Quantitative trading portfolio optimization based on machine learning

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Abstract. The capital asset pricing model (CAPM) is to study the quantitative relationship between the expected return rate of assets and risky assets in the securities market. With the development of computer network technology and the trading market, quantitative trading has gradually become a new way of investment. Its essence lies in the decision of trading strategy. This paper obtains the relative price data of the subsequent short-term investment period through the time series method, then uses the PA algorithm in machine learning to update the portfolio and verify the effectiveness of the strategy in the data set. Based on the exchange rate change trend of the US dollar, gold, and bitcoin within five years given in the question, we constructed dynamic portfolio models for the three assets to ensure maximum profits. First, we use the time series model to predict the relative price data of the next period in a continuous trading period. Then, we implement the PA passive attack algorithm by calling the Sklearn package in Python to update the asset components in real-time every day and calculate the average annual return rate in 5 years. Based on the above analysis, we used the ADF unit root to test the stationarity of the time series. Meanwhile, we also carried out a white noise test of time series. The specific test indexes include: According to CAPM's theory, this product has a high yield, with an annual interest rate of 195.849%, which is entirely objective. However, the annual standard deviation reached 0.919164, and the maximum retractable rate exceeded 50%, indicating a sizeable overall risk. Despite this, the composite product's Calmar ratio is 3.556, Sharpe ratio is 2.104623, and its return per unit risk is still high. Compared with other quantitative trading models, this paper combines time series, machine learning, Monte Carlo, and other algorithms to effectively solve the portfolio investment optimization problem of nonlinear time-varying correlation structure among multiple assets. It enriches the results of portfolio investment structure decisions.

Keywords: Portfolio Investment, Time Series, PA Algorithm, Machine Learning.

1. Introduction

The Capital Asset Pricing Model (CAPM) is developed by William Sharpe, John Lintner, Jack Treynor, and Jan Mossin) et al. in 1964. The model mainly studies the relationship between the expected return rate of assets and risk assets in the securities market and how equilibrium price is formed, which is the pillar of the price theory of the modern financial market and is widely used in investment decision-making and corporate finance.

1.1 Research Background

The primary content of the capital asset pricing model (CAPM) is to study the quantitative relationship between the expected return rate of assets and risky assets in the securities market [1] [2].

With the development of computer network technology and the trading market, quantitative trading has gradually become a new way of investment, and its essence lies in the decision of trading strategy.

1.2 Restatement of the Problem

The essence of the problem can be considered as a five-year investment product consisting of three assets: U.S. dollars, gold, and Bitcoin, adjusting the proportion of the assets randomly according to the fluctuation of the market.

1.3 Methods put forward

In the primary model establishment, we use the time series model to predict the relative price data of the next period in a continuous trading period. Then, we implement the PA passive attack algorithm by calling the Sklearn package in Python to update the asset components in real-time every day and calculate the average annual return rate in 5 years.

Considering the stationarity of our time series, we use the Augmented Dickey-Fuller test and the white noise verification, and then through the dynamic portfolio model, we can figure out the maximum profit of our investigation. Moreover, the model introduces several parameters, such as the Sharpe ratio, the Calmar ratio, etc.

To allow for the robustness of our model, we use the Monte-Carlo Simulation to adjust the proportion of transaction costs between two assets. Experiments show that the proportion of the gold transaction does not affect the result too much. Therefore, the decision of preference for bitcoins transaction has specific stability.

2. Assumptions and Explanations

It ia assumed that we do not consider the existence of tax costs in the market.

Assume that assets are divisible infinitely.

Assume that the exchange rate of the US dollar against bitcoin and gold was not affected by unexpected factors

Assume that net income and overall risk are only related to exchange rates and transaction costs and are not affected by other external factors

3. Notations

The following table 1 shows the variable factor used in this paper.

Table 1. Variable Description Model Preparation

Symbols	Description
x_t	*N*-dimensional vector, the input example in the t^{th} loop
y_t	Correct class acceptance label, the collection of its value= {+1, -1)
τ_t	Step length
w	Weighted vector with dimensions
$\lvert w \cdot x \rvert$	Predictive confidence
$y_t\left(w_t \cdot x_t\right)$	Marginal profit in the t^{th} loop
x_j	The investment ratio of the j^{th} asses
r	return on assets
μ	expected return

3.1 Model Establishment

To look from a longer time, the economy will run under a particular rule. However, due to some external factors (market demand, regulation, control, etc.), the rate between bitcoins, gold, and US Dollars may change from day to day, which could create some difficulties for us to predict. Therefore, the AR auto-regressive and Moving Average model provided a relatively reliable approach to predicting the value of bitcoins and gold in the short term because it considered the dependence of economic phenomena on time series and the interference of random fluctuation.

