \sum_{k} X_{k,i,j} = 1, \forall j \tag{6}$$

$$\sum_{j} X_{k,0,j} = 1, \forall k \tag{7}$$

$$\sum_{i} X_{k,i,0} = 1, \forall k \tag{8}$$

$$\sum_{i,0\le i\le n} X_{k,i,j} = \sum_{g,0\le g\le n} X_{k,j,g}\ , \forall k, j \tag{9}$$

$$\sum_{i,j} X_{k,i,j} \cdot C_{i,j} \le D, \forall k \tag{10}$$

$$X_{k,i,j} \in 0{,}1, \forall k, i, j \tag{11}$$

In the mTSP model, the objective function is to minimize the maximum total distance traveled by all campus safety vehicles. To avoid the model becoming nonlinear, this study introduces a new variable, T, which represents the maximum total distance traveled by any vehicle. Constraints (5) are included to ensure that T is greater than or equal to the total distance traveled by each vehicle. This guarantees that the objective function is minimized while also ensuring that each vehicle travels a feasible distance.

First, constraints (6) ensure that each location is entered by a vehicle exactly once in the sum of all vehicles, while constraints (7) guarantee that each location is left by a vehicle exactly once in the sum of all vehicles. Moreover, constraints (8) ensure that each campus safety vehicle starts patrolling from location 0 (the campus safety center), and constraints (9) ensure that each vehicle returns to location 0, thereby ensuring that each vehicle is used in a patrol. Additionally, constraints (10) limit the working time of each campus safety officer in the vehicles to a certain maximum value, reflecting real-life constraints. Constraints (11) impose binary restrictions on the variable $X_{k,i,j}$, allowing it to take only two values, 0 and 1.

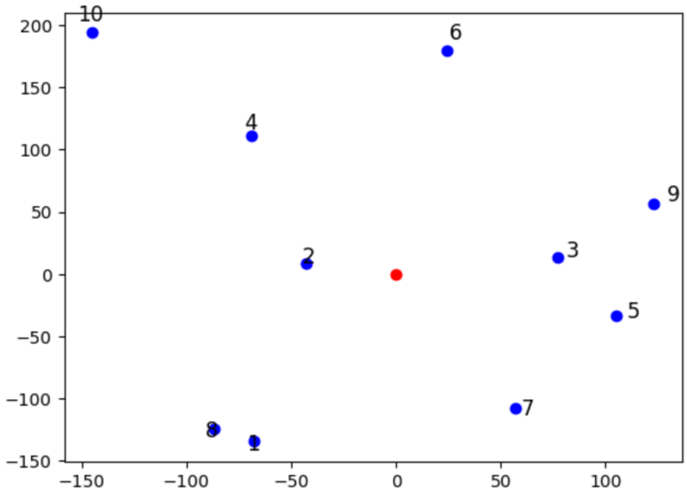

Fig. 1 Map of the10 locations.

To ensure that the proposed model does not create sub-tours, one has included a sub-tour elimination constraint. This constraint is essential for ensuring that the solution obtained is valid and does not violate the constraints specified in the model. The sub-tour elimination constraint is presented in detail in the third part, along with a step-by-step procedure for its implementation. By including this constraint in the formulation, one can guarantee that the solution obtained is a valid solution that satisfies all the constraints specified in the model. Together, these constraints create a comprehensive and realistic model for solving the mTSP problem in a patrol context, which effectively tracks the patrolling path of each vehicle.

3. Simulation with Random Data

In this part, the paper establishes a computer program to simulate the model introduced earlier. Python and Jupyter Notebook are used to write the computer program, and Gurobi is employed as the solver. In the simulation, random locations is generated by python stored by a list. The map of location is given in Fig. 1.

During the simulation, it is assumed that the roads between locations are strictly vertical or horizontal. As a result, this study calculates the total cost for traveling between two locations by summing their vertical and horizontal differences. This simplification allows us to focus on the main aspects of the problem and obtain results efficiently. However, it is important to note that real-world road networks may have more complex structures and different cost metrics, which could affect the accuracy of our simulation results. Once, one has all the necessary data and parameters, one proceeds to convert the mathematical model into an LP file that is readable by Gurobi [10]. It is important to note that the process of converting the model into an LP file may vary depending on the solver used and the specific circumstances of the problem. Therefore, it does not provide a detailed illustration of this step in this paper as it does not generalize to all circunstances. After converting the model into an LP file, one calls Gurobi to solve it. The results obtained indicate that Vehicle 1 travels from Location 0 to Location 4 and then returns to Location 0, as well as visiting Location 3 to Location 9 and then returning to Location 3. However, such a patrolling path is not realistic, as the patrol path for vehecle 1 is not consecutive and this path containing only location 3 and locatin 9 is called a sub-tour.

