Research on Interest Rate Risk Management Based on Duration, Convexity and Immunization

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Abstract. For bond investors and fundraisers, interest rate risk is an inescapable issue that requires attention. Therefore, efficiently measuring and preventing interest rate risk is a topic worthy of in-depth research. Duration, modified duration and convexity are tools that can be used to measure different interest rate risks. Based on the concepts of duration, immunization is introduced as an effective strategy to shield investors' overall financial status. This paper mainly reviews the development and evolution of duration, modified duration, convexity and immunization strategy. In addition, the application scenarios of duration and convexity and improved immunization strategies are discussed and served as a reference for investors and financiers when hedging interest rate risks.

Keywords: Duration; Convexity; Immunization strategy; Interst rate risks

1. Introduction

For investment institutions and investors, they are investing now in the hope that they can match their cash flow with future capital commitments. At the same time, most of the available bonds promise to pay off over a period of time. In order to realize the agreed amounts on the maturity, these amounts must be reinvested at an unknown future interest rate. To minimize the risk of failing to meet future commitments, duration immunization is adopted. Duration is a weighted average of income payment dates, where the weights add up to a uniform number, related to the present value of the income stream [1]. Given the limitations imposed on it, modified duration and convexity are discussed and studied in anticipation of breaking these limitations. Based on these studies, optimal duration models are proposed to minimize the risk of potential interest rate fluctuations. This paper mainly summarizes and analyses duration, modified duration, convexity, and the existing immunization strategy. Real-life investment case applications are also discussed.

2. Research on duration and convexity

2.1 Research on Duration

In 1938, the American economist F. R. Macaulay first proposed the concept of duration when studying the term structure of interest rate. It means the weighted average value of the time of the future cash flow generated by the bond, in which the weight is the proportion of the present value of the cash flow of each period in the price of the bond [2]. In the period of interest rate regulation before the 1970s, due to the slight fluctuation of interest rate, Macaulay duration was not fully developed and applied in commercial banks' interest rate risk management. In addition, Macaulay duration implies at least four strict assumptions: first, there is a linear relationship between price and yield curve; Second, the term structure of interest rate is flat which means the discount rate of each single period is equal; Third, the future cash flow does not change with the fluctuation of interest rate; Fourth, there is a parallel shift of the yield curve. These assumptions are inconsistent with reality, which

seriously restricts the practical application of Macaulay duration. Hicks put forward the concept of modified duration based on Macaulay duration, making the sensitivity of bond price to interest rate change more obvious [3]. However, the modified duration cannot modify the assumptions of Macaulay duration, which is limited in applications like Macaulay duration.

Fisher and Weil proposed the F-W duration, which discounts future cash flows using the estimated interest rate for each period. At this time, the term structure of interest rate can be of any shape, which overcomes the defect of the flat term structure of Macaulay duration hypothesis [4]. Although the F-W model effectively prevents the assumption that the yield curve is flat, it is still considered based on the premise that the yield curve moves parallel.

Toevs proposed the duration gap model, which has been used in the interest rate risk management of commercial banks for a great extent. It measures how the change of interest rate affects the net assets of banks [5]. The duration of asset portfolio and liability portfolio is calculated through the weighted method, and then the duration gap of bank assets and liabilities is calculated. According to the bank's objectives, the gap size is changed by adjusting assets and liabilities to manage interest rate risk.

Cox et al. defined the stochastic duration under the term structure of interest rate in CIR equilibrium model [6]. Since then, many scholars have begun defining and applying the random duration method combined with the term structure of dynamic interest rate under the idea of CIR duration. Because the stochastic duration model relaxes the relevant assumptions about the term structure of interest rate in the traditional duration model, it can effectively measure the situation when the yield curve moves non-parallel. However, due to the complexity of interest rate term structure modelling, the problem of model setting error in practical application will significantly affect the accuracy of random duration. The implementation cost is high, so it has not been applied in interest rate risk management.