The p-order auto-regressive model can be described as:

$$r_t = \emptyset_0 + \emptyset_1 r_{t-1} + \ldots + \emptyset_p r_{t-p} + a_t \tag{1}$$

The expression can be represented by ARMA (*p*,0, *q*).

After we predict the rate between bitcoins, gold, and US Dollars, we should aim at the best earnings of our investigation. Therefore, we decided to use the Passive-Aggressive Algorithm to figure out the maximum value of our investment.

In order to make our overall solution clearer, we use a flow chart to visualize our process.

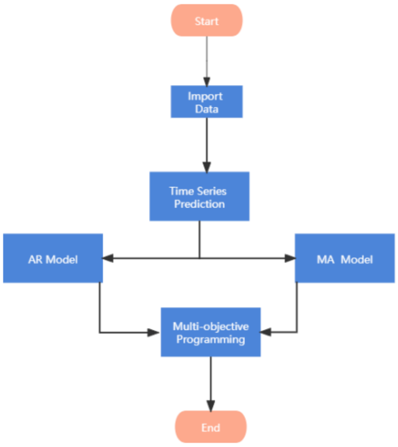

Figure 1 The overall framework

3.2 The solution of the AR Model

Since the essence of the problem is a class of maximum(minimum) value problems, the common solution is the least square method. The method can be described as the following statement:

We can add the error to the input vector if the error is measurable. Thus, we can form an augmented vector:

$$\emptyset_{t-1} = [-y_{t-1}, \ldots, -y_{t-p}, \epsilon_{t-1}, \ldots, \epsilon_{t-p}] \tag{2}$$

Thus, we have:

$$y_t = \emptyset_{t-1}^T \theta + \epsilon_t \tag{3}$$

Using the recursive least square method, we can figure out the coefficient θ.

3.3 The solution of the MA Model

In terms of the MA Model, the self-covariance function can be discussed in the following three scenarios:

$$\gamma_k = \begin{cases} \sigma_\varepsilon^2(1+\theta_1^2+\ldots+\theta_q^2) & \text{when k} = 0 \\ \sigma_\varepsilon^2(-\theta_k+\theta_1\theta_{k+1}+\ldots+\theta_{q-k}\theta_q) & \text{when } 1 \le \text{k} \le \text{q} \\ 0 & \text{when k} > q \end{cases} \tag{4}$$

According to the formula, we have:

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \begin{cases} \frac{\theta_k+\theta_{k+1}\theta_1+\ldots+\theta_q^2}{1+\theta_1^2+\ldots+\theta_q^2}, k = 1, \ldots, q \\ 0, k > q \end{cases} \tag{5}$$

The equation can be calculated in terms of the value of q:

When $q=1$, we have the degradation:

$$\theta_1 = \frac{-1 \pm \sqrt{1-4\rho_1^2}}{2\rho_1} \tag{6}$$

However, when q is larger than 1, the expression can be very complicated, so we can use the iteration method to find the approximate solution:

$$\hat{\theta}_k = -\left(\frac{\hat{\gamma}_k}{\hat{\sigma}_\varepsilon^2} - \hat{\theta}_1\hat{\theta}_{k+1} - \ldots - \hat{\theta}_{q-k}\hat{\theta}_q\right) \tag{7}$$

Where one group of the initial value of the parameter is:

$$\sigma_\varepsilon^2(0) = \hat{\gamma}_0, \hat{\theta}_1(0) = \widehat{\hat{\theta}}_2(0) = \ldots = \hat{\theta}_q(0) = 0 \tag{8}$$

3.4 The principle of the PA Algorithm

After obtaining the predicted value of the time series by ARIMA model fitting, we automatically use the PA passive attack algorithm to update the portfolio. The online passive attack algorithm is a lazy updating algorithm. When the lowest standard line is met, the parameters are not updated. The parameters are updated to form a better learner [3].

