Multi-attribute decision-making model based on triangular fuzzy numbers and prospect theory

Junqiang Xie *

School of Economics Management, Taiyuan University of Technology, Taiyuan, China

* 1945578034@qq.com

Abstract. This article combines the theory of triangular vague numbers and prospects to build a multi -attribute decision -making optimization model, and the application of the decision -making problem of the project construction plan has achieved the choice of the construction plan of the project. Among them, the fuzzy treatment of the scheme attribute value through the vague number of the triangle reflects the uncertainty and risk in the actual project. Through the comprehensive weights composed of the objective weight determined by attribute differences and the subjective weight of Delphi -based, the error of attribute weights is effectively reduced. On the basis of the optimization model of this decision, the BIM technology -related application platform and expert assessment method are applied to determine the reference point of the plan attributes and build attribute expectations. The value function of the prospect theory is combined with the comprehensive right vector calculation of the comprehensive prospective value matrix to achieve multi -plan multi -attribute decision -making optimization.

Keywords：triangular fuzzy number; Prospect theory; Degree of difference; Multi-attribute decision optimization

1. Introduction

In engineering construction management activities, the optimization of engineering construction decision is of great significance for improving project efficiency, shortening construction period, reducing risks, improving quality and safety and promoting sustainable development. Therefore, it is of great significance to apply prospect theory reasonably. As the primary link and important stage of management activities, the decision-making optimization of project construction scheme plays a key role in project cost, construction period and quality. This paper intends to construct a multi-attribute decision-making model based on triangular fuzzy number and prospect theory, which provides an effective practical application means for decision optimization of construction scheme. For a long time, China has always focused on the construction stage of project cost management, and paid more attention to the audit of construction drawing budget, settlement and other construction and installation project prices. In fact, the decision-making stage of construction project is one of the most important parts of project cost management. According to the empirical conclusion, the cost generated in the decision-making stage of the project is only less than 1% of the whole life-cycle cost of the project, but the cost of less than 1% has an impact on the whole project cost of more than 75%. Therefore, the construction project decision-making optimization scheme is of great significance for reducing costs, improving project quality and reducing risks.

2. Literature review

Multi-attribute decision-making model refers to the decision-making analysis method model of choosing the most suitable target scheme in the case of multiple target schemes and multiple attribute indexes. Up to now, this method has been widely used in engineering, economy, biology, psychology and other fields, providing a comprehensive and effective analysis method for things evaluation and decision-making. This paper combines fuzzy numbers with multi-attribute decision-making, which shows that in the specific decision-making process, the relevant factors affecting decision-making are flexible and cannot be simply defined by absolute values. As a mathematical theory and method to study and deal with fuzziness, fuzzy number has obvious advantages in dealing with incomplete information in group multi-attribute decision-making.

The research on triangular fuzzy numbers has been mature at home and abroad: Verma[1] first proposed the quasi-fuzzy priority weighted average (QFPWA) operator and the quasi-fuzzy priority weighted ordered weighted operator, and studied the multi-attribute decision-making problem in triangular fuzzy environment with different priorities for attributes and experts; Guo and Fan[2] applied triangular fuzzy numbers to the network security evaluation of computer network information systems by deeply developing triangular fuzzy geometric Bonferroni average (TFGBM) operator and triangular fuzzy weighted geometric Bonferroni average (TFWGBM) operator. Radulescu and Constantina Zoie [3] proposed the combination of fuzzy simple weighted (SAW) method and maximization, which provided an effective means to help decision makers evaluate and rank cloud provider services. Scholars Shao Guoxia and Cao Zhengguo [4] combined triangular fuzzy number with TOPSIS method to construct an optimization model of adjacent foundation reinforcement scheme; Deng and Zhang[5] introduced grouping calculation method and applied triangular fuzzy analytic hierarchy process (TFAHP) to the performance evaluation of earthquake disaster supply chain. Ebrahimnejad and Luis Verdegay[6] used triangular intuitionistic fuzzy numbers (TIFN) to propose a method to solve the initial basic feasible solution and optimal solution of intuitionistic fuzzy transportation problems, which overcame the shortcomings of traditional transportation algorithms. Kumar[7] extends the concept of triangular fuzzy sets to the intuitive fuzzy correlation between the idea of intuitive fuzzy sets and the performance of attributes and overall satisfaction; Dong and Wan[8] put forward the concepts of fuzzy consistency index and fuzzy consistency ratio, and obtained a new fuzzy optimization method (BWM) based on triangular fuzzy numbers. Cao Kamen [9] and others combined triangular fuzzy number with analytic hierarchy process to establish a model for missile system performance evaluation; BIsht and Srivastava[10] use the minimum demand supply algorithm to solve the optimal cost, and study the optimization method of a kind of fuzzy triangular cost fuzzy transportation problem with demand and supply as clear numbers.

This paper also applies the cumulative prospect theory to the multi-attribute decision-making model. Prospect theory (PT) was first put forward by Kahneman and Tversky. They pointed out that people are different in sensitivity to loss and gain. Compared with loss prospect, people are more miserable about loss prospect than they are happy about gain prospect, that is, prospect theory has the characteristics of "reference dependence" and "risk aversion". In the description of value function in prospect theory, it is emphasized that the carrier of subjective value is the change of gain or loss relative to the reference point, not the absolute quantity of the final state of subjective value. In the multi-attribute group decision-making model, the deviation of decision-making attribute values from the expected reference point under different scheme sets is helpful for us to choose the best scheme.