The multiple duration theory was put forward and developed by scholars in their attempt to overcome the defects of the above random duration. Reitano gave the directional duration model and partial duration model [7]. The directional duration model proposes to break down the yield curve into a limited number of segments, and then express the operation idea of interest rate change at the level of these decomposition The yield to maturity, relevant maturity time and several parameters are used to represent the yield curve of a specific form, and then the factors that determine the parameters are calculated, which are the factors that determine the interest rate. In this way, we can get the form of the yield curve and the law of its change. The measurement effect of combining each partial duration is the sensitivity of the bond price to the fluctuation in interest rate, this is the basic operation idea of partial duration.

For financial assets with default risk, Chance gave a closed-form solution, which can be used to measure the duration of zero-coupon bond with systematic risk [8]. The basic idea is a zero-coupon bond without default risk such as a zero-interest treasury bond and a put option of a company's assets can be combined to copy the zero-coupon bond with default risk. The weighted average obtained by weighting the duration of the bond without default risk and the duration of the put option is the duration of the zero- coupon bond with default risk. Chance's conclusion is that the default risk will reduce the interest rate sensitivity of bonds, that is, the duration of bonds with default risk is smaller than that of bonds without default risk. Biewag and Kaufman gave various duration expressions based on expected default after risk adjustment according to the expected probability of default under the assumption that the term structure of interest rate is flat [9]. Through their research, it can be proved that for the duration model adjusted for default risk, the timing of default and how to recover the cash flow after default are very important.

If financial assets have implicit options, their future cash flow will be uncertain. At this time, their interest rate risk cannot be measured correctly by the traditional duration model. Frank gave the effective duration to deal with this defect [10]. The effective duration is the percentage change in a bond's price when the level of interest rates changes. Because it directly uses the bond prices under different yield changes in the calculation, and these bond prices already include the value of implied

options, the effective duration is more effective in measuring the interest rate risk of financial assets with implied options.

La Grandville proposed the directional duration of no horizontal yield curve under no parallel change and studied the influence of multiple factor curve yield model on duration [11].

Since Macaulay duration is interpreted as the maturity of zero-coupon bonds with the same price sensitivity, Thomas et al. proposed approximate duration [12]. Their specific idea is to find a zero-coupon bond with a maturity of D for a given interest bearing bond, so that the two bonds have the same present value, and their prices are "approximate" to the overall sensitivity of the changes of various parameters of the spot interest rate curve. Referring to the third meaning of Macaulay duration, they defined the maturity D of zero-coupon bonds as the approximate duration of interest bearing bonds.

The above theories are based on the assumption that the yield curve moves in parallel, but in fact, the translated yield curve rarely appears in real life. What is more common is that the shape and slope of the yield curve change. Therefore, with the deepening of the research on the duration model, new duration models have been proposed successively, such as partial duration [13], key interest rate duration [14], approximate duration and risk adjusted duration [15], and redemption terms [16] are added to the research, it has effectively promoted the development and progress of duration model research.

2.2 Research on Convexity

Modified duration implies the assumption that bond prices and yields have a linear relationship. Thus, for small fluctuation of the bond's yield to maturity, the duration rule is a quite good method to use. However, for larger fluctuation of yield, there is increasly more "daylight" between the two plots, which means modified rule becomes progressively inaccurate as interest rates increase. To adjust this problem, Bodie et al. proved that the relationship between bond price and yield is nonlinear, and introduced convexity into the duration model, which overcomes the defect of modified duration assumption by adding the modification for convexity term to the original modified formular [17]. This convexity theory effectively corrects the price error of Macaulay duration estimation.

For financial instruments with implied options, their future cash flows usually have unstable changes with interest rate fluctuations. The Macaulay duration model cannot measure the interest rate risk of financial instruments whose cash flows are vulnerable to changes in interest rates. Aiming at the limitation of Macaulay duration model, Frank proposed effective convexity model [10].

Dunetz and Mahoney measure the interest rate risk of redeemable bonds on the basis of Macaulay duration by combined duration and convexity [18]. Sarkar pointed out that duration and convexity are vitally necessary for bond portfolio managers and fixed income investors, and highly praised the duration and convexity model [19].

Miles and Zhou believed that using the modified convexity method to measure the price change would underestimate the price's decline and overestimate the price's rise, which would mislead the market behaviour of risk-averse investors [20]. Therefore, he proposed the index duration. By establishing the index model, the changed price will be slightly lower than the real price to avoid the defect of overestimating the price with the traditional duration and convexity.