The larger the margin is, the better the effect is. When the marginal profit Margin is positive, the prediction of the algorithm is correct. In addition, we also want the margin to be at least 1, in which case the prediction confidence of the algorithm is good. There is a momentary loss in each cycle when the margin is less than 1. We describe this loss quantitatively, which is the hinge loss function:

$$\ell(b_t;(x,y)) = \begin{cases} 0, & \text{if } y\left(b_t^T \cdot x\right) \geq 1 \\ 1 - y\left(b_t^T \cdot x\right), & \text{otherwise} \end{cases} \tag{9}$$

The loss is 0 when the margin is greater than 1; otherwise, it is the difference between 1 and margin.

We expect the weight vector to be as close as possible before and after the update because we want to match the maximum interval theory so that the generalization performance is good and that each update retains the information learned before. Also, it avoids a trading volume that is a too significant loss of too much poundage [4].

The updated formula is:

$$b_{t+1} = \arg\min_{b \in R^n} \frac{1}{2}\|b - b_t\|^2, \text{ s.t. } l(b;(x_t,y_t)) = 0 \tag{10}$$

The purpose of the algorithm is to solve this optimization.

The model can be solved as follows:

There is a simple closed-form solution.

$$b_{t+1} = b_t + \tau_t y_t x_t \text{ where } \tau_t = \frac{l_t}{\|x_t\|^2} \tag{11}$$

When $l_t = 0$ $b_{t+1} = b_t$ is the optimal solution; when $l_t > 0$the Lagrange function of the optimization problem in the formula is:

$$\mathcal{L}(b,\tau) = \frac{1}{2}\|b - b_t\|^2 + \tau(1 - y_t(b \cdot x_t)) \tag{12}$$

Where $\tau \geq 0$ is the Lagrange multiplier. It takes the partial derivative of this concerning *b* and sets the partial derivative equal to 0.

$$\partial_w \mathcal{L}(b,\tau) = b - b_t - \tau y_t x_t = 0 \tag{13}$$

Then we have:

$$b = b_t + \tau y_t x_t \tag{14}$$

Combine the formulas above because we can gain.

$$\mathcal{L}(\tau) = -\frac{1}{2}\tau^2 \|x_t\|^2 + \tau(1 - y_t(b_t \cdot b_t)) \tag{15}$$

Take the part of the concern, and set the partial to be 0.

$$\frac{\partial \mathcal{L}(\tau)}{\partial_\tau} = -\tau \|x_t\|^2 + (1 - y_t(b_t \cdot x_t)) = 0 \tag{16}$$

Which finally forms:

$$\tau = \frac{1 - y_t(b_t \cdot x_t)}{\|x_t\|^2} = \frac{l_t}{\|x_t\|^2} \tag{17}$$

However, the data often carries noises, and the sample label may be wrong, resulting in unreasonable data updating. Therefore, the optimization model can be improved as follows:

$$b_{t+1} = \arg\min_{b \in R^n} \frac{1}{2}\|b - b_t\|^2 + C\zeta, \text{ s.t. } l(b; (x_t, y_t)) \leq \zeta \text{ and } \zeta \geq 0 \tag{18}$$

A slight modification of the above procedure results in:

$$\tau_t = \min\left\{C, \frac{l_t}{\|x_t\|^2}\right\} \tag{19}$$

Here τ is the non-negative relaxation variable, and *C* is the aggressiveness factor.

Now we bring the predicted results from the ARIMA model into the PA automatic attack algorithm. First, we use the ARIMA model to get the forecast price $\tilde{p}_{t+1}$ to calculate the forecast relative price.

$$\tilde{x}_{t+1} = \frac{\tilde{p}_{t+1}}{\tilde{p}_t} \tag{20}$$

When ε is the sensitive coefficient, receiving

$$\tau_t = \min\left\{0, \frac{\tilde{x}_{t+1}^T \cdot b_t - \varepsilon}{\|\tilde{x}_{t+1} - \bar{x}_{t+1} \cdot 1\|^2}\right\} \tag{21}$$

From this, we can obtain the dynamic update mode of b_t:

$$b_{t+1} = b_t - \tau_{t+1}(\tilde{x}_{t+1} - \bar{x}_{t+1} \cdot 1) \tag{22}$$

3.5 Analysis and Evaluation of results

The traditional model supposes that the datum in the research accord to a static random process. However, online learning infers its statistical properties by observing the numeric value. Based on maximum margin clustering, Hetmblod et al. Proposed the passive-aggressive algorithm, which could turn each step of the dynamic update into an easier optimization problem.

