The Efficiency between Markowitz Model and Single Index Model

Zeheng Cai[1, †], Yantong Long[2, *, †]

[1]Xiamen Shuangshi Middle School of Fujian, Fujian, China.

[2]Vanke Meisha Academy, Guangdong, China.

*Corresponding author: 2270909347jacky@gmail.com

longyantong@stu.vma.edu.cn

†These authors contributed equally.

Abstract. It is important to investigate the different impact factors on the establishment of investment portfolio. In order to maximize the profit of a portfolio, this research selects six stocks: Adobe (ADBE), International Business Machines Corp (IBM), Bank of America Corporation (BAC), Citigroup (C), Southwest Airlines Co (LUV) and Alaska Air Group Inc. (ALK) as an empirical case to conduct investment decision. This research compares different results of two models (Markowitz model and Index Model) by analyzing the data of the portfolio such as minimum risk portfolio and maximum Sharpe Ratio portfolio. This research finds that the Markowitz Model is more appropriate to the portfolio establishment, which is presented by its sharper efficient frontier. Under all the conditions the study considered, the Markowitz Model always gets more returns than the Index Model when the risk increases by the same amount. Therefore, this research concludes that the Markowitz Model is more suitable for this portfolio. By analyzing the portfolio in two models, the study has found a better model for this portfolio, thus make more rational investments.

Keywords: Portfolio, Markowitz model, Index Model, minimum risk, maximum Sharpe Ratio, efficient frontier.

1. Introduction

The Economic Times [1] explains that a stock market is a place where consumers can purchase shares of pubic listed companies for trading. It is the collection of markets and exchanges. and by issuing stocks to society, companies quickly obtain a large number of funds to achieve scaled production operations. Buying and selling activities are conducted through institutionalizing formal exchanges, and countries could have multiple stock trading venues [2]. For the United States, the leading stocks are New York Stock Exchange (NYSE), Nasdaq, and the Chicago Board Options Exchange (CBOE). The US stock market is formed by these primary national exchanges and other exchanges in the states [2].

The aim of this research is to find out the optimal portfolio within the given assets including six risky assets and one risk free asset. This will be determined through the use of Markowitz model and Single Index model.

Edelweiss [3] explains that When investors purchase groups of assets, it is called an investment portfolio. This portfolio includes stock and other options such as bonds, stocks, mutual funds, pension plans, real estate, etc. The optimal portfolio is to have the assets that obtain the highest return with the lowest risk, thus understanding how to manage the portfolio and distribute each asset's portions. Often time, the investor has issues with portfolio selection; thus, portfolio optimization model comes into place. Adopting the Markowitz Model and Single Index Model can help the investor find their optimal portfolio when investing.

Markowtiz once mentioned that asset allocation is the only free lunch in the investment market. For any rational investor, they all want to reduce risk and maximize return in the investment. Thus, Markowitz Model is being applied, which can find the efficient frontier and help to find the highest return under a specified level of risk [4]. Dr. Harry Markowitz creates this model, and he presents the methodology of portfolio selections. Radović *et al*., [5] found that this could be set in three stages:

first, it is the forming of possible portfolios, then determining the set of efficient portfolios, and lastly, depending on investment preferences which will use the utility function. Therefore, portfolio optimization is reached. In 2018, a faculty of legal and business studies wanted to find a portfolio with the lowest risk under a given return in the Serbian capital market. The optimization portfolio model they adopted is the Markowitz Model. Using the Markowitz model, researchers can find the set of efficient portfolios, which can reduce a firm's unsystematic risk by diversification and find its minimal risk. At the same time, the researchers found that the correlation between return and portfolio's beta coefficient appears to be positive [5]. From 2013 to 2016, there is another practical study of Markowitz portfolio optimization which focuses on the Bulgarian stock market. By using Markowitz's theory, the efficient frontiers and optimal portfolio of dataset consists of the closing price of 50 companies traded on the Bulgarian Stock Exchange in these three years will be determined. Dospatliev *et al*., [6] said that as a result, a Markowitz model-based efficient portfolio did better performance than individual domestic security within the study timeframe.