3. Application of durationan & convexity

Based on the concepts of duration and convexity, Wen discussed the interest rate risk immunity of national debt when the term structure of interest rate is subject to linear impact and nonlinear impact, and studied the interest rate risk immunity of national debt under random term structure of interest rate [21]. He believes that the duration immunization strategy is strictly effective when the income curve is flat; When the term structure of interest rate is impacted by non linearity, the Taylor series expansion of bond price on the change of interest rate needs to be used to improve the accuracy of

interest rate risk immunity of national debt; When the term structure of interest rate is random, we can prevent interest rate risk by constructing a zero position immune bond portfolio.

Liu et al. investigated how the spot rate of return and its change affect the discounted cash flow through the directional duration [22]. Taking the immune condition of the directional duration as the constraint and maximizing the return of the bank's asset portfolio as the objective function, they gave an asset liability portfolio optimization model based on the interest rate risk immunity of the directional duration. It is an important innovation to use the direction of cash flow and long-term portfolio immunity to manage the portfolio risk of assets and liabilities, which solves the problem of interest rate immunity with different changes of yield in different periods in the real world.

Li and Yang mainly studied how to measure random duration and duration matching immune strategy under HJM framework, A new model to measure stochastic interest rate risk under the HJM framework and the corresponding interest rate risk immunity strategy are given. and theoretically obtained a more sensible interest rate risk measurement immune method by using random duration. Their empirical results show that the interest rate risk immunization method proposed by them has good immune effect and good avoidance effect on interest rate risk [23].

Liu and Tang calculated the term of total assets and total liabilities of commercial banks respectively, constructed the term gap model according to the duration theory, and put forward the immune strategy of interest rate risk of commercial banks [24]. Through the empirical analysis of a sample bank, this paper shows that properly adjusting the asset liability structure can realize the interest rate risk immunity of commercial banks to a certain extent, which is helpful to the interest rate risk management of commercial banks.

Xia et al. analyzed the deposit and loan risk with implicit options by constructing the jump model of benchmark interest rate of deposits and loans, and investigated the effectiveness of the matching strategy of duration convex gap model [25]. It is found that when there are implicit options in deposits and loans, the duration matching strategy without considering implicit options will make banks face the risk of duration mismatch and convexity mismatch. If the effective duration matching strategy is adopted, the negative convexity gap of implied options will bring additional interest rate risk to banks.

4. Traditional immunization strategy

4.1 Definition and Principle

Duration immunization is a passive approach for protecting assets from the hazards associated with interest rate fluctuations. Bond investments are considered immune if their real rate of return is certain to be at least equal to or greater than the properly estimated rate of return until maturity [26]. The strategy is founded on the idea that, when viewed from the standpoint of investment duration, price risk and coupon payment reinvestment risk cancel each other out completely. Interest rates falling in the short term will increase the value of bonds and short-term bond investors will benefit from the decrease in interest rates by earning capital gains. In the long run, however, the fall in the interest rate results in a decline in the reinvestment yield of bond coupon payments. Due to the polar opposite outcomes of interest rate fluctuations, it follows that there exists a “middle term” between the long term and short term. A discounted bond with a maturity equal to this medium-term and holding it for the medium-term would be similar to doing this. Consequently, interest rate fluctuations have no impact on the return on investment. By constructing a bond portfolio with a duration equal to or greater than the intermediate term of the bond portfolio, a duration-immune return can be attained [27].

4.2 Formation

Redington was the first to introduce the term “immunization” to signify a method of investing in assets that protects the existing business from broad changes in interest rates. He used the example of a life insurance firm to demonstrate that such a corporation will be immune to tiny swings in interest rates if two requirements are met. For starters, changes in the net present value of assets and liabilities have no effect on the net present value of assets and liabilities. If the interest rate is fixed at zero, the

first order derivative of the net present value of the asset and liabilities is also fixed at zero. Second, for changes in the interest rate in any of its neighborhoods, the present value of the asset must be greater than the present value of the liability, i.e., its second-order derivative with respect to the interest rate should exceed zero in order for the asset and liability to be profitable [28]. It is true that the mean term, which is the idea of duration and convexity proposed by Macaulay and is represented by the first and second order derivatives of the present value with respect to the interest rate, is the mean term.