It is also observed the existence of two additional sub-tours in the solution. The first sub-tour contains Location 6 and Location 10, while the second sub-tour contains Location 1 and Location 8.

To avoid sub-tours in the solution, one needs to add sub-tour elimination constraints. Theoretically, there are 2^n possible sub-tours that can exist, which could significantly increase the running time for Gurobi to solve it. Therefore, one manually adds sub-tour elimination constraints to the program each time one oberseves a sub-tour in the solutin until one gets an optimal solution. The list T is generated in the computer program to store the sub-tours observed during the solution process. The five attempt results are given in Table. 1.

Table. 1 Results of five attempts.

Attempt 1		Attempt 2		Attempt 3		Attempt 4		Attempt 5	
T	538	T	606	T	726	T	726	T	726
X_{1,0,4}	1	X_{1,0,5}	1	X_{1,0,6}	1	X_{1,0,6}	1	X_{1,0,7}	1
X_{1,3,9}	1	X_{1,3,9}	1	X_{1,4,0}	1	X_{1,4,0}	1	X_{1,1,8}	1
X_{1,4,0}	1	X_{1,5,3}	1	X_{1,6,10}	1	X_{1,6,10}	1	X_{1,7,1}	1
X_{1,9,3}	1	X_{1,9,0}	1	X_{1,10,4}	1	X_{1,10,4}	1	X_{1,8,0}	1
X_{2,0,2}	1	X_{2,0,2}	1	X_{2,0,5}	1	X_{2,0,7}	1	X_{2,0,5}	1
X_{2,2,0}	1	X_{2,2,0}	1	X_{2,3,0}	1	X_{2,1,8}	1	X_{2,3,0}	1
X_{2,6,10}	1	X_{2,4,10}	1	X_{2,5,9}	1	X_{2,7,1}	1	X_{2,5,9}	1
X_{2,10,6}	1	X_{2,6,4}	1	X_{2,9,3}	1	X_{2,8,0}	1	X_{2,9,3}	1
X_{3,0,5}	1	X_{2,10,6}	1	X_{3,0,2}	1	X_{3,0,2}	1	X_{3,0,6}	1
X_{3,1,8}	1	X_{3,0,1}	1	X_{3,1,8}	1	X_{3,2,0}	1	X_{3,2,0}	1
X_{3,5,7}	1	X_{3,1,8}	1	X_{3,2,0}	1	X_{3,3,5}	1	X_{3,4,2}	1
X_{3,7,0}	1	X_{3,7,0}	1	X_{3,7,1}	1	X_{3,9,3}	1	X_{3,6,10}	1
X_{3,8,1}	1	X_{3,8,7}	1	X_{3,8,7}	1			X_{3,10,4}	1

After adding the three sub-tours identified in the solution process, one re-runs Gurobi to obtain an updated solution. The solution generated by the updated model indicates that Vehicle 1 travels from Location 0 to Location 5, then to Location 3 and finally to Location 9 before returning to Location 0. Vehicle 2 travels from Location 0 to Location 2 and back to Location 0. However, one observes the appearance of a new sub-tour containing three locations: Location 4, Location 10, and Location 6. Hence, one needs to update the sub-tour list and generate a new solution. Upon examining the updated solution, one observes the appearance of a new sub-tour containing Location 1, Location 7, and Location 8. This paper then updates the subttour list and generate a new solution. Subsequently, one adds a new subtour containing location 3, location 5 and location 9, and the runs for a new solution. The solution generated by the model indicates that Vehicle 1 travels from Location 0 to Location 7, then to Location 1, Location 8 before returning to Location 0. Vehicle 2 travels from Location 0 to Location 5, then to Location 9, Location 3 before returning to Location 0. Vehicle 3 travels from Location 0 to Location 6, then to Location 10, Location 4, Location 2 before returning to Location 0. Remarkably, the maximum traveling distance among all vehicles is minimized to 726. No sub-tours are observed in this solution, indicating that the solution generated is valid and satisfies all the constraints specified in the model. Therefore, one considers this as our final optimal solution to the MTSP problem in a patrol scenario.