The application research of prospect theory has always been a hot spot in various fields: Peng[11] combines the prospect preference algorithm with the stochastic multi-criteria decision-making algorithm, that is, considers the regret and disgust of decision makers in the decision-making process, and on this basis, puts forward two models to solve the stochastic multi-criteria decision-making problem; Xiao and Xie[12] applied the prospect theory to the establishment of mathematical model, and proposed a defense scheme for cloud storage system against advanced persistent threat attacks; Dong and Luo[13] introduced the preference-recognition structure to determine the reference point in the situational theory, and used the foreground value function to obtain the individual and collective foreground values. Based on this, a new MPDM problem framework with heterogeneous preference representation structure was proposed. Li and Zhao[14] put forward an evaluation method combining entropy theory and cumulative prospect theory, which overcomes the inaccuracy or misjudgment in evaluating crane safety. Liu and Song[15] and others capture the irrational psychological aspects of decision-making units under risk by introducing the prospect value of decision-making units, and put forward a prospect cross-efficiency evaluation model based on the prospect value; Xu and Xiao[16] applied the cumulative prospect theory to study the scenario of cloud storage defense system under advanced persistent threat attack, and deduced the Nash equilibrium of advanced persistent threat defense game based on the cumulative prospect theory. Guan and Annaswany[17] and others put forward a dynamic pricing model of shared mobile on-demand service based on the behavior modeling of cumulative prospect theory (CPT), and introduced the derivation of passenger behavior model in the shared mobile on-demand service environment in detail. Wang and Sun[18] applied cumulative prospect theory to study the path selection behavior of travelers in degradable transportation networks, and established a user equilibrium model based on cumulative prospect theory. Gabedo-Peris and Gonzalez-Sala[19] systematically summarized the relationship between addiction behavior and prospect theory, and studied the level of loss and risk aversion of addicts. Ghader and Darzi[20] studied the influence of travel time reliability on travel mode selection by using cumulative prospect theory, and introduced a mode selection framework based on CPT. Clark and Lisowski[21] studied the theoretical contribution of prospect theory, which provided a new understanding for the decision-making of immigration or residence; Rouyard and Attema[22] and others applied the decision-making framework based on prospect theory to the trade-off between health outcomes and behavior changes brought by chronic disease management, and implemented intervention measures for behavior changes of specific patient groups.

In this paper, triangular fuzzy number and prospect theory are combined, and the weights are innovatively divided into supervisor weight and objective weight. BIM technology is applied to determine the reference point of attribute value, and combined with value function to calculate the comprehensive prospect value, so as to construct a mathematical model for decision optimization. The model is suitable for a series of multi-attribute decision-making problems, such as construction scheme optimization decision, engineering material procurement, design scheme optimization and so on.

3. Theoretical basis

Definition 1[23] If $\tilde{a}=[a^L,a^M,a^U]$ exists in the positive real number set, and $0\le a^L\le a^M\le a^U\le+\infty$, then a is called triangular fuzzy number. Where a^U is the upper limit of the triangular fuzzy number, a^L is the lower limit of the triangular fuzzy number, and a^M is the possible maximum value. The membership degree of triangular fuzzy numbers can be expressed as:

$$\mu_{\tilde{a}}(x)\begin{cases}(x-a^L)/(a^M-a^L), a^L\le x\le a^M;\\(a^U-x)/(a^U-a^M), a^M\le x\le a^U;\\0 \qquad ,else\end{cases}$$

Definition 2[24] The form of Fuzzy semantic term set is $S=\{S_t|t=1,2,...,T\}$, and in general, T is an odd number. When a > b, there is sa > sb. Each Sξ element corresponds to a semantic term, such as when T=7, $S=\{S_1$= "extremely poor", S_2= "very poor", S_3= "poor", S_4= "moderate", S_5= "good", S_6= "very good", S_7= "excellent"}. For the convenience of calculation, the fuzzy semantic term set S_t is transformed into triangular fuzzy number Ht, and the transformation relationship can be expressed as:

$$H_t=S_t=\left[max\left\{\frac{t-1}{T},\ 0\right\},\ \frac{t}{T},\ min\left\{\frac{t+1}{T},\ 1\right\}\right]$$

Definition 3 To compare the sizes of two triangular fuzzy numbers $\tilde{a}=[a^L,a^M,a^U]$ and $\tilde{b}=[b^L,b^M,b^U]$, refer to the practice of reference [24], and then get the size of the sum by comparing the fuzzy expected values of $\tilde{a}$ and $\tilde{b}$:

$$EX(\tilde{a})=\frac{a^L+4a^M+a^U}{6},\ EX(\tilde{b})=\frac{b^L+4b^M+b^U}{6}$$

Among them, when $EX(\tilde{a})>EX(\tilde{b})$, $\tilde{a}>\tilde{b}$; When $EX(\tilde{a})<EX(\tilde{b})$, $\tilde{a}<\tilde{b}$; When $EX(\tilde{a})=EX(\tilde{b})$, $\tilde{a}=\tilde{b}$.

Definition 4[25] When triangular fuzzy numbers $\tilde{a}$=[a^L,a^M,a^U] and $\tilde{b}$=[b^L,b^M,b^U] are canonical triangular fuzzy numbers, then s($\tilde{a}$, $\tilde{b}$) is called the similarity between $\tilde{a}$ and $\tilde{b}$, and $s^{-}(\tilde{a},\ \tilde{b})$ is the difference between $\tilde{a}$ and $\tilde{b}$.

The calculation formula of similarity is as follows:

$$s(\tilde{a},\ \tilde{b})=\frac{a^L b^L+\ a^M b^M+a^U b^U}{MAX\{(a^L)^2+(a^M)^2+(a^U)^2,\ (b^L)^2+(b^M)^2+(b^U)^2\}}$$

Among them, the greater the similarity, that is, the closer the distance between and, the more similar they are, and $\tilde{a}=\tilde{b}$ if and only if $s(\tilde{a},\ \tilde{b})=1$.