A decade earlier, Fisher and Weil proposed the standard hypothesis of duration immunization, which was based on their previous studies [4]. There are three specific assumptions that were made in order for this theory to be formalized: the portfolio is examined over a set period of time, with no cash inflows or outflows, and interest rates move forward in a parallel shift. According to these assumptions, it is possible to conclude that a portfolio is successfully immunized if its value at the end of the holding time is at least as large as it would have been if the interest rate had remained unchanged for the whole holding period. The duration of the portfolio is aligned with the investment horizon of the future liability in order to obtain this level of protection against the future liability.

According to their findings following a series of empirical experiments, immunization was a successful method for protecting investors' overall financial situation from vulnerability to interest rate volatility [4].

4.3 The flaws

The traditional duration immunization technique, as an idealistic and hypothesized model, is unable to completely solve the bond hedging problem under a variety of yield curve characteristics.

On the one hand, market interest rates do swing up and down over the course of an investment term, in reality, they do not. As a result, the portfolio's McCauley length will fluctuate in reaction to market interest rate changes. The length of time that McCauley lasts likewise changes with the passage of time. As time passes and the term structure of interest rates shifts, it is possible that the duration matching conditions that were previously met will no longer be met [2]. As a result, in order to make the bond portfolio immune to interest rate changes, the investors must rebalance it such that the portfolio's McCauley duration equals the duration of the remaining investment period.

On the other, the immunization strategy implicitly implies that the flat yield curve moves in parallel and calculates all cash flows of different maturities at the same discount rate. In other words, the future reinvestment yield, which is determined on the basis of this assumption, is far from being the case in reality.

Furthermore, as previously stated, when it comes to bond duration, convexity is a more accurate estimate of interest rate risk than other measures. Even though duration presupposes that interest rates and bond prices have a linear connection, convexity enables extra factors to be considered, resulting in a slope. Although conventional duration immunization does not take into account convexity, it is merely an approximate assessment of bond prices, and as a result, the interest rate risk cannot be totally removed.

5. improvements on immunization

As a result of the realization that traditional interest rate risk immunization strategies cannot completely eliminate interest rate risk in practice, researchers have investigated new approaches along two distinct lines: one exploits the advances in modern financial theory on the stochastic process of interest rates to develop duration immunization; the other postulates alternative models and develops different measures of duration to generalize the application of immunization and minimize interest rate risk..

5.1 Optimization models based on stochastic duration interest rate immunity

Many studies on the dynamic behavior of short-term interest rates that have grown increasingly relevant to the realities of financial markets have established the groundwork for the creation of modern immunity theory.

Ingersoll, Skelton, and Weil presented a stochastic duration measure (duration of ISW/CIR), which was later refined by Ingersoll and colleagues. For interest rates, it takes into account the change in a single underlying instantaneous interest rate as a potential risk factor [29].

Wu proposed a new measure of bond price sensitivity to compensate for the shortcoming that ISW/CIR duration underestimates the price sensitivity of long coupon bonds relative to short coupon bonds. This measure was designed to compensate for the shortcoming that ISW/CIR duration underestimates the price sensitivity of long coupon bonds relative to short coupon bonds. Changes in bond-specific zero-curve yields are taken into consideration as a risk factor in this model [29]. In the same way that the ISW/CIR maturity measure is derived from Vasicek and CIR's term structure theory, the new measure can help to maintain the theoretical and empirical tractability of the term structure model as well. An empirical test utilizing Belgian government bond data demonstrates that this new approach improves on immunity and outperforms Macaulay's maturity, and the results are encouraging [30].

The Vasicek stochastic duration measure, for example, has been utilized to produce an interest rate risk measure despite the fact that these models have not been proposed specifically for immune purposes.

5.2 Alternative models

Even though stochastic duration immunization based on equilibrium models is theoretically more appealing, Nelson and Schaefer demonstrated that the performance of stochastic duration immunization based on equilibrium models is closer to that of traditional duration immunization models without demonstrating a statistically significant improvement [30]. A number of researchers have as a result turned their attention to work on other alternative models as a result.