Another useful portfolio optimization model is the Single- Index Model, which helps solve complexity in the Markowitz Model. William F. Sharpe is the creator of the single index model [7]. This reduces the parameter estimation significantly and derives the systematic and firm-specific risk components of the total risk in the portfolio, respectively. In 2014, a study examined the utility of Sharp's Single-Index Model for portfolio construction. By the end of the study, the optimum portfolio using Sharp's Single Index Model, including fifteen sample combines, has only chosen four for optimum investment. Nalini [8] explains that the present study's findings will be more beneficial to fund managers in merged economies like India. Another portfolio analysis using the Single-Index Model analyzed ten selected stocks of the Kuala Lumpur Stock Exchange (KLSE). Daily and weekly research using a single- index model was conducted in this paper. The results show that entrance of 5 stocks is optimal when setting up a daily portfolio, and only two stores are optimal when setting up a weekly portfolio. Therefore, weekly portfolio will be the optimal portfolio as it will bring higher returns with lower risk.

In this paper, this research uses the Markowitz model and Index Model to analyze a specific stock portfolio. This research considers the differences of their efficient frontier, minimum risk point, and maximum Sharpe Ratio point in the analysis. In addition, the study constructs three conditions to ensure the accuracy and precision of the comparison. Firstly, the study introduces a free constraint, which means investors have no restriction in their investment plan. Secondly, the study adds a constraint that simulates the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions. Thirdly, this research introduces an additional constraint that the weight of Standard & Poor's 500 (SPX) index equal to zero. We mainly focus on the horizontal comparison and vertical comparison in these three conditions. Firstly, in the vertical comparison figure, the study finds that the free constraint condition has the sharpest efficient frontier; meanwhile, the effect of shorting is not so effective as the SPX in these two models so the portfolio with SPX equal to zero is risker. Secondly, after three times horizontal comparing, this research has a significant finding that the Markowitz model always presents better than Index Model. This means when this research uses the Markowitz model to predict the return and standard deviation, this research will always get a large Sharpe Ratio due to the sharper efficient frontier of a portfolio in Markowitz model. Therefore, this research concludes that in this specific portfolio, Markowitz model is more appropriate than Index Model to forecast the return and standard deviation.

The remainder of the paper is 6 sections. In section 2, this research describes the sample and data; In Section 3, this research performs and compares two models; In section 4, this research introduces the information (efficient frontier, minimum risk point and maximum Sharpe Ratio point) that present in the figure of two models; In section 5, this research presents the result analysis of each figure; In section 6, this research shows the discussion of advantage and disadvantage of two models. The last section presents this research’s conclusions.

2. DATA

This project aims to calculate the optimal portfolio from the given seven stocks. Daily total return data from six stocks Data are collected from January, 2000 to December, 2000 including Adobe (ADBE), International Business Machines (IBM), Bank of America (BAC), Citigroup (C), Southwest Airlines (LUV), and Alaska Air Group (ALK); which belong to three different industry groups: aviation, technology, and financial service. The other one is Standard and Poor's 500 (S&P 500) equity index and a proxy for the risk-free rate.

SPX-Also name Standard & Poor 500 Index. This is a stock composed of the New York Stock Exchange's (NYSE) 500 largest US companies. Being a theoretical index, its price is calculated as an actual portfolio. This stock is a risk-free asset. Thus, there is relatively low risk when investing in this stock.

ADBE-Adobe is an American multinational computer software company that was founded in 1982. In the early time, it focuses on the development of desktop PC software. In nowadays, it has provided a series of cloud-based products for the photograph, video editing, digital media, etc.
IBM is an International Business Machines Corporation founded in 1911 and provided hardware, software, cloud-based services, and cognitive computer.

BAC-Stands for Bank Of America. This is a multinational investment bank and holds financial services.

C-stands for Citigroup, an investment banking company that offers financial services and is a multinational investment bank.

LUV-This is an aviation stock that stands for Southwest Airlines.

ALK- Another aviation stock, named Alaska Air Group.

Then this research presents the following figure of different stocks' price with time.

2.1 Graphs

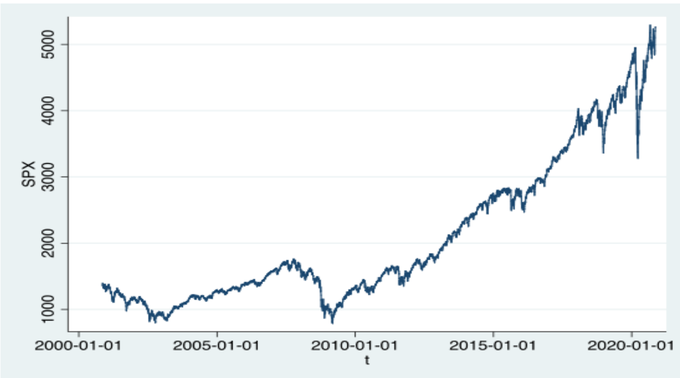

Fig. 1 The price of SPX

In figure 1, the whole graph appears to be an increasing upward trend in these 20 years. The price started with approximately 1400, and it experiences a small degree of decreasing price from 2001 to 2004. From 2004 to around 2009, the stock price increases to a small level. After experiencing a few months of decrease in share price, starting from 2010 to 2020, the stock increase rapidly in these ten years with slight fluctuation. There is a sharp decline at the beginning of 2020 in price but recovery quickly. The lowest share price in these 20 years is approximately 500 to 600 occur in the first five years, and its highest price is above 5000 which is in the year 2020.