It is easy to draw that the difference between $\tilde{a}$ and $\tilde{b}$ is expressed as:

$$s^{-}(\tilde{a},\ \tilde{b})=1-\ s(\tilde{a},\ \tilde{b})$$

Definition 5[26] For any two triangular fuzzy numbers $\tilde{a}$=[a^L,a^M,a^U] and $\tilde{b}$=[b^L,b^M,b^U], the expression of distance $D\left[\tilde{a},\tilde{b}\right]$ is:

$$D\left[\tilde{a},\tilde{b}\right]=\int_0^1 d[\mu_{\tilde{a}}(\lambda),\mu_{\tilde{b}}(\lambda)]\,d\lambda$$

$$d[\mu_{\tilde{a}}(\lambda),\mu_{\tilde{b}}(\lambda)]$$
$$=\int_0^1\left|\left[\mu_{\tilde{a}}^L(\lambda)+x(\mu_{\tilde{a}}^U(\lambda)-\mu_{\tilde{a}}^L(\lambda))\right]-\left[\mu_{\tilde{b}}^L(\lambda)+x(\mu_{\tilde{b}}^U(\lambda)-\mu_{\tilde{b}}^L(\lambda))\right]\right|dx$$

In this paper, the distance between two triangular fuzzy numbers is calculated in the form of calculus by using the nature of membership degree of triangular fuzzy numbers. The greater the distance, the weaker the relationship between the two triangular fuzzy numbers; On the contrary, the smaller the distance, the stronger the relationship between them.

Definition 6 Based on the prospect theory put forward by Kahneman and Tversky(1979)[27], the comprehensive prospect value is calculated by the value function and the decision probability weight. The value function is reflected in the loss or gain between a certain value and the reference point, and its specific expression is as follows:

$$v(x)=\begin{cases} x^{\alpha}\ , & x\geq 0 \\ -\theta(-x)^{\beta}\ , & x<0 \end{cases}$$

In this formula, α and β represent risk aversion coefficient and risk preference coefficient respectively, and $\alpha,\beta\in(0,1)$. θ is called loss aversion coefficient and $\theta>1$, which makes decision makers more sensitive to risk loss. Based on the research results of reference [28], this paper chooses $\alpha=\beta=0.88$, and θ takes 2.25.

The comprehensive prospect value multiplies the loss or gain value of each attribute with its relative reference point by the probability weight, and obtains specific values that can compare the advantages and disadvantages with each other, which provides a convenient means for screening the optimal scheme. Assuming that ω is the probability weight of decision attributes, the comprehensive prospect value of each attribute under each decision scheme set can be calculated by the following formula:

$$U=\sum\omega v(x)$$

4. Construction of decision-making optimization model

4.1 Problem description

When decision makers make multi-scheme and multi-attribute decisions, in order to idealize the decision-making optimization model, here are some assumptions:

①Principle of self-interest: when decision makers choose the best scheme, they all make decision optimization based on the principle of maximizing their own interests;

②Principle of scheme independence: any scheme to be decided is independent from other schemes, and the formulation of this scheme has no direct influence on other schemes;

③Principle of truthfulness: Any scheme should be formulated with reasonable and reliable data and theoretical support to ensure the feasibility of the scheme.

When a decision maker makes a multi-attribute decision, there are *N* schemes to be decided, and there are *M* scheme attributes that affect the decision, then the *N* schemes to be decided constitute a decision scheme set $SC=\{SC_1,SC_2,...,SC_N\}$, $N\in Z^+$, Attribute set $AT=\{AT_1,AT_2,...,AT_M\}$, $M\in Z^+$;composed of *M* attributes that affect decision-making; The initial decision matrix *X* is composed of decision scheme set and attribute set, and its specific expression is:

$$X=\begin{vmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{12} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{vmatrix}$$

Among them, x_{ij} represents the decision value of the *i*-th attribute under the *j*-th scheme, which is expressed by triangular fuzzy numbers or natural semantic terms. $W_{sb}=\{\omega_1,\omega_1,\ldots,\omega_m\}(m\in Z^+)$ represents the subjective weight vector set for the ATtribute set at, which is often subjectively given by the decision-making team according to the actual situation to meet their own interests to the greatest extent; $W_{ob}=\{\omega_1',\omega_2',\ldots,\omega_m'\}$ $(m\in Z^+)$ represents the objective weight vector set for Attribute set *AT*, which is to reduce the error of subjective weight and assign the weight to the attribute set in a more objective and fair way.

4.2 Decision optimization method steps

4.2.1 Normalized initial evaluation matrix

When the decision-maker formulates several feasible schemes through communication and consultation with the consulting expert team, the initial evaluation matrix X is obtained. Firstly, the initial evaluation matrix is normalized: if the negotiation attribute value is a natural semantic term, it is converted into a triangular fuzzy number $x_{ij}=\left[x_{ij}^L,x_{ij}^M,x_{ij}^U\right]$ according to *Definition 1*, and it is converted into a standardized decision-making attribute value according to formulas (1) and (2); If it is a numerical initial negotiation attribute value, it is directly converted into a normative decision attribute value. The specific normalized formula [25] is as follows:

When the decision makers communicate and consult with the team of consulting experts and formulate multiple feasible solutions to obtain the initial evaluation matrix *X*, first of all, the initial evaluation matrix is normalized: if the attribute value of the negotiation is a natural term Vague number $x_{ij}=\left[x_{ij}^L,x_{ij}^M,x_{ij}^U\right]$, and transform into a standard decision-making attribute value according to the formula (1) and (2); if it is the initial negotiation attribute value of the value type, it will be directly transformed into a normative decision-making attribute value. Specific standardized formula [25] as follows:

1）When x is an beneficial attribute, then:

$$\bar{x}_{ij}=\left[\frac{x_{ij}^L}{V_c},\frac{x_{ij}^M}{V_c},\frac{x_{ij}^U}{V_c}\right],i\in M,\ j\in N; \qquad (1)$$

Among them

$$V_c=\left\{\sum_{i=1}^{M}\left[\left(x_{ij}^L\right)^2+\left(x_{ij}^M\right)^2+\left(x_{ij}^U\right)^2\right]\right\}^{\frac{1}{2}}.$$

2）When it is a cost-type attribute, then:

$$\bar{x}_{ij}=\left[\frac{1}{x_{ij}^L\cdot V_b},\frac{1}{x_{ij}^M\cdot V_b},\frac{1}{x_{ij}^U\cdot V_b}\right],i\in M,\ j\in N; \qquad (2)$$

Among them

$$V_b=\left\{\sum_{i=1}^{M}\left[\left(1/x_{ij}^L\right)^2+\left(1/x_{ij}^M\right)^2+\left(1/x_{ij}^U\right)^2\right]\right\}^{\frac{1}{2}}.$$

The initial evaluation matrix X in all the initial evaluation value $x_{ij} = \left[x_{ij}^L, x_{ij}^M, x_{ij}^U\right]$ is changed to normalized evaluation value $\bar{x}_{ij} = \left[\bar{x}_{ij}^L,\ \bar{x}_{ij}^M,\ \bar{x}_{ij}^U\right]$, and the normalized evaluation matrix $\bar{X}$ is obtained. The specific form is as follows:

$$\bar{X} = \begin{vmatrix} [\bar{x}_{11}^L, \bar{x}_{11}^M, \bar{x}_{11}^U] & [\bar{x}_{12}^L, \bar{x}_{12}^M, \bar{x}_{12}^U] & \cdots & [\bar{x}_{1n}^L, \bar{x}_{1n}^M, \bar{x}_{1n}^U] \\ [\bar{x}_{21}^L, \bar{x}_{21}^M, \bar{x}_{21}^U] & [\bar{x}_{22}^L, \bar{x}_{22}^M, \bar{x}_{22}^U] & \cdots & [\bar{x}_{2n}^L, \bar{x}_{2n}^M, \bar{x}_{2n}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [\bar{x}_{m1}^L, \bar{x}_{m1}^M, \bar{x}_{m1}^U] & [\bar{x}_{m2}^L, \bar{x}_{m2}^M, \bar{x}_{m2}^U] & \cdots & [\bar{x}_{mn}^L, \bar{x}_{mn}^M, \bar{x}_{mn}^U] \end{vmatrix}$$

4.2.2 Select the expected vector

Assuming the expected vector $E(X) = \{e_1, e_2, \ldots, e_m\}$ is the attribute reference point. By applying *BIM* technology, the cost, construction progress, risks and quality of engineering projects are used to make reasonable simulation presumes, and then the experts are evaluated to determine the expected vector value that is most consistent with its own interests. In order to facilitate calculations, the specific expectations of the expected vector value have been blurred by factors such as the time value and engineering accidents of the funds and transformed into a triangle blurred number.

4.2.3 Calculation distance difference matrix *D*

The distance D_{ij} between the standardized evaluation value and the expected attribute value is calculated according to the *Definition 5*, and the distance difference matrix D is obtained as follows:

$$D = \begin{vmatrix} D_{11} & D_{12} & \cdots & D_{1n} \\ D_{21} & D_{12} & \cdots & D_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ D_{m1} & D_{m2} & \cdots & D_{mn} \end{vmatrix}$$

4.2.4 Build a prospect value matrix

Substitute the calculated distance difference matrix D into the following foreground value calculation formula according to *Definition 6.*

$$v(\bar{x}_{ij}) = \begin{cases} {D_{ij}}^{\alpha}\ , & \bar{x}_{ij} \geq e_i \\ -\theta(-D_{ij})^{\beta}\ , & \bar{x}_{ij} < e_i \end{cases} \quad ,i \in M, j \in N$$

Select the $\alpha=\beta=0.88$, $\theta = 2.25$ here, and to build the prospect value matrix $v(\bar{x})$ from the calculated $v(\bar{x}_{ij})$:

$$v(\bar{x}) = \begin{vmatrix} v(\bar{x}_{11}) & v(\bar{x}_{12}) & \cdots & v(\bar{x}_{1n}) \\ v(\bar{x}_{21}) & v(\bar{x}_{22}) & \cdots & v(\bar{x}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ v(\bar{x}_{m1}) & v(\bar{x}_{m2}) & \cdots & v(\bar{x}_{mn}) \end{vmatrix}$$

4.2.5 Determine the attribute weight vector

The subjective weight vector $W_{sb} = \{\omega_1, \omega_1, \ldots, \omega_m\}$ is based on the Delphi method, and the decision-maker team formulates the attribute weights that are most in line with their own interests based on their own needs and actual conditions; the objective weight vector $W_{ob} = \{\omega_1', \omega_2', \ldots, \omega_m'\}$ calculates the difference between different schemes for the same attribute, and obtains the same attribute as A vector of objective weights independent of the nature of the attribute itself. If the decision variable has a greater similarity between different schemes of the same attribute, that is, the smaller the difference, the less important it is for the weight decision; on the contrary, if the decision variable has a smaller similarity between different schemes of the same attribute , that is, the greater the difference, the higher the importance of weight determination. Therefore, based on the above definition 4, the following calculation form is obtained:

$$\omega_i' = \frac{\frac{1}{\sum_{j=1}^{n}\sum_{k=1,k>j} s^-(\bar{x}_{ij},\ \bar{x}_{ik})}}{\sum_{i=1}^{M}\frac{1}{\sum_{j=1}^{n}\sum_{k=1,k>j} s^-(\bar{x}_{ij},\ \bar{x}_{ik})}}, i \in M,\ j,k \in N. \tag{3}$$