4. Limitations & Prospects

Despite the promising results of the proposed formulation and simulation for mTSP problems, there are still several limitations that need to be addressed. Firstly, it is not always possible to obtain a plausible solution by running the computer program just once. In practice, one needs to manually observe the sub-tours and update the sub-tour lists to obtain a better solution without sub-tours. Furthermore, if one includes all possible sub-tour elimination constraints in the model, the running time would increase sharply. Secondly, the current model focuses on a simple patrolling scenario, and it may not be suitable for more complex real-world applications. Therefore, modifications are necessary to adapt the model to more realistic problems.

Despite these limitations, the proposed formulation and simulation provide a solid foundation for further research on mTSP problems. There are several potential prospects for future research in this area. Firstly, future research can focus on the development of advanced algorithms for solving the NP-hard problem. This would improve the efficiency and accuracy of the solution obtained, and enable the model to be applied in more complex real-world applications. Secondly, new ways of formulating mTSP problems can be explored to enhance the flexibility and adaptability of the model to a wider range of applications. Thirdly, variations of the current model can be developed, such as dynamic locations and multi-objective optimization, to improve the realism and applicability of the model to a wider range of real-world scenarios. Overall, the proposed formulation and simulation provide a strong foundation for further research on mTSP problems, and the prospects are promising for enhancing the efficiency and effectiveness of mTSP problems in a wide range of real-world applications.

5. Conclusion

In conclusion, this study proposes a formulation for the mTSP problem in a patrolling scenario, and demonstrates its effectiveness through a computer simulation using Gurobi as the solver. The simulation results show that the proposed formulation can efficiently minimize the maximum traveling distance among all vehicles, and generate a valid and optimal solution without sub-tours. However, the proposed model has some limitations that need to be addressed, e.g., the need for manual observation and updating of sub-tours to obtain a plausible solution, and the need for modifications to adapt the model to more complex real-world problems. Nevertheless, the proposed formulation and simulation provide a strong foundation for future research on mTSP problems, and the potential prospects for future development are promising. With the advancement of algorithms and the exploration of new ways, this research has the potential to significantly improve the efficiency and effectiveness of patrolling operations in a wide range of real-world applications.

References

[1] Voigt B F. der Handlungsreisende, wie er sein soll und was er zu thun hat, um Aufträge zu erhalten und eines glücklichen Erfolgs in seinen Geschäften gewiss zu sein. Commis-Voageur, Ilmenau, 1831: 50.

[2] Flood M, Savage L J. A Game theoretic study of the tactics of area defense. RAND Research Memorandum, 1948, 51.

[3] Khanra A, Maiti M K, Maiti M. Profit maximization of TSP through a hybrid algorithm. Computers & Industrial Engineering, 2015, 88: 229-236.

[4] Su F, Kong L, Wang H, et al. Modeling and application for rolling scheduling problem based on TSP. Applied Mathematics and Computation, 2021, 407: 126333.

[5] Kumar R, Luo Z. Optimizing the operation sequence of a chip placement machine using TSP models. IEEE Transactions on Electronics Packaging Manufacturing, 2003, 26(1): 14-21.

[6] Wex F, Schryen G, Feuerriegel S, et al. Emergency response in natural disaster management: Allocation and scheduling of rescue units. European Journal of Operational Research, 2014, 235(3): 697-708.

[7] Gonzalez-Longatt F M. Optimal offshore wind farms' collector design based on the multiple travelling salesman problem and genetic algorithm. 2013 IEEE Grenoble Conference. IEEE, 2013: 1-6.

[8] Wang C, Lan H, Saldanha-da-Gama F, et al. On optimizing a multi-mode last-mile parcel delivery system with vans, truck and drone. Electronics, 2021, 10(20): 2510.

[9] Homepage of Gurobi, Retrieved from: https://www.gurobi.com/

[10] IBM documentation, Retrieved from: https://www.ibm.com/docs/en/icos/12.9.0?topic=cplex-lp-file-format-algebraic-representation