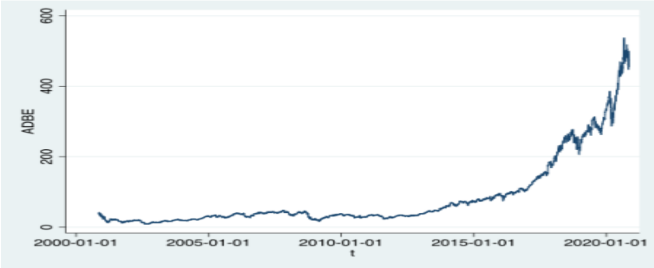

Fig. 2 The price of ADBE

In figure 2, the whole graph appears to be constant in the first 15 years and occurs upward increasing trends in the following five years. From the year 2000 to 2015, there is slight fluctuation in its share prices. Thus, it does not experience sharp grows or falls. The increasing tendency becomes more noticeable start with the year 2015 to 2020. The share price climbs significantly in the following five years, shown by its steep slope. The lowest price was in 2009, and the highest price is in 2020.

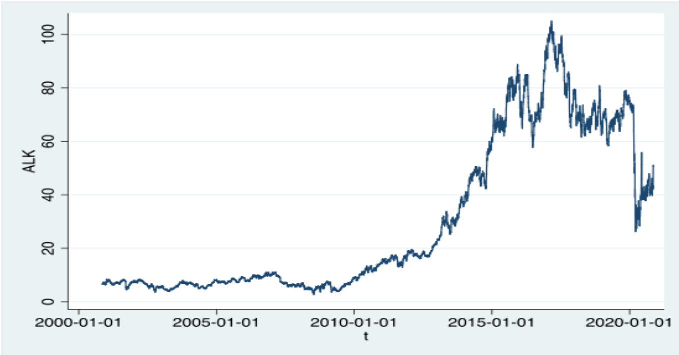

Fig. 3 The price of ALK

Fig.4 The price of BAC

In figure 3, the whole graph appears to be not stable with its share price, and it experiences a dramatic increase and decrease in these 20 years. From 2000 to 2010, the price remains relatively stable with small fluctuation. Started from 2010 to 2015, the graph shows a sharply increasing. However, by 2016 the price volatility decreases then went up and met its peak after a short period. Later, the graph indicates a consistent decline in the rest of the years. There is a sharp fall in 2020 that has to get almost the same price six years ago in 2014. The lowest price of the graph is in 2009 and it reach its peak around 2016.20 that had gone higher than 500.

In figure 4, the whole graph indicates the volatility of the share prices in 2020, which experience a dramatic rise and fall in its price. From 2000 to 2008, the price fluctuated sharply and is continuously increasing. The graph shows a sudden decline in 2009 with almost a vertical slope and continues to the next year with a significant drop in its price. From 2010 to 2020, the price did not remain very stable as there is a noticeable increase and decrease in price. The stock has its lowest price from around 2018 to 2019, and its highest price is in 2018 that has gone above 60.

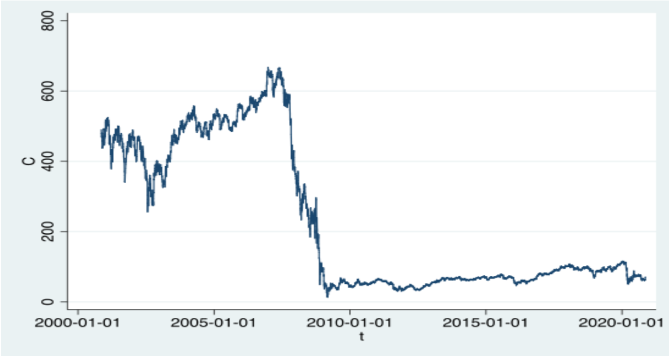

Fig. 5 The price of C

In figure 5, the whole graph does not follow an aggregate increase or aggregate decrease in these 20 years, the half of the graph's share prices remain at a higher price level while the other half remains at a lower price level. From 2000 to 2008, the graphs appear to be in a growth trend overall, except there is a sharp fluctuate in 2003 but soon recovery back up. After the graph reached its maximum point in 2007, there is a sharp decline and consistently dropped in the next two years. From 2009 and to 2020, the share price remains relatively stable with small fluctuation. The lowest point of the graph appears in 2009, and the highest point appears in 2007 that had gone above 600.

