Timing Profits to Realize Gains with $1,000: Evidence from Five-Year Prices of Bitcoin and Gold

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Abstract. Quantitative investment methods use mathematical models to estimate the price trends of financial assets based on their historical movements and to accurately choose the best time to trade, enhancing returns and their stability. In this paper, we construct quantitative investment models and strategies and practice them in a five-year portfolio of cash, gold and bitcoin, achieving considerable returns. To form a complete investment strategy, two main models are required, namely, one to forecast prices of gold and bitcoin, and one to decide on specific trading operations. To forecast prices, ARIMA-GARCH models group is firstly built. In the method of price forecasting using time series analysis, the ARIMA time series model can grasp the dynamic pattern and trend of prices. However, according to the empirical data, it is found that it is difficult to reflect the price volatility characteristics using ARIMA model forecasting. Therefore, we consider the conditional heteroskedasticity GARCH model to reflect the volatility of the time series in practice. In this section, we use the data given in the topic to construct a hybrid model ARIMA (4,1,4)-GARCH (1,1) by combining the ARIMA model with the GARCH model to forecast the price of gold and the price of bitcoin. After forecasting of the future prices of gold and bitcoin, we apply a dynamic program model to design the specific daily investment strategies based on the prediction. We divide the investment strategies into whether to trade a certain asset and how much to trade. In the specific discussion of each case, we introduce qualifying conditions from the perspective of risk and trading rules, and determine the amount of trading by solving the objective function under the qualifying conditions to form a complete investment strategy. Setting a 10% cap on our gold position and a 90% cap on our bitcoin position without changing transaction fees, our portfolio ends up at $37370.91 after 5 years, with an ideal ROI of 3637.09%, an IRR of 205.19%. For complementary, we conduct sensitivity analysis. To study the impact of transaction costs on total returns, we change the transaction costs of gold and bitcoin 1% at a time simultaneously and calculate the change in expected returns, while using two models, the normalized coefficient method and the shapley value decomposition method, to determine the sensitivity of the transaction costs of both to total returns. It is finally shown that the higher the transaction cost, the lower the return; bitcoin's transaction cost is more sensitive and contributes more. At the end of the paper, we present evidence that our model provides a better strategy by proving the ARIMA-GARCH model has the ability to forecast prices more accurately.

Keywords: ARIMA-GARCH Model; Dynamic Planning; Quantitative Investments.

1. Introduction

1.1 Background

The prices of financial assets such as stocks, futures and virtual currencies on the public trading market are influenced by a large number of factors. In the long term, the price trend of a particular asset may be inevitable, but in the short to medium term, price changes tend to be more volatile and uncertain. Traditional investors tend to rely on qualitative and subjective judgments when making decisions, including fundamental information, macro information and news. But such investments often miss the best time to trade and are also susceptible to subjective psychological factors such as fear and greed, and information that non-professional investers can ackwowledge is ususally considered inadaquate. However, if we correctly use computer technology and mathematical models from a quantitative point of view, we can reasonably estimate their price trends and a better time to trade, based on the historical price trend of financial assets, namely, quantitative investment. In this

way, we can effectively avoid the shortcomings of traditional investment methods and improve the rate of return and its stability.

One of the most common financial assets is "money", of which gold and bitcoin are the two extremes. The former is the original form of modern money, which has long played the role of a value preserver throughout its long history, while the latter is an emerging form of money born from computers and blockchain technology. In this study, we apply quantitative investment tools to investments in gold and bitcoin to generate higher returns for investors who adopt our investment model over a total period of 5 years from 9/11/2016 to 9/10/2021.

1.2 Restatement of the Problem

To achieve our goals to provide higher returns for investors, specifically, we need to:

- Designing price prediction models for gold and bitcoin, predicting the price movements of both over a certain period of time in the future based on past historical data.
- Developing trading strategies to decide under what conditions to buy or sell financial assets and what the amount of the transaction should be, given the known transaction costs and forecasts of future price movements.
- Analyzing how different transaction costs will affect our strategies and investment results.
- Presenting our models, strategies and investment results to traders or investors.

1.3 Model Overview

To solve the problems mentioned above, we will proceed as follows.

To make complete investment strategies, the ARIMA-GARCH models group is firstly built to achieve predictions for gold and bitcoin prices. In this section, GARCH models are combined with ARIMA madels to reflect the volatility of the time series in practice. We first take the natural logarithm of the original data and perform differencing to verify that it is consistent with a smooth time series. Secondly, we compare the statistics of ARIMA models with different lag orders and finally choose ARIMA (4,1,4) and test it for heteroskedasticity. Next, we construct different ARIMA-GARCH models using different distributions and finally choose ARIMA (4,1,4)-GARCH (1,1) by comparing different statistics as our model for forecasting the price of bitcoin and gold.

Once the short-term prices of gold and bitcoin are predicted, we substitute them into the investment strategy model that follows and use them as an important basis for trading. Using a dynamic programming model, we divide the investment strategy into whether to trade a certain financial asset and how much to trade. In determining whether to trade a certain financial asset, we use as a judgment condition whether the expected volatility of that asset on the next day exceeds its transaction fee percentage, and accordingly classify it into three cases: both trading, one trading, and neither trading. In the specific discussion of each case, we introduce qualifying conditions from the perspective of risk and trading rules, and determine the amount of trading by solving the objective function under the qualifying conditions to form a complete investment strategy.