Finally, the comprehensive weight vector $\bar{W} = \{\varpi_1, \varpi_2, \ldots, \varpi_m\}$ is obtained according to the formula $\bar{W} = \xi W_{sb} + (1-\xi)W_{ob}$. Where $\xi \in (0,1)$, the closer ξ is to 1, it indicates that the subjective weight vector of the expert team accounts for a larger proportion, and the nature of the attribute itself is more important in influencing the purchaser's choice; if ξ is closer to 0, it indicates that the objective weight vector accounts for a larger proportion , the difference between different schemes for the same attribute is more important to influence the purchaser's choice.

4.2.6 Calculate comprehensive prospects

The outlook value matrix $v(\bar{x})$ and the comprehensive right vector $\bar{W}$ are into the following formulas according to the definition:

$$U_j = \sum_{i=1}^{m} \varpi_i v(\bar{x}_{ij}),\ i \in M,\ j \in N.$$

The maximum value of the comprehensive prospect value of the calculation, that is, *U*=MAX*{ U_1, U_2, …, U_n }, the maximum comprehensive prospect value *U** corresponding scheme *SC** is the best solution *SC**.

5. Case verification

5.1 Case background description

In this case, it is intended to apply a project to verify the construction plan decision-making optimization model based on the triangular fuzzy number and prospect theory. For the decision-making optimization of the construction plan, this article combines BIM related project management and cost platforms to include the decision attributes of cost, construction progress, risks and quality into the plan. And other costs are represented by Cost1, Cost2, Cost3, Cost4,Cost5, respectively. Decision makers communicate with the expert team. For example, the experts of the consulting company formulate multiple construction and construction plans. The decision makers formulate the expectations of their own financial expenditure capabilities and engineering construction standards based on the digital project management platform and the corresponding cost software. Multi-attribute decision-making optimization model selects the optimal construction plan.Here is a systematic description of the above four attribute indicators:

Engineering cost is a cost-type attribute. The price of engineering materials and the cost of workers will fluctuate according to the market conditions, and in the planned construction period, the project accident and currency fluctuations are unavoidable. Therefore, the cost of engineering can be described by triangular fuzzy numbers.

The construction progress is also a cost-type attribute. The actual construction period will fluctuate due to a series of inevitable factors such as design errors, natural environmental factors, and social environmental factors. Therefore, quantifying the expected construction period with triangular vague number can more accurately express the actual construction period of construction progress.

The level of risk is also a cost-oriented attribute. The decision makers naturally describe the risk of the implementation of the plan based on the formulation of the construction plan, such as lower risk.

The quality of the engineering is an beneficial attribute. The decision makers comprehensively evaluate the quality of the project after a series of factors such as construction period, engineering material price, technical level, and construction machinery and equipment. The quality of the project is unified by the decision maker and the expert team. Of these, 10 points indicate the highest quality of the project, and 1 point indicates the lowest quality of the project. Among the triangular vague numbers describing the quality of the project, the upper limit represents the highest score of the decision maker and expert team in the team, and the lower limit represents the lowest score, while the intermediate item indicates the average value of all evaluation scores.

5.2 Example verification analysis

(1) According to the description of the above-mentioned example, decision makers have developed multiple construction project plans through communication with the expert team to build an initial evaluation matrix X ：

Table 1 Initial evaluation matrix

		SC_1	SC_2	SC_3	SC_4	SC_5
Cost	Cost1	[15164786.68, 16849762.98, 18534739.27]	[10808888.81, 12009876.46, 13210864.10]	[13203628.30, 14670698.12, 16137767.93]	[14111420.08, 15679355.65, 17247291.21]	[12205348.40, 13561498.23, 14917648.053]
	Cost2	[813588.87, 903987.64, 994386.40]	[886188.50, 984653.89, 1083119.28]	[861508.68, 957231.87, 1052955.05]	[901949.22, 1002165.81, 1102382.39]	[903081.39, 1003423.77, 1103766.14]
	Cost3	[358169.90, 397966.56, 437763.21]	[386769.09, 429743.44, 472717.78]	[331971.32, 368857.03, 405742.73]	[383671.78, 426301.98, 468932.17]	[277751.26, 308612.52, 339473.77]
	Cost4	[476880.71, 529867.46, 582854.20]	[548070.89, 608967.66, 669864.42]	[503878.56, 559865.07, 615851.57]	[546078.70, 606754.12, 667429.53]	[538079.21, 597865.79, 657652.36]
	Cost5	[178762.70, 198625.23, 218487.75]	[170810.01, 189788.91, 208767.80]	[212081.18, 235645.76, 259210.33]	[206878.57, 229865.08, 252851.58]	[188528.81, 209476.46, 230424.10]
Construction Schedule		[208.8,232.0, 255.2]	[230.4,256.0, 281.6]	[233.1,259.0, 284.9]	[260.1,289.0, 317.9]	[216,240,264]
Risk		lower risk	lower risk	lower risk	lower risk	lower risk
Quality		[8.00,8.23, 8.40]	[7.00,7.26, 7.40]	[7.40,7.69, 7.90]	[7.80,8.06, 8.20]	[6.80,7.09, 7.21]