Fig. 6 The price of IBM

In figure 6, the graph shows significant fluctuation in these 20 years, and it is volatility. From 2000 to 2009, the graphs show one time of sharp fluctuation in 2003 and a noticeable increase in 2007. Overall, the price remains at a lower level. From 2010 to 2020, it first experiences a sharp increase and eventually reaches its highest point in the first four years showing on the graph with a steep slope. There is another sharp decrease in the following two years, and volatility remains large and continued.

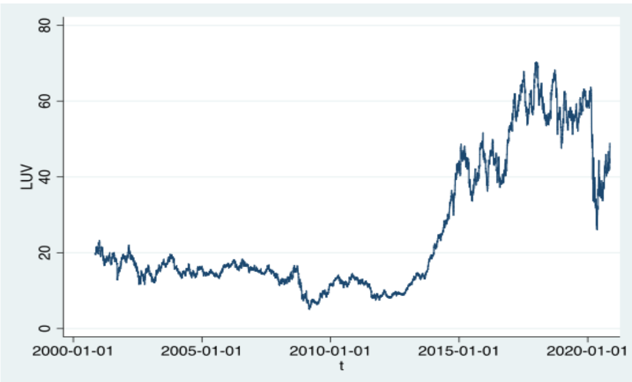

Fig. 7 The price of LUV

In figure 7, the whole graph does not follow an aggregate increase or aggregate decrease in these 20 years, the half of the graph's share prices remain at a lower price level while the other half remains at a higher price level. There is a slight fluctuation in share price from 2000 to 2012 and experience its lowest point in 2009. From 2012 to 2017, there is a sharp increase in its price and volatility increase and continue until 2020 and eventually reach its lowest point in 2020.

3. Method

This paper aggregated the daily data to the monthly observation and calculate all proper optimization inputs using the Markowitz model (MM) and Index model (IM). Through the calculation of these two models, the return, standard deviation, and sharp ratio of stock's minimum risk and maximum sharp would be found.

3.1 Three cases of the additional constraint

Using the optimization inputs for MM and IM, this research finds the regions of permissible portfolio for three cases of the additional constraints: free constrain, constrain $w \geq 0$, and $w = 0$, The free constraint can long or short freely, constrain $w \geq 0$ does not allow any short positions but can allocate money into SPX, and $w = 0$ cannot allocate money into SPX.

3.2 Markowitze Model

The Markowitz model was developed by Harry Markowitz, which helps determine the minimum variance for an expected return. It assumes that investors are more likely to choose a portfolio with a lower risk lever than a higher risk level when gaining the same return. Thus, this portfolio of assets will be efficient.

The formula for calculating the expected return on a portfolio using the Markowitz model is as follows: X_i is the fraction of investment that investors put in stock i, and $\mathrm{E}(\mathrm{R}_t)$ is the expected rate of return of stock i.

$$E(\mathrm{R}_p) = \sum_{i=1}^{n} \mathrm{X}_i \mathrm{E}(\mathrm{R}_t) \tag{1}$$

3.3 Single Index Model

The single-index model assumes that the stock prices move together and systematically due to the common co-movement with the market. Also, it assumes that the unique return has no relationship with a market index. The formula for calculating the expected return is as following: R_i is the expected return on security I, a_i is the intercept of a straight line or alpha coefficient; β_i is the slope of the line or beta coefficient and multiplying R_m, which is the expected return on the index, and e_i is the unexpected component or return due to unforeseen events that only affect this specific security.

$$\mathrm{R}_i = \mathrm{a}_i + \beta_i \mathrm{R}_m + \mathrm{e}_i \tag{2}$$

4. Portfolio investment

Based on the Markowitz model and Index Model presented above, we consider two specific points in the figure of portfolio set: the minimum variance point and the maximum Sharpe Ratio point. By calculating these two points in both figures of portfolio set, we can analyze the efficiency and difference between different models in different constraints. We present the formula of the minimum variance frontier, Minimal Risk Portfolio, and Maximum Sharpe Ratio.

4.1 Some Common Mistakes

The minimum variance portfolio or minimum risk portfolio is a so-called risk-based approach to portfolio construction. This means that the portfolio is constructed using measures of risk. The reason why investors want to use the risk-based approach is that the risk is easier to estimate than the estimate of future return.