In our sensitivity analysis, in order to study the impact of transaction costs on total returns, we change the transaction costs of gold and bitcoin 1% at a time simultaneously and calculate the change in expected returns, while using two models, the normalized coefficient method and the shapley value decomposition method, to determine the sensitivity of the transaction costs of both to total returns. It is finally shown that the higher the transaction cost, the lower the return;

Bitcoin’s transaction cost is more sensitive and contributes more.

At the end of the paper, we present evidence that our model provides a better strategy by proving the ARIMA-GARCH model has the ability to forecast prices more accurately.

In conclusion, the structure of the whole paper is shown in Fig. 1.

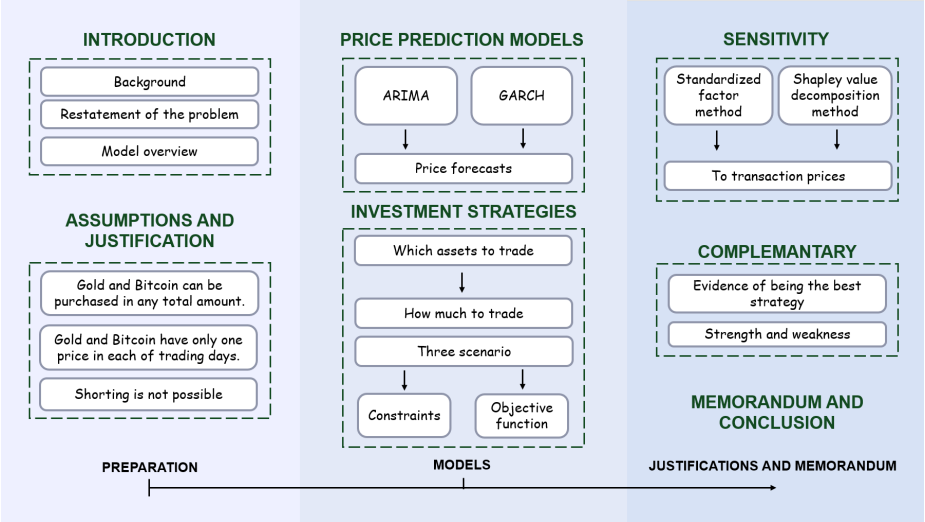


Fig. 1 The structure of the whole paper

2. Assumptions and Justification

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

● Gold and Bitcoin can be purchased in any total amount, instead of unit by unit. According to trading rules, the minimum trading unit of gold is 100 grams, which is about 3 troy ounces, worthing nearly $3000 exceeding the starting capital. This means we can't buy gold until we make a certain amount of money from investing in bitcoin. However, in real quantitative investing, very few projects have an initial capital of only $1000. Adequate starting capital will hardly be limited by the minimum trading unit. We mimic this situation so that our model can be applied to larger investments.

● Gold and Bitcoin are considered to have only one price in each of their trading days. Gold and Bitcoin have continuously changing prices during each of their trading days. However, since we are offering a daily trading strategy and not a mid-day trading strategy,

we assume that there is only one price within a trading day and use that as the basis for our strategy.

● Shorting is not possible This means that there is no operation to sell assets first and then sell them after they have fallen.

3. Price Prediction Models Based on ARIMA-GARCH

In this section, the process of building prediction model of gold prices will be presented in detail. Since we use the similar method to predict bitcoin prices, we will only present the main result of its model at the last part of the section. The structure of the section is shown in Fig 2.

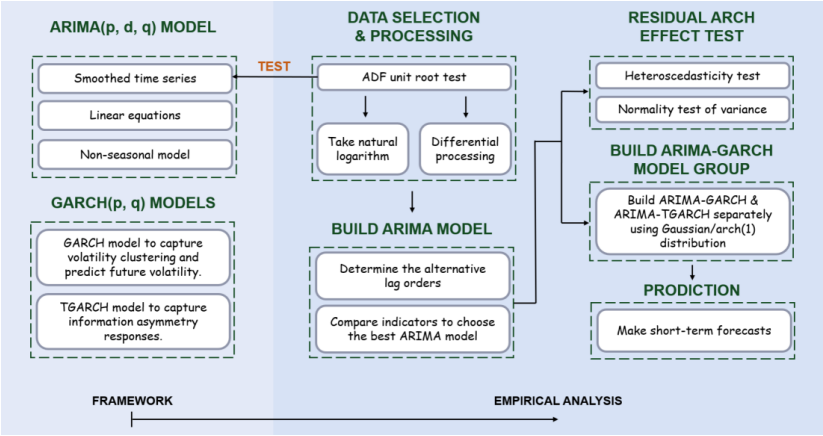


Fig. 2 The structure of building ARIMA-GARCH models

3.1 The Framework of ARIMA(p, d, q)-GARCH Model Group

3.1.1 ARIMA(p, d, q) Model

The ARIMA model is a general model for predicting time series and can be used to obtain a smooth time series by differencing. The ARIMA prediction equation for a smooth time series is a linear equation in which the predictors consist of the dependent variable lagged term and the prediction error lagged term. The non-seasonal ARIMA model is classified as an ARIMA(p, d, q) model, where p is the number of autoregressive terms, d is the number of differences, and q is the number of lagged forecast error terms in the forecasting equation. The ARIMA(p, d, q) equation expressed by Hyndman and Khandakar can be written as:

$$\emptyset_p(B)(1-B)^d y_t = \delta + \theta_q(B)\varepsilon_t\emptyset \tag{1}$$