The M2 model developed by Fong and Varsicek created a portfolio with limited sensitivity in the face of interest rate movement by lowering the risk measure under the following conditions: the present value of assets equals the present value of liabilities, the length of the portfolio equals that of liabilities, and the range of durations for individual bonds in the portfolio must have a span that is greater than the span of durations for individual liabilities [31].

When a portfolio's characteristic is used to calculate the portfolio's exposure to any type of interest rate fluctuation, according to Fong and Varsicek, "the characteristic might be regarded a measure of immunization risk" [31]. It was proposed that a lower limit be placed on the fluctuation in the value of an immunized portfolio at the end of horizon in the event of a fluctuation in interest rates that is purely arbitrarily determined. This lower limit results from two terms being added together. One of these terms is completely dependent on the type and degree of the rate shift, whilst the other term is solely dependent on the structure of the portfolio, as previously stated. When combined with the first term, the second term offers the required measure of immunization risk. Given that rates will always shift by a non-infinitesimal amount before a portfolio can be reconstructed in actuality, this appears to be a more relevant strategy for application development.

When it comes to specific circumstances, Nawalkha and Chamber introduced the M-Absolute model, which is derived as a first-order lower bound immunity risk model [32]. This model is powerful and practical because it provides a single measure of risk immunity that is both powerful and practical. When it comes to bond valuation, The weighted average of the absolute distance between the bond's cash flows and a horizontal point is known as the M-Absolute. However, despite the fact that M-Absolute is a single measure of risk, it may be used to successfully mitigate the effects of multiple different types of interest rate risk, rather than hedging only one type of term structure change. According to empirical experiments, M-Absolute reduces the interest rate risk inherent in typical term models by more than half, independent of the exact time period under consideration.

As a means of compensating for the shortcomings of just doing duration immunization while neglecting convexity, Wu and Chi developed an immunization method based on directional duration as well as directional convexity to be used in conjunction with directional convexity [33]. Using the concept of directional duration to form the convexity, this study proposes that the second-order derivatives of asset or liability prices to interest rates are also affected by different time periods in order to form the convexity. This is accomplished by forming the directional duration through the first-order derivatives of asset or liability prices to interest rates that are affected by interest rate movements at different time periods. This alters the thinking of the prior study, which was based solely on the matching of directional duration in the two directions. A further result of this research was the development of an expression for directional convexity, which is based on the relationship between the convexity of asset or liability cash flows affected by different interest rate changes [33]. This expression is used to reflect the impact of spot interest rates and the amount of their change on discounted cash flows. As a result, the problem of earlier research failing to account for directional convexity is addressed, and optimal strategies are no longer exposed to interest rate risk until when interest rates vary by a modest amount.

6. Potential future research direction

The ultimate purpose of theoretical research is to translate it into practical applications in the actual world. In order to develop immunization strategies to hedge interest rate risk, the majority of the present research focuses on specific conditions such as duration and convexity themselves. It is possible, however, that the efficacy of an immunization strategy will provide significantly different results in the setting of different investment eras, different investment markets, and/or other investing philosophies. When it comes to the developing Chinese bond market, the unique nature of the interest rate environment implies that interest rate risk manifests itself in a variety of ways and has constantly changing features. It is also important to note that China's relatively small market size and illiquid micro-market structure are considerably different from those of mature foreign markets that are very liquid [34]. As a result of the vastly diverse application settings, it is possible that there may be considerable discrepancies in the use of an immunization strategy in local and international markets. What immunization method is most appropriate for the domestic bond market will be a subject of future investigation, and further research efforts are being conducted to that end.

7. Conclusion

This paper briefly reviews the literature on the definition, development process and applications of duration and immunization strategy. By listing several application cases of duration and convexity and analyzing the shortcomings of the traditional duration and immunization theory, it introduces the modified duration, convexity, and the asset-liability optimization immunization models. The increasingly accurate estimation and persuasive applications can be of great benefit to future investors in better hedging interest rate risk. As a summary of the previous significant research on duration and immunization, the paper also provides directions for future research with literature references and potential research perspectives.

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