(2) Convert the fuzzy synonym in the initial evaluation matrix based on *Definition 1* into a triangular vague number. S = {S_1 = "major risk", S_2 = "larger risk", S_3 = "general risk", S_4 = "lower risk", S_5 = "risk-free"}, the triangular fuzzy number is obtained as follows:

(3)

Table 2 Fuzzy semantic term conversion

Fuzzy semantic term	Triangle blurred number
general risk	[0.8,0.6,0.4]
lower risk	[0.6,0.4,0.2]
risk-free	[0,0,0]

(4) According to formula (1) and formula (2), the normalized evaluation matrix $\bar{X}$ is obtained：

Table 3 Normalized evaluation matrix

		SC_1	SC_2	SC_3	SC_4	SC_5
Cost	Cost1	[0.5237,0.5699, 0.6332]	[0.5497,0.5588, 0.6209]	[0.5465,0.5602, 0.6225]	[0.5471,0.5600, 0.6222]	[0.5564,0.5751, 0.5997]
	Cost2	[0.5475,0.5598, 0.6220]	[0.5451,0.5608, 0.6232]	[0.5196,0.5716, 0.6351]	[0.5373,0.5642, 0.6269]	[0.5616,0.5730, 0.5970]
	Cost3	[0.4946,0.5814, 0.6460]	[0.5300,0.5830, 0.6159]	[0.4928,0.5821, 0.6468]	[0.5349,0.5884, 0.6064]	[0.5314,0.5777, 0.6196]
	Cost4	[0.5539,0.5570, 0.6189]	[0.5431,0.5617, 0.6241]	[0.5386,0.5636, 0.6263]	[0.5432,0.5617, 0.6241]	[0.5517,0.5755, 0.6036]
	Cost5	[0.4715,0.5899, 0.6555]	[0.5196,0.5716, 0.6351]	[0.5147,0.5880, 0.6239]	[0.4910,0.5828, 0.6475]	[0.5271,0.5798, 0.6212]
Construction Schedule		[0.5411,0.5626, 0.6251]	[0.5477,0.5597, 0.6219]	[0.5459,0.5605, 0.6228]	[0.5274,0.5801, 0.6207]	[0.5511,0.5787, 0.6012]
Risk		[0.2857,0.4286, 0.8571]	[0.2857,0.4286, 0.8571]	[0.3841,0.5121, 0.7682]	[0.3841,0.5121, 0.7682]	[0.3841,0.5121, 0.7682]
Quality		[0.5625,0.5786, 0.5906]	[0.5596,0.5804, 0.5916]	[0.5573,0.5792, 0.5950]	[0.5614,0.5801, 0.5902]	[0.5580,0.5818, 0.5917]

(5) The corresponding expert team and technical talents of the project through BIM technology platforms such as digital project management and cost software to obtain initial expected vector $E(X)$, specifically as shown in the table below:

Table 4 Initial expected vector

Cost	Cost1	[12943112.55,14381236.17,15819359.78]
	Cost2	[845091.30,938990.34,1032889.37]
	Cost3	[350571.89,389524.33,428476.76]
	Cost4	[509424.91,566027.68,622630.44]
	Cost5	[238680,265200,291720]
Construction Schedule		[247.5,275.0,302.5]
Risk		risk-free
Quality		[8.8,9.0,9.2]

In the same way, the expected vector is standardized, and the standardized expected vector $\hat{E}(X)$ is obtained:

Table 5 Standardized expectations

Cost	Cost1	[0.5304,0.5835,0.6150]
	Cost2	[0.5426,0.5796,0.6080]
	Cost3	[0.5344,0.5742,0.6203]
	Cost4	[0.5464,0.5721,0.6117]
	Cost5	[0.5196,0.5716,0.6351]
Construction Schedule		[0.4912,0.5403,0.6832]
Risk		[0,0,0]
Quality		[0.5644,0.5773,0.5901]

(6) Calculate the objective weight of the decision attribute. By calculating the differences between the decision-making values of the same attributes and different schemes, the impact of the attribute

on decision-making on the decision-making of the attribute will then obtain the heavy vector of objective rights. Calculate the difference between each attribute and different schemes as follows:

Table 6 Different matrix

Cost	Cost1	0.003800209
	Cost2	0.005073132
	Cost3	0.008323552
	Cost4	0.002227873
	Cost5	0.00842729
Construction Schedule		0.003294117
Risk		0.04918365
Quality		0.000381277

The difference in the computational differences, calculate the objective right vector of objective rights according to the formula (3):

Table 7 Objective power vector

Cost	Cost1	4.708%
	Cost2	6.286%
	Cost3	10.313%
	Cost4	2.760%
	Cost5	10.441%
Construction Schedule		4.081%
Risk		3.689%
Quality		0.032%

(7) Determine the subjective weight vector of the decision-making attribute. The subjective weight is given by the team's expert team, and discuss the importance of each attribute value according to actual needs. Each expert scores the corresponding attributes, and finally obtains the subjective weight of the attribute value. Weighing various factors, the subjective weight vector of the case is as follows:

Table 8 Subjective weight vector

Cost	Cost1	22%
	Cost2	14%
	Cost3	5%
	Cost4	8%
	Cost5	4%
Construction Schedule		16%
Risk		12%
Quality		19%

(8) Determine the integrated weight vector. Take ξ=0.65, calculate the comprehensive weight vector $\overline{W}$ as shown below:

Table 9 Comprehensive rights vector

Cost	Cost1	15.948%
	Cost2	11.300%
	Cost3	6.859%
	Cost4	6.166%
	Cost5	6.254%
Construction Schedule		11.828%
Risk		9.091%
Quality		12.361%