3.1.2 GARCH(p, q) Model

Not all time series errors satisfy the assumption of homoscedasticity. Sometimes, the vari- ance is time-varying and conditional, so Engle developed an autoregressive conditional het- eroskedasticity (ARCH)-like model to describe series with time-varying conditional variance. Bollerslev extended it to develop the GARCH model, which captures volatility clustering and predicts future volatility. In the GARCH model, both the past variance and the predicted value of the past variance can be used to predict the future variance. The expression of the standard GARCH model is: yt = μ + εt , where yt is time-series data

$$.\mu_t = \varepsilon_t\sigma_t^2 = \varepsilon_t\sqrt{h_t}, \varepsilon_t \sim N(0,1) \tag{2}$$

$$h_t = \delta + (\alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots) + (\beta_1 h_{t-1}^2 + \beta_2 h_{t-2}^2 + \cdots) = \delta + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \tag{3}$$

where δ = α0 (1 - β1), ht = σt̂ . For time-series data, α1 + β1 < 1, α0 , β0 < 0.

Good news and bad news can have different effects on volatility, with bad news thought to have a greater impact on future return volatility. When this happens, the symmetric GARCH model cannot capture the asymmetry of the volatility response. The TGARCH model proposed by Carnero and Pérez is able to handle the different effects of positve and negative effects, with εt −i > 0 for positive news or good news and εt −i < 0 for negative news or bad news. TGARCH can capture the impact of positive news shocks on financial markets when the market is in equilibrium, which is modeled as:

$$h_t = \delta + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \gamma\varepsilon_{t-1}^2 d_{t-1}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \tag{4}$$

where $h_t = \sigma_t^2$, γ is the leverage term, $d_t = \begin{cases} 1, & \varepsilon_t < 0, \\ 0, & \varepsilon_t \geqslant 0 \end{cases}$

3.2 Emperical Analysis Based on ARIMA(p, d, q)-GARCH Model Group

3.2.1 Data Processing

Before building the ARIMA-GARCH model, the original data of gold price needs to be tested for the smoothness of the time series. The result is shown in Fig. 3.

Dickey-Fuller test for unit root				Number of obs = 1244
		Interpolated Dickey-Fuller		
	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-0.732	-3.430	-2.860	-2.570

Machinnon approximate p-value for Z(t)=0.8382

Fig. 3 The result

It can be seen that the values of test statistics are greater than the critical values of t-statistics at 1%, 5%, and 10% test levels, indicating that the original data have unit roots, i.e., the data are not stable. So we do a first order differential on the price data of gold, as shown in Fig. 4.

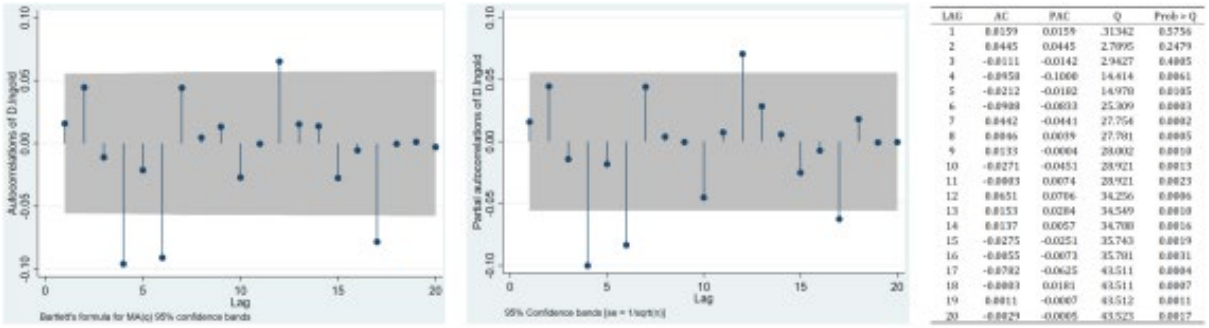
Dickey-Fuller test for unit root				Number of obs = 1244
		Interpolated Dickey-Fuller		
	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-34.596	-3.430	-2.860	-2.570
Machinnon approximate p-value for Z(t)=0.0000				

Fig. 4 Unit root test of D. Lngold

At this point the DF value of D(lngold) is -34.596, which is much less than the critical value of the t-statistic at 1%, 5%, and 10% test levels, with a concomitant probability near 0. Therefore, the data can be considered as smooth time series data and can be analyzed.

3.2.2 Build ARIMA(p, d, q) Model

The autocorrelation and partial autocorrelation plots are then established, as shown in Fig. 5. The autocorrelation coefficients are significant for lag orders p = 2, 4. The partial autocorrelation coefficients are significant for lag orders q = 2, 4.