The change in total assets is shown as follows:

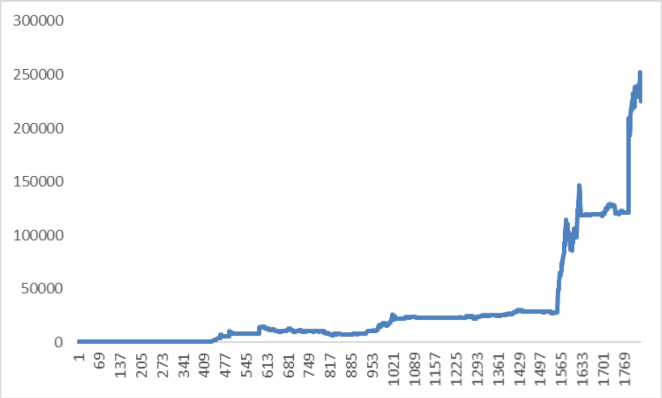

Figure 2 The change chart of total assets

It is shown that $1,000 will be worth $226,648.42 in five years.

4. Model Test

4.1 Stability test

4.1.1. ADF unit root test

In order to prove that the ARIMA model is suitable for time series analysis, we must prove that the data is stationary. Stationarity assumes that some time series is generated by some random process, assuming every value in a time series X_t all comes out randomly from a probability distribution, then the mean, variance, and covariance are all independent of time [5][6]. Here we use the ADF unit root test. If the sequence is stationary, there is no unit root. Otherwise, there would be a unit root. If the data is not stable, you can do a different transformation to check whether it is stable after the transformation.

$$\begin{gathered}\Delta X_t = \delta X_{t-1} + \sum_{i=1}^{m} \beta_i \Delta X_{t-i} + \varepsilon_t \\ \Delta X_t = \alpha + \delta X_{t-1} + \sum_{i=1}^{m} \beta_i \Delta X_{t-i} + \varepsilon_t \\ \Delta X_t = \alpha + \beta t + \delta X_{t-1} + \sum_{i=1}^{m} \beta_i \Delta X_{t-i} + \varepsilon_t\end{gathered} \tag{23}$$

It is assumed that a time series is generated by a random process, also as a hypothetical time series,

The zero hypothesis $H_0: \delta = 0$(X_t as a random changing series)

The alternative hypothesis $H_1: \delta < 0$(X_t as a stationary series)

When inspecting, we start from the third model, then the second, then the first, using the ***t*** inspection in the OLS (ordinary least squares) method to finish the inspection. With the increase in the number of delay periods, the auto-correlation coefficient of the stationary series will soon decay to 0, indicating good stability.

4.2 Representation of model

Using the classical Markowitz model to invest on n assets, supposing that x_j represents the investment ratio of the j^{th} asset, the return on assets is a random variable r, and the expected return μ is known.

$$\mu = \mathbb{E}r \tag{24}$$

The variance of the asset is:

$$\Sigma = \mathbb{E}(r-\mu)(r-\mu)^T \tag{25}$$

The expected return on the portfolio is:

$$\mathbb{E}y = \mu^T x \tag{26}$$

The combination of variance is:

$$\mathbb{E}(y-\mathbb{E}\mathbf{y})^2 = x^T \Sigma x \tag{27}$$

In the case of limiting risks, the portfolio return rate maximization model is:

$$\begin{aligned} &\max \mu^T x \\ &\text{s.t.} \begin{cases} e^T x = w + e^T x^0 \\ x^T \Sigma x \leq \gamma^2 \\ x \geq 0 \end{cases} \end{aligned} \tag{28}$$

The variance-covariance matrix Σ is a semi-positive definite matrix, so there is a matrix G that satisfies.

$$\Sigma = GG^T \tag{29}$$

It can use the Cholesky decomposition to find, and for a given, we have

$$x^T \Sigma x = x^T G G^T x = \left\| G^T x \right\|_2^2 \tag{30}$$

The risk constraints can be expressed as

$$\gamma \geq \left\| G^T \chi \right\| \tag{31}$$

Also, as

$$\left(\gamma, G^T, x\right) \in Q^{n+1} \tag{32}$$

Among it Q^{n+1} is the second-order cone of (n+1) dimension so that the model can transform to the max $\mu^T x$.

$$\text{s.t.} \begin{cases} e^T x = w + e^T \chi^0 \\ \left(\gamma, G^T, x\right) \in Q^{n+1} \\ x \geq 0 \end{cases} \tag{33}$$

5. Model Test

According to the results of *Q* statistics, if the *p*-value of Q_6 is greater than 0.1, the null hypothesis cannot be rejected at the significance level of 0.1. The residual of the model is white noise, and the model meets the requirements.