(9) Calculate each decision attribute value and expected vector worth of distance d. The calculation results are shown in the table below:

Table 10 Distance matrix

		SC_1	SC_2	SC_3	SC_4	SC_5
Cost	Cost1	0.0996	-0.0536	-0.0344	-0.038	-0.1652
	Cost2	0.0364	0.0508	0.2004	0.0968	-0.12
	Cost3	0.262	0	0.2724	-0.0576	0.0092
	Cost4	-0.0012	0.0628	0.0896	0.0624	-0.0536
	Cost5	0.274	0	-0.0252	0.164	-0.0856
Construction Schedule		-0.432	-0.4712	-0.4604	-0.3948	-0.5676
Risk		2.2856	2.2856	1.5364	1.5364	1.5364
Quality		0.0096	0.0252	0.048	0.0124	0.032

(10) The distance matrix obtained by the calculated is to build a prospect value matrix:

Table 11 Prospect value matrix

		SC_1	SC_2	SC_3	SC_4	SC_5
Cost	Cost1	0.131361536	-0.17133557	-0.115972214	-0.126587867	-0.461350883
	Cost2	0.054171295	0.072637363	0.243035654	0.128106249	-0.348226658
	Cost3	0.307684158	0	0.318406739	-0.182538427	0.016148541
	Cost4	-0.006051486	0.087539586	0.119682609	0.08704873	-0.17133557
	Cost5	0.320051963	0	-0.088189117	0.203733566	-0.258677962
Construction Schedule		-1.074999384	-1.160387832	-1.1369506	-0.993103254	-1.366909678
Risk		2.069761003	2.069761003	1.459230111	1.459230111	1.459230111
Quality		0.016764812	0.039195163	0.069102266	0.020999603	0.048365088

(11) Combining the comprehensive rights vector and the decision-making value of each attribute, calculate the comprehensive prospect value U_j under the SP_j of each plan, and obtain the comprehensive prospect value of the following construction decision-making optimization schemes:

Table 12 Comprehensive prospect value

SC_1	SC_2	SC_3	SC_4	SC_5
0.130900363	0.042033489	0.039390316	0.017662993	-0.161607757

It can be seen from the table that the comprehensive prospect value of the construction scheme SC_1 is the largest, so the optimal construction plan is SC_1.

6. Conclusion

Multi-attribute decision-making optimization model based on triangular vague numbers and prospects is a method that uses the application of triangular fuzzy and foreground theory to multi-attribute decision-making issues. This model combines the vagueness of the vague number and the subjectivity of the prospect theory, which can more accurately reflect the decision maker's attitude towards uncertainty and risk. By introducing triangular vague numbers, this model can handle fuzzy and uncertainty. Triangle blur consists of three parameters, which respectively represent the lower limit, center, and upper limit of the attribute value, which can better uncertain value of attribute values. In multi-attribute decision-making, we often face various uncertainty such as incomplete data and inaccurate information. The number of vaginal bangs can solve these problems to a certain extent. The prospect theory takes into account the subjective attitude and preferences of the decision makers. The prospect theory believes that people's perception of interests is asymmetric, and the feeling of losses is stronger than the income. Multi-attribute decision-making optimization models based on prospects can fully consider the subjective preferences of decision makers, making the decision results more in line with the actual situation.

In practical applications, this model can help decision makers better face complex multi-attribute decision-making problems and provide scientific and reasonable decision-making basis. It can be applied to various fields, such as engineering management, financial investment, environmental assessment, etc., and can meet different decision scenarios and the needs of decision makers. However, there are still some challenges and limitations in the model. First of all, the calculation complexity of this model is high, and a large amount of calculation and reasoning need to be performed. Secondly, the parameter settings and weight of the model have a greater impact on the results, and need to rely on the experience and knowledge of professionals. In addition, the explanatory and operability of the model also needs to be improved further. With the continuous development and improvement of related theories and methods, it is believed that the model will have a greater role in practice and provide decision makers with more scientific and accurate decision-making support.

References

[1] Verma, R.; Sharma, B., Prioritized Information Fusion Method for Triangular Fuzzy Information and Its Application to Multiple Attribute Decision Making. International Journal of Uncertainty Fuzziness and Knowledge-Based Systems 2016, 24 (2), 265-289.

[2] Guo, J.-c.; Fan, D.; Che, H.-y.; Duan, Y.-n.; Wang, H.-S.; Zhang, D.-W., An approach to network security evaluation of computer network information system with triangular fuzzy information. Journal of Intelligent & Fuzzy Systems 2015, 28 (5), 2029-2035.

[3] Radulescu, C. Z. In Cloud providers' services evaluation using triangular fuzzy numbers, 21st International Conference on Control Systems and Computer Science (CSCS), Univ Politehnica Bucharest, Bucharest, ROMANIA, 2017 May 29-31; Univ Politehnica Bucharest, Bucharest, ROMANIA, 2017; pp 123-128.

[4] Shao, G. X.; Cao, Z. G., Preferred reinforcement scheme for proximate foundations of existing high-speed railways based on triangular fuzzy number-TOPSIS %J Railway Standard Design. 2019, 63 (02), 40-44.

[5] Deng, F.; Zhang, X.; Liang, X.; Guo, Z.; Bao, C.; Ieee In Earthquake Disaster Emergency Supply Chain Performance Evaluation Based on Triangular Fuzzy Numbers, IEEE International Conference on Industrial Engineering and Engineering Management (IEEM), Bali, INDONESIA, 2016 Dec 04-07; Bali, INDONESIA, 2016; pp 1483-1487.