Autocorrelation plot Partial autocorrelation plot Detailed values

Fig. 5 Autocorrelation and partial autocorrelation plots

Finally we choose ARIMA(4, 1, 4) model, of which the fitting result is shown below. According to Fig. 6, the specific form of the model is:

D (Lngold) = 0.9402198AR (2) -0.6137607AR (4) -0.9154429MA (2) +0.5125399MA (5)

		OPG				
D.lngold	Coef.	Std. Err.	z	P > | z |	[95% Conf.	Interval]
lngold						
_cons	.0001646	.0002326	0.71	0.479	-.002913	.0006205
ARMA						
ar						
L1.	-.1788713	.1250075	-1.43	0.152	-.4238814	.0661389
L2.	.9402198	.1090264	8.62	0.000	.726532	1.153908
L3.	-.0850534	.122353	-0.70	0.487	-.3248608	.1547541
L4.	-.6137607	.1160547	-5.29	0.000	-.8412238	-.3862977
ma						
L1.	.1984676	.1379428	1.44	0.150	-.0718954	.4688305
L2.	-.9154429	.1219317	-7.51	0.000	-1.154425	-.6764611
L3.	.0618514	.1369171	0.45	0.651	-.2065012	.330204
L4.	.5125399	.1269714	4.04	0.000	.2636806	.7613993
/sigma	.0085645	.0001039	82.39	0.000	.0083608	.0087682

Fig. 6 ARIMA (4, 1, 4) model

3.2.3 Residual ARCH Effect Test

Then the residuals of the model are tested for heteroskedasticity below. According to the z-test and p-value, the residuals are a smooth time series; the residuals are plotted as follows, which can also be visualized as a smooth time series.

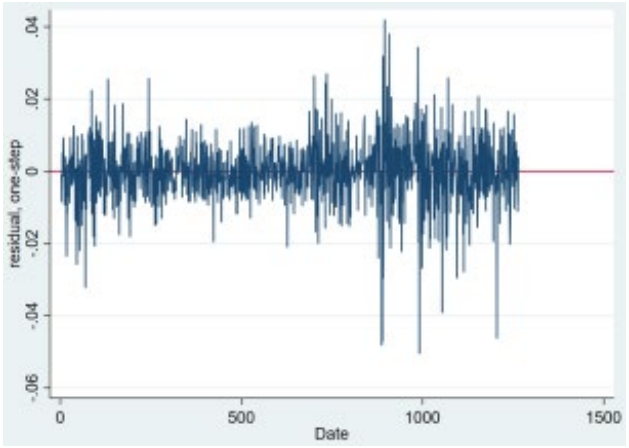


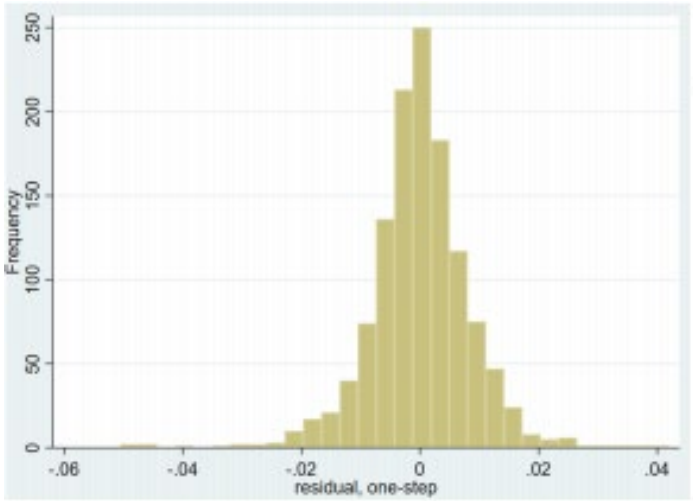
Fig. 7 Residuals plot of the ARIMA (4, 1, 4) model

After the coefficient significance test is passed, the validity test of the model is performed, which is to check whether the residuals are white noise. The white noise test is performed with the Q statistic. As the result in Fig. 8, P = 1.0000 > 0.05, passing the white noise test.

Portmanteau (Q) statistic	=	1.5643
Prob > chi2(20)	=	1.0000

Fig. 8 Portmanteau test for white noise

In order to determine whether the distribution of the residuals is normal, a descriptive statistical test is performed on the residuals, and the test results are shown in Fig. 9, which indicates that the model residuals fit well with the normal distribution.



Variable	lngold_RES
Obs	1244
Mean	-4.42e-06
Std. Dev.	0.0085693
Min	-.0504418
Max	.0418636

Fig. 9 The descriptive statistics of residuals

3.2.4 Build ARIMA(p, d, q)-GARCH Model Group

For the perturbation term εt in ARIMA model, specifically in Equation (1), we build ARIMA-GARCH model saparately adopting t distribution, Gaussian distribution and arch(1) distribution. The results are shown in Fig. 10.

The cons statistics of the 3 models are all below 0.05, indicating ideal fitting effect. So, we turn to compare the aic statistics and bic statistics of the 3 models in the figure below, observing that model adopting t distribution owns the lowest aic and bic statistics, which is also the model we choose, namely GARCH (1,1) model. Together with the ARIMA model, the ARIMA (4, 1, 4)-GARCH (1, 1) model is final model we use to predict the prices of gold.