We treat our dynamic portfolio like a fund product that can change its composition dynamically. We can also use common quantitative indicators to measure the merits and demerits of this portfolio product, just like other fund products. Meanwhile, the ratio of each asset can change over time, which provides the product with a fantastic character. It can be seen as either a short-term project with a cycle of 1 or a five-year project. It makes the method legal.
Several criteria for inspection:

5.1 Annualized interest rate

The annualized interest rate refers to the annual interest rate discounted by the inherent rate of return of the product, which is the most intuitive way to view the return on investment quantitatively. The formula is:

$$\text{SharpeRatio} = \frac{E(R_P) - R_f}{\sigma_P} \tag{34}$$

If we did what our plan said, our $1,000 would end up as high as $226,648.42, at which point the annualized interest rate would be 195.849%. That is a staggering number, even higher than the 136.884% annualized rate. You'd get it if you put all your dollars into Bitcoin at once, regardless of risk, and far higher than gold's 6.262% annualized risk-free rate of 2.4%.

The reason is that the price of Bitcoin was relatively depressed during an intermediate period, and the dynamic investment strategy enabled us to get timely feedback and withdraw from bitcoin and turn to the relatively stable gold and US dollar markets, and timely catch the bottom when the price of Bitcoin rose to maximize the benefits. Perhaps this can inspire us to pay attention to the market at any time, constantly adjust our investment strategy, and make rational investments even better than luck and vision to achieve the best situation.

5.2 The average daily rate of return

The average daily rate of return is the sequential rise and fall range by subtracting 1 from the adjusted closing price of the previous trading day in the following trading day. It is pooled into a line chart we can have:

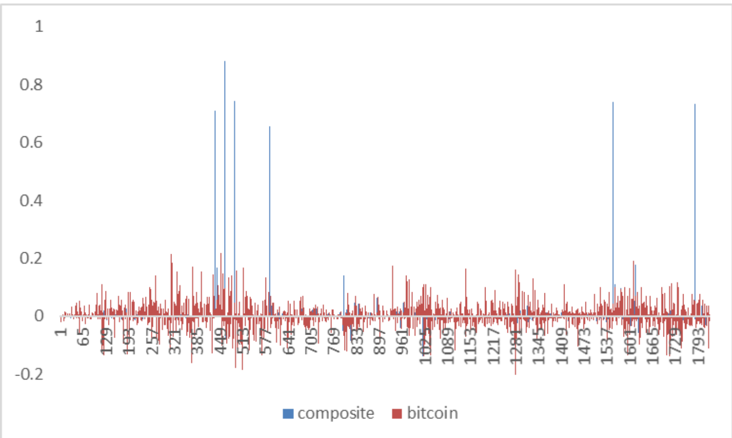

Figure 3 The experimental results

We can discover from this chart that bitcoin's daily returns are mixed. Portfolio projects look sparse, but there are several spikes. We hedge our risk with gold and the dollar so that the daily movement of assets is not as significant as bitcoin, which is close to zero. Moreover, those peaks are the points at which we reinvest when the day's returns change dramatically due to changes in the proportion of assets.

5.3 Maximum withdrawal rate

Maximum retracement rate: This is an important risk indicator for hedge funds and quantitative strategy trading. It represents the maximum return retracement at any historical point in the selected cycle when the net value of the product has reached its lowest point. A maximum retracement is used to describe the maximum loss that any investor may face, which can effectively measure the anti-risk ability.

The formula can be expressed like this:

P is the net value of the day, x is the day, y is the day after x, P_x is the net value of the product on the day after x, and P_y is the net value of the product on the day after P_x. So, we have the formula

$$\text{drawdown} = \max(P_x - P_y)/P_x \tag{35}$$

It essentially evaluates the retracement rate for each net value and finds the largest.