[6] Ebrahimnejad, A.; Luis Verdegay, J., An efficient computational approach for solving type-2 intuitionistic fuzzy numbers based Transportation Problems. International Journal of Computational Intelligence Systems 2016, 9 (6), 1154-1173.

[7] Kumar, M., Evaluation of the intuitionistic fuzzy importance of attributes based on the correlation coefficient under weakest triangular norm and application to the hotel services. Journal of Intelligent & Fuzzy Systems 2019, 36 (4), 3211-3223.

[8] Dong, J.; Wan, S.; Chen, S.-M., Fuzzy best-worst method based on triangular fuzzy numbers for multi-criteria decision-making. Inf. Sci. 2021, 547, 1080-1104.

[9] Jiang, B.; Liang, S. A.; Gao, Y. J.; Zhang, Y. M., A triangular fuzzy number multi-attribute decision-making method based on determining attribute weights based on reliability %J Fuzzy Systems and Maths. 2021, 35 (04), 113-123.

[10] Bisht, D.; Srivastava, P. K. In A Unique Conversion Approach Clubbed with a New Ranking Technique to Optimize Fuzzy Transportation Cost, 2nd International Conference on Recent Advances in Mathematical Sciences and its Applications (RAMSA), Jaypee Inst Informat Technol, Noida, INDIA, 2017 Dec 12-14; Jaypee Inst Informat Technol, Noida, INDIA, 2017.

[11] Peng, X.; Dai, J., Approaches to Pythagorean Fuzzy Stochastic Multi-criteria Decision Making Based on Prospect Theory and Regret Theory with New Distance Measure and Score Function. International Journal of Intelligent Systems 2017, 32 (11), 1187-1214.

[12] Xiao, L.; Xu, D.; Xie, C.; Mandayam, N. B.; Poor, H. V., Cloud Storage Defense Against Advanced Persistent Threats: A Prospect Theoretic Study. Ieee Journal on Selected Areas in Communications 2017, 35 (3), 534-544.

[13] Dong, Y.; Luo, N.; Liang, H., Consensus building in multiperson decision making with heterogeneous preference representation structures: A perspective based on prospect theory. Applied Soft Computing 2015, 35, 898-910.

[14] Li, A.; Zhao, Z., Crane Safety Assessment Method Based on Entropy and Cumulative Prospect Theory. Entropy 2017, 19 (1).

[15] Liu, H.-h.; Song, Y.-y.; Yang, G.-l., Cross-efficiency evaluation in data envelopment analysis based on prospect theory. European Journal of Operational Research 2019, 273 (1), 364-375.

[16] Xu, D.; Xiao, L.; Mandayam, N. B.; Poor, H. V.; Ieee In Cumulative Prospect Theoretic Study of A Cloud Storage Defense Game Against Advanced Persistent Threats, IEEE Conference on Computer Communications (IEEE INFOCOM), Atlanta, GA, 2017 May 01-04; Atlanta, GA, 2017; pp 541-546.

[17] Guan, Y.; Annaswamy, A. M.; Tseng, H. E.; Ieee In Cumulative Prospect Theory Based Dynamic Pricing for Shared Mobility on Demand Services, 58th IEEE Conference on Decision and Control (CDC), Nice, FRANCE, 2019Dec 11-13; Nice, FRANCE, 2019; pp 2239-2244.

[18] Wang, W.; Sun, H.-j., Cumulative prospect theory-based user equilibrium model with stochastic perception errors. Journal of Central South University 2016, 23 (9), 2465-2474.

[19] Cabedo-Peris, J.; Gonzalez-Sala, F.; Merino-Soto, C.; Pablo, J. a. C.; Toledano-Toledano, F., Decision Making in Addictive Behaviors Based on Prospect Theory: A Systematic Review. Healthcare 2022, 10 (9).

[20] Ghader, S.; Darzi, A.; Zhang, L., Modeling effects of travel time reliability on mode choice using cumulative prospect theory. Transportation Research Part C-Emerging Technologies 2019, 108, 245-254.

[21] Clark, W. A. V.; Lisowski, W., Prospect theory and the decision to move or stay. Proceedings of the National Academy of Sciences of the United States of America 2017, 114 (36), E7432-E7440.

[22] Rouyard, T.; Attema, A.; Baskerville, R.; Leal, J.; Gray, A., Risk attitudes of people with 'manageable' chronic disease: An analysis under prospect theory. Social Science & Medicine 2018, 214, 144-153.

[23] Starczewski, J. T.; Goetzen, P.; Napoli, C., TRIANGULAR FUZZY-ROUGH SET BASED FUZZIFICATION OF FUZZY RULE-BASED SYSTEMS. Journal of Artificial Intelligence and Soft Computing Research 2020, 10 (4), 271-285.

[24] Wang, S. L.; Qu, S. J.; Ma, G., Supplier selection decision for online multi-attribute procurement auctions based on prospect theory and fuzzy theory %J Control and Decision. 2020, 35 (11), 2637-2645.

[25] Chen, Xue; Huang, Zhi; Luo, Key, A triangular fuzzy number type uncertain multi-attribute decision-making method based on relative similarity relationship %J Control and Decision. 2016, 31 (12), 2232-2240.

[26] Zhu, Zhang-Ya; Cao, Bing-Yuan, Linear programming problems with fuzzy variables %J Fuzzy Systems and Mathematics. 2008, (01), 115-119.

[27] Kahneman, D.; Tversky, A., Prospect Theory: An Analysis of Decision under Risk. Econometrica 1979, 47 (2), 263-291.

[28] Tversky, A.; Kahneman, D., Advances in prospect theory: Cumulative representation of uncertainty. Journal of Risk and Uncertainty 1992, 5 (4), 297-323.