As for predicting prices of bitcoin, we adopted the same process of predicting gold prices. The final result of the prediction model for bitcoin is also ARIMA (4, 1, 4)-GARCH (1, 1).

t distribution		Gaussian distribution		arch(1) distribution	
	(1)		(1)		(1)
	_000003		lngold		lngold
		lngold		**lngold**	
		_cons	7.159***	_cons	7.158***
			(12031.79)		(11835.80)
		ARCH		**ARCH**	
L._000003	0.992***	L.arch	0.993***	L.arch	0.993***
	(230.31)		(6.24)		(6.30)
L._000004	0.00360	L.garch	-0.000718		
	(0.11)		(-0.32)		
_cons	0.00305*	_cons	0.0000368***	_cons	0.0000363***
	(1.96)		(8.88)		(9.77)
N	1053	N	1255	N	1255

t statistics in parentheses
* p < 0.05, ** p < 0.01, *** p < 0.001

Fig. 10 Parameters of the 3 models

Parameters	t distribution	Gaussian distribution	arch(1) distribution
N	1265	1265	1265
ll	-7795.5	-8202.6	-8565.6
aic	15601.0	16413.2	17137.2
bic	15626.7	16433.8	17152.6

Fig. 11 Statistics of the 3 models

3.3 Prediction

Based on the models mentioned above, we predict the daily prices of gold and bitcoin using historical prices to that date. Price forecasts for selected periods, namely, from 09/11/2016 to 09/11/2019 for gold and from 09/11/2016 to 09/11/2017 for bitcoin , are showned in Fig. 12.

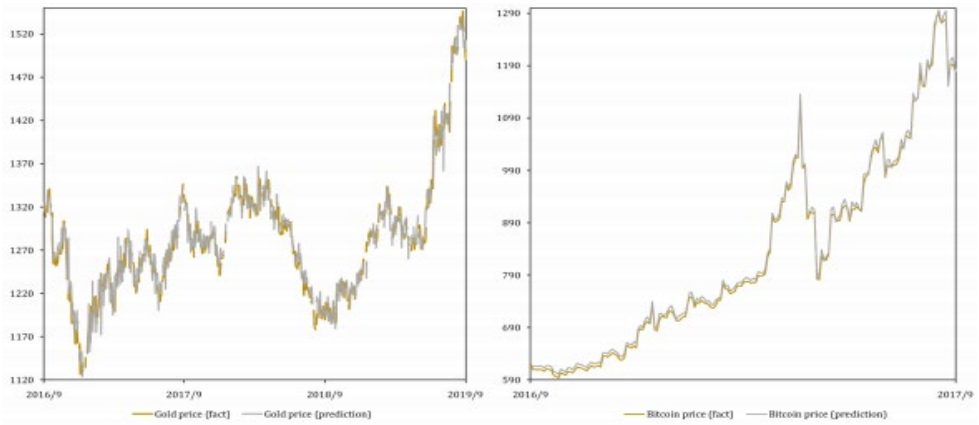


Fig. 12 Prices forcasts for gold and bitcoin

4. Investment Strategy

4.1 Notations

In this section, we use the nomenclature in in the model construction. Other none frequent- used symbols will be introduced once they are used.

In the notations above, LG = (YG - PG) × 100%/PG, and LB = (YB - PB) × 100%/PB. Under general conditions, CG = 1%, CB = 2%, and the beginning value of T is 1000 dollars.

In addition, x1 and x2 represent the amount of gold and bitcoin sold during each trading day, respectively, with positive values indicating assets sold and negative values indicating assets bought. They are the key variables that express the daily trading behavior.

Symbol	Definition	Symbol	Definition
T	Amount of total assets	V_G	Percentage of gold position cap
x_1	Amount of gold sold	V_B	Percentage of bitcoin position cap
x_2	Amount of bicoin sold	L_G	Fluctuation range of gold price
$Cash$	Amount of cash available	L_B	Fluctuation range of bitcoin price
P_G	Price of gold	Y_G	Next day's price forecast of gold
P_B	Price of bitcoin	Y_B	Next day's price forecast of bitcoin
C_G	Percentage of gold Trading Cost	G	Amount of gold available
C_B	Percentage of bitcoin Trading Cost	B	Amount of bitcoin available

Fig. 13 Notations in this section

4.2 Dynamic Programming Model

On each trading day, we design a trading strategy for that day based mainly on the ARIMA-GARCH model's forecast of the price for the next trading day, using the dynamic programming model. The trading strategy is divided into two main components, namely, which assets to trade and how much to trade, i.e., what is value of x1 and x2 . We condition whether the absolute value of the predicted volatility of an asset on the next trading day exceeds its corresponding percentage of transaction costs to determine whether we trade this asset. If it exceeds, we trade this asset; if it does not exceed, we do not trade this asset, because in this case, both the expected gain from buying and the expected loss avoided from selling will be less than the inevitable transaction fee paid, then obviously not trading can reduce the loss instead.

The structure of this section is shown below.