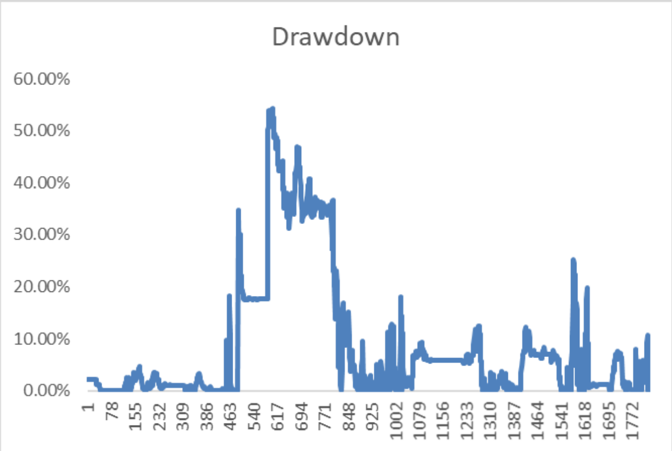

Figure 4 The the retracement rate

The higher the retracement rate, the greater the risk at that point. The maximum retracement rate reached 54.48%. It is a high number, reflecting the volatility of the bitcoin market. However, after the third year, the withdrawal rate of the portfolio has been at a low level, basically below 20%, reflecting a good overall development trend of the asset.

5.4 The Sharpe Ratio

The ratio reflects the "cost performance" of investment return and risk. Sharpe ratio in modern investment theory research shows that the size of the risk in determining portfolio performance has a fundamental role. The risk-adjusted rate of return is a comprehensive index that can consider both return and risk and eliminate the adverse effects of risk factors on performance evaluation in the long run. There is a general characteristic in investment: the higher the expected return of the investment object, the higher the volatility risk investors can tolerate. Conversely, the lower the expected return, the lower the volatility risk. Therefore, the primary purpose of rational investors to choose investment targets and portfolios is: to pursue the maximum return under the fixed risk they can bear; Or at a fixed expected reward, the pursuit of the lowest risk. The mean represents a return in the CAMP model, and variance represents risk. Thus, the ratio of annualized return to portfolio standard deviation is a good proxy for this "price-performance."

The Sharpe ratio requires a risk-free rate of return, and the risk-free rate of return is generally the yield on the 10-year Treasury, which is 2.4%. The Sharpe Ratio of this product is 2.104623, which performs well. Note that the Sharpe Ratio is closely related to the length of time. The short-term Sharpe Ratio is likely to reach a vast number, but it is significant for a five-year product. While the maximum retracement rate suggests many risks, Sharpe still suggests it is a risk-worthy product.

As a classic indicator, the Sharpe ratio is easy to use, but it also has some disadvantages: using standard deviation as a risk indicator is not reliable when the sample size is large. The Sharpe ratio has no reference point. Although it has a reasonable range by convention, its size is meaningless and only valuable compared to other combinations; The Sharpe ratio is linear, but at the efficiency frontier, the transformation between risk and return is not. As a result, the Sharp index is biased in measuring the performance of funds with large standard deviations. The most important point is that the calculation result of the Sharpe index is related to the selection of the period and the time interval of the return calculation [7].

5.5 The The Calmar Ratio

The Calmar Ratio is a modified version of the Sharpe Ratio but is also used to measure a fund's risk-return ratio. The Sharpe Ratio equates standard deviation with risk. The Calmar Ratio equates risk with maximum retracement:

$$\text{The Calmar Ratio } = \frac{\text{excess return}}{\text{maximum retracement}} \tag{36}$$

The reliability of this model is based on the fact that the maximum withdrawal rate is not too small. Equate "cost performance" with the return on a potential loss. In general, we consider the product to be better if the Calmar ratio reaches 3. The Calmar Ratio of this portfolio reached 3.556334223. For the same category of products, the Calmar Ratio's absolute numbers are more convincing and less time-dependent.

6. Conclusion

The arima model is practical and straightforward. Fewer parameters are required in the Arima model. Compared to the single AR model and the MA model, the Arima model combines the methods of auto-regression and moving average method, which has better accuracy. The model can be considered a five-year investment product consisting of three assets: U.S. dollars, gold, and Bitcoin, adjusting the proportion of the assets randomly according to the fluctuation of the market. We use the Arima model to do linear regression over time series and assess how the value of various products changes over time. Based on this, we use online learning with a passive attack algorithm to do simple optimization for each day and automatic update of the weight vector.

After running the model, it was found that the effect was good, the return was almost always increasing in the long run, and the final asset was $18,269.06. The above is the model of the team. From the perspective of the final assets, the model meets the requirements of the actual situation.

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