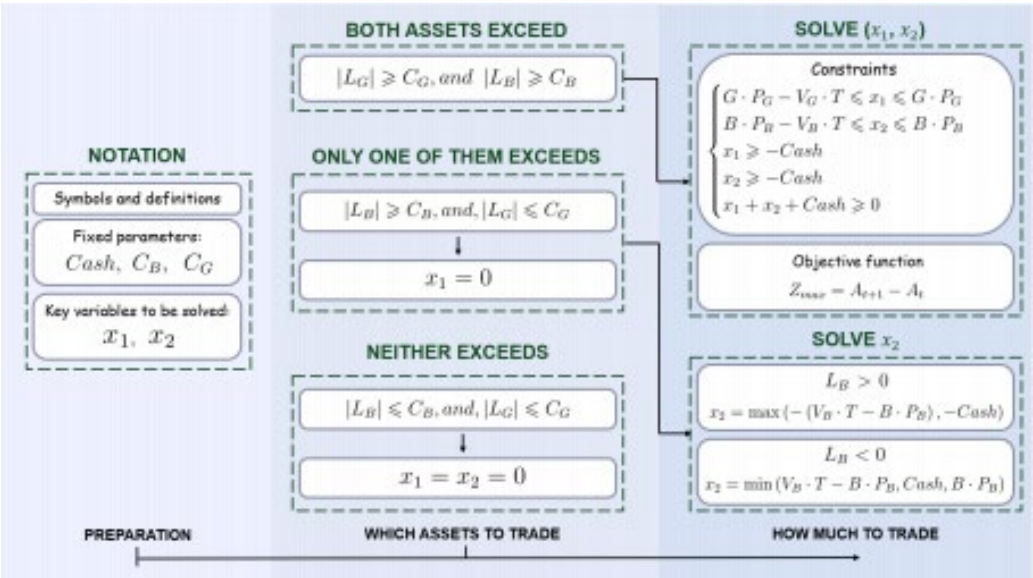


Fig. 14 The structure of the investment strategy

4.2.1 Both assets exceed the transaction fee

The condition for this case can be expressed symbolically as |LG | > CG , and |LB | > CB .

In designing a dynamic programming model, we must firstly specify the constraints on the solution of the objective function. In our model, there are two main constraints:

- Since shorting is not possible, the value of cash, gold, and bitcoin cannot be negative after any transaction has occurred.
- At no time should positions in gold and bitcoin exceed the position limits required by risk management.

The constraints can be expressed in equations as follows:

$$\begin{cases} G \cdot P_G - V_G \cdot T \leqslant x_1 \leqslant G \cdot P_G \\ B \cdot P_B - V_B \cdot T \leqslant x_2 \leqslant B \cdot P_B \\ x_1 \geqslant -Cash \\ x_2 \geqslant -Cash \\ x_1 + x_2 + Cash \geqslant 0 \end{cases} \tag{6}$$

The top two formulas indicate position limits on the left side, and the right side indicates that the amount of gold and bitcoin held cannot be negative. The three formulas below state that cash cannot be negative.

Subject to constraints above, the objective function of this dynamic programming is to maximize the expected return from each day's transaction, i.e., to maximize the difference between tomorrow's expected asset value and today's asset value. Tomorrow's expected asset value At+1 and today's asset value At can be expressed as:

$$\begin{cases} A_{t+1} = \left(G - \frac{x_1}{P_G}\right) Y_G + \left(B - \frac{x_2}{P_B}\right) Y_B + Cash + x_1 + x_2 - 0.01x_1 - 0.02x_2 \\ A_t = Cash + G \cdot P_G + B \cdot P_B \end{cases} \tag{7}$$

Then, the objective function will be:

$$\text{Zmax} = \text{At+1} - \text{At} \tag{8}$$

Simplifying the objective function, we get

$$Z_{\max} = G\left(Y_G - P_G\right) + B\left(Y_B - P_B\right) - \left(\frac{Y_G}{P_G} - 0.99\right) x_1 - \left(\frac{Y_B}{P_B} - 0.98\right) x_2 \tag{9}$$

Finally, x_1 and x_2, which are solved in the computer to maximize the expected return, are the amounts of gold and bitcoins that need to be sold on that day, respectively

4.2.2 Only one of them exceeds the transaction fee

In this case, we assume that the volatility of bitcoin exceeds the transaction fees, but the volatility of gold does not, i.e., |LB | > CB , and, |LG | ← CG . The case of the opposite condition is symmetrical to the case we will discuss, and will not be repeated.

In this case, no matter how much bitcoin is traded, gold is not traded, i.e., x1 = 0. As for x2 , if bitcoin is predicted to rise, i.e., LB > 0, we buy bitcoin for the smaller of the amount between cash available and the difference from position limit:

$$\text{x2} = \max\left(-\left(\text{VB}, \text{T} - \text{B}, \text{PB}\right), -\text{Cash}\right) \tag{10}$$

If bitcoin is predicted to fall, i.e., LB < 0,we sell bitcoin for the smaller of the amount among cash available, the difference from position limit, and the value of bitcoin available, in order that the values of cash and bitcoin are positive:

$$\text{x2} = \min\left(\text{VB}, \text{T} - \text{B}, \text{PB}, \text{Cash}, \text{B}, \text{PB}\right) \tag{11}$$

4.2.3 Neither exceeds the transaction fee

If none of the assets are forecasted to fluctuate more than the transaction fee, i.e., |LB | ← CB , and, |LG | ← CG , then we do not place any trades, namely, x1 = x2 = 0.

4.3 Results

Using the daily trade strategies mentioned above and setting a 10% cap on our gold position and a 90% cap on our bitcoin position without changing transaction fees, our portfolio ends up at $37370.91 after 5 years, with a net return of $36370.91 and an ROI of 3637.09%. The net movement of the various asset classes in the portfolio is shown below.

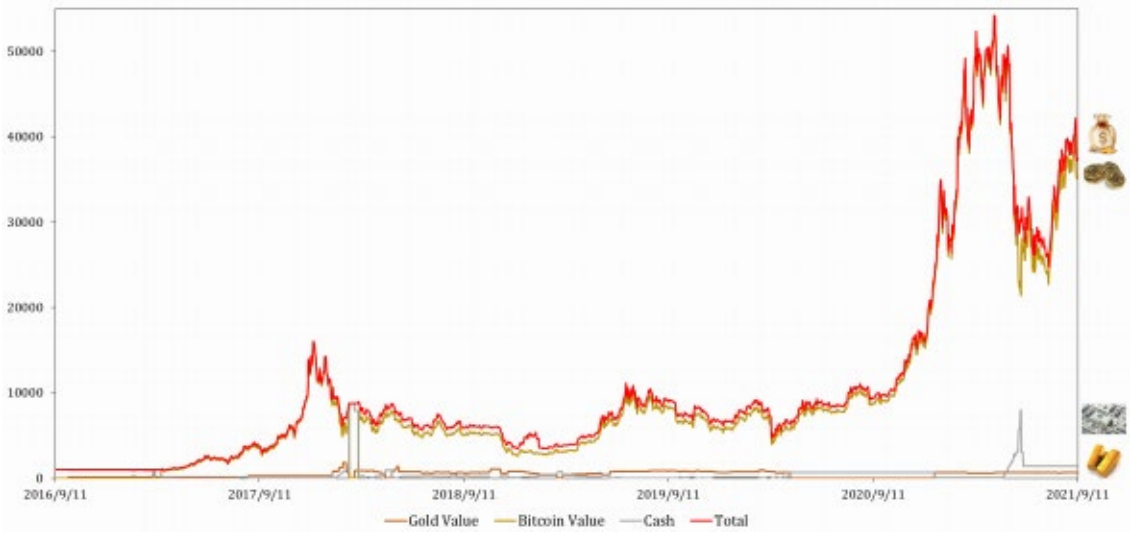


Fig. 15 Changes in assets in the portfolio

5. Sensitivity Analysis

In order to study the impact of transaction costs on total returns, we change the transaction costs of gold and bitcoin 1% at a time simultaneously, calculate the change in expected returns, and simultaneously use two models, the standardized coefficient method and the shapley value decomposition method, to determine the sensitivity of the transaction costs of both to total returns.

The standardized regression coefficients can eliminate the self-influence of the dependent and independent variables to obtain the standardized regression coefficients, which directly reflect the impact of the respective transaction costs on the total returns by the magnitude of the absolute values. However, since only the impact of changes in transaction costs on returns is considered in this paper, it is not possible to know the inter-influence relationship between variables. Moreover, the comparison results of standardized regression coefficients are only applicable to a specific environment and not absolutely correct. Therefore, this paper considers the effect of both variables on total returns by using the Shapley value decomposition method at the same time.

5.1 Standardization Factor Method

Firstly, we analyze the overall characteristics of the data with different transaction costs of returns. The transaction cost of gold varies from 0.5% to 1.5%, and the transaction cost of bitcoin varies from 1.5% to 2.5%, and the range of variation is 0.5% up or down from the original transaction cost, totaling 100 sets of data.

The overall characteristics of the total revenue (TR) data are as follows.

Variable	Obs	Mean	Std. Dev.
TR	69	11131.06	4442.353

Fig. 16 Total revenue data

The standard deviation of the explanatory variables was obtained as 4442.353.

After decentralization, the resulting data were subjected to standardized coefficient regres- sion analysis and the results were as follows.

TR	Coef.	Std. Err.	t	P > |t|	Beta
BIT_Cost	0.125221	.1221432	1.03	0.309	.125221
Gold_Cost	-0.005184	.1221431	-0.04	0.966	-.005184
_cons	-8.93e-09	.1212312	-0.00	1.000	

Fig. 17 Results of regression analysis of standardized coefficients

The beta values corresponding to each variable show that the transaction cost of bitcoin is more sensitive to the total return and its importance to the final total return is greater than the importance of

5.2 Conclusion

In summary, by using two sensitivity analysis methods we get that the sensitivity of the transaction cost of bitcoin to the total return is much greater than the sensitivity of the transaction cost of gold to it.

As seen by the higher price variance of bitcoin than gold, the price volatility of bitcoin is higher than that of gold, so when the transaction cost is raised, i.e., the minimum increase or decrease in trading, bitcoin's exchanges are more affected, and the total return is more sensitive to the higher transaction cost of bitcoin because the return from bitcoin is higher than the return from gold.

6. Complementary

6.1 Evidence that our model being the best strategy

In our investment strategy model, forecast prices for gold and bitcoin are important input variables and a key decision basis. The forecast price determines whether to trade an asset and how much to trade, which in turn affects the final return. Therefore, the accuracy of the forecast price will largely affect the return.

In our scenario, we choose a hybrid ARIMA-GARCH model instead of a single ARIMA model to predict prices. We compare the forecasts of the two models separately for gold prices

from 2/1/2017 to 2/5/2017, as well as their differences from the actual values, and the results of the comparison are as follows.

Date	Actual value	Prediction		Relative Error	
		ARIMA	ARI-GAR	ARIMA	ARI-GAR
2017/2/1	1203.65	1211.64	1199.595	0.00663	0.00337
2017/2/2	1221.95	1210.47	1228.198	0.00939	0.00766
2017/2/3	1215.20	1209.31	1220.640	0.00485	0.00448
2017/2/4	1226.75	1208.14	1216.438	0.01517	0.00841
2017/2/5	1231.00	1206.98	1243.507	0.01952	0.01016

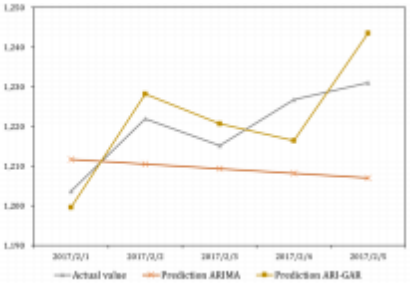


Fig. 18 Prediction of different models

As can be seen, the ARIMA-GARCH hybrid model has a higher forecasting accuracy compared to the ARIMA model can better assist the dynamic planning model for investment decisions. From this perspective, our strategy is superior to the other strategies.

6.2 Strength and Weakness

6.2.1 Strength

Our model has successfully achieved short-term price forecasts for assets such as gold and bitcoin, and has used dynamic programming models to design daily investment strategies that have yielded substantial returns. The main advantages of our model are as follows.

● Being based on "future" information. We base our daily investment strategies on fore- casts of future prices rather than relying solely on historical trading data. Many traditional technical investment approaches are based on variables such as K-charts, averages, and volume, which are also quantitative in perspective, but are based on historical information only. We use the ARIMA-GARCH model to predict future prices from existing prices and trade based on "future" information.

● Automating the entire investment process. If our model is linked to an electronic trading system, our model does not require frequent human intervention and judgment to complete the investment. The trader only needs to set the starting capital and position limits.

● Offering a high degree of flexibility. Traders and investors can easily manage risk by setting position limits for different assets based on their different risk appetites and expected return targets. In addition, we have demonstrated in our sensitivity analysis that our model can achieve significant returns even with changing transaction costs. The above factors allow the model to meet the needs of a wider range of people and adapt to different trading scenarios.

● Avoiding high transaction costs.Our model uses transaction cost as the threshold, and will not trade when the expected asset volatility is less than the transaction cost, which can effectively avoid excessive transaction costs from frequent operations.

6.2.2 Weakness

However, there is room for improvement in our model due to the limitations of data availability and the methodology used.

● The consideration of risk factors can be more diversified. Our model manages risk by setting upper limits on positions in different assets. However, there are many specialized measures of risk, including standardized variance of historical prices, deviations, and excess returns relative to average market returns. Our model can optimize the investment strategy by incorporating these risk indicators.

● The ability to forecast long-term asset price movements is limited.The ARIMA- GARCH model has good predictive power for short-term asset prices, but has limited predictive power for long-term asset price movements. If the model can achieve accurate prediction of long-term trend, it can further reduce the number of trades and improve the return.

7. Conclusion

In this paper, we have designed a complete quantitative investment model and simulated and analyzed it in real historical price data for gold and bitcoin from 9/11/2016 to 9/11/2021, acheiving ideal return. Our model consists of two parts, the price forecasting model and the trading strategy model based on this together form the complete investment strategy.

We first apply the ARIMA-GARCH model set to forecast the prices of gold and bitcoin. By processing the raw data and analyzing some of the statistics, we determine the relevant parameters of the ARIMA model. Then, we combined it with different GARCH models using different distributions and compared the forecasting results separately to determine the parameters of the GARCH model. The final forecasting model is: ARIMA (4,1,4) - GARCH (1,1). This flexibility of parameters will also allow our model to be used to predict other types of assets.

After obtaining the price forecasts, we introduce them into the trading strategy model. We divide the trading strategy into two parts: what to trade and how much to trade. In the first part, an asset will be traded if its forecasted volatility exceeds the transaction cost; otherwise it will not be traded. After determining who to trade, we construct an objective function and solve for it under constraints to calculate the exact amount to be traded, forming a complete trading strategy. One of the constraints is the upper limit of each asset position, which is used for risk management and can be adjusted by the user at any time as needed.

In addition, we analyze the sensitivity of the returns generated by the model to the transac- tion costs in order to investigate the impact of studying the transaction costs on the total returns. We made judgments using both the normalized coefficient method and the shapley value decom- position method of the model, and ultimately showed that the degree of contribution obtained from the analysis of both methods is more sensitive and higher for Bitcoin's transaction costs; and the total return is lower when transaction costs are higher.

Also, we demonstrate that our strategy is superior by comparing the accuracy of different price prediction models. The data analysis shows that the hybrid ARIMA-GARCH model can bring more accurate prediction results than the single ARIMA model, which also means that it provides a more accurate basis for the trading strategy model.

Finally, we have included a memo for traders or investors who may use this model to make it easier for them to understand and apply our strategy.

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