Under the modern portfolio theory and the efficient frontier, this research can find a portfolio with minimum variance in the efficient frontier. The minimum portfolio with the lowest standard deviation is a portfolio that can be constructed from the set of securities that the investor can invest in.

Minimal Risk or Variance Frontier:

$$\begin{cases} \sigma(\vec{w}) \to min \\ subject\ to\colon r(\vec{w}) = const \end{cases} \tag{3}$$

Minimal Risk *Portfolio*:

$$\sigma(\vec{w}) \to min \tag{4}$$

4.2 Maximum Sharp Ratio

Based on the Markowitz model and Index Model presented above, this research considers two specific points in the figure of portfolio set: the minimum variance point and the maximum Sharpe Ratio point. By calculating these two points in both figures of portfolio set, this research analyzes the efficiency and difference between different models in different constraints. The study presents the formula of the minimum variance frontier, Minimal Risk Portfolio and Maximum Sharpe Ratio.

$$Sharpe\ Ratio = \frac{E(R_P)-R_f}{\sigma_p} \tag{5}$$

$E(R_P)$: The expected annualized return of the portfolio

R_f: Annual risk-free rate

σ_p: Standard deviation of the portfolio's annualized rate of return

The purpose of the Sharpe Ratio is to calculate the excess return of a portfolio for each unit of total risk it takes. The Sharpe index represents the number of excess return investors can get for every extra minute of risk they take. If it is greater than 1, it means that the return rate of the fund is higher than the volatility risk; If it is less than 1, it means that the operational risk of the fund is greater than the rate of return. In this way, each portfolio can calculate the Sharpe Ratio, which is the Ratio between investment returns and excessive risks. The higher this ratio is, the better the portfolio is.

5. Result Analysis

First, the study considers three conditions to this portfolio to simulate the market circumstance. This paragraph will introduce these three conditions.

(1) The first condition has a free constraint. A “free” problem, without any additional optimization constraints, to illustrate how the area of permissible portfolios in general and the efficient frontier, in particular, look like if the portfolio has no constraints.

(2) The second condition has additional optimization constraints is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions. Therefore, this research considers to add an additional optimization constraint:

(3) The third condition has the 0 weight of the broad index. This research would like to estimate if the inclusion of the broad index into the portfolio has positive or negative effect. Therefore, this research considers to add an additional optimization constraint:

Under three conditions, the study analyzed the data in two aspects: vertical analysis and horizontal analysis.

Table 1. Data of stocks' weight, return, standard deviation and Sharpe Ratio

			SPX	**ADBE**	**IBM**	**BAC**	**C**	**LUV**	**ALK**	**Return**	**StDev**	**Sharpe**
free constraint	MM	MinRisk	1.355	-0.125	-0.024	0.057	-0.228	0.011	-0.046	0.070	0.131	0.535
		MaxSharpe	1.003	0.262	-0.153	0.468	-0.719	-0.091	0.230	0.190	0.215	0.883
	IM	Min-StD	1.431	-0.108	-0.018	-0.089	-0.174	-0.022	-0.020	0.064	0.129	0.500
		Max-SP	1.049	0.265	-0.140	-0.008	-0.411	0.034	0.210	0.150	0.196	0.761

			SPX	**ADBE**	**IBM**	**BAC**	**C**	**LUV**	**ALK**	**Return**	**StDev**	**Sharpe**
W>0	MM	MinVar	0.995	0.000	0.000	0.000	0.000	0.000	0.005	0.067	0.151	0.444
		MaSharpe	0.083	0.511	0.000	0.000	0.000	0.000	0.407	0.158	0.263	0.600
	IM	Min-StD	0.000	0.000	0.030	0.000	0.000	0.000	0.000	0.067	0.151	0.444
		Max-SP	0.156	0.440	0.000	0.000	0.000	0.057	0.348	0.146	0.250	0.584

			SPX	**ADBE**	**IBM**	**BAC**	**C**	**LUV**	**ALK**	**Return**	**StDev**	**Sharpe**
W1 = 0	MM	MinVar	0.000	0.123	0.548	0.183	-0.156	0.217	0.085	0.100	0.210	0.476
		MaxSharpe	0.000	0.709	0.044	0.872	-1.083	-0.065	0.523	0.304	0.368	0.826
	IM	Min-StD	0.000	0.105	0.502	0.053	-0.049	0.241	0.149	0.092	0.209	0.442
		Max-SP	0.000	0.685	0.024	0.132	-0.543	0.216	0.487	0.235	0.333	0.704

According to Table 1,this research gets the precise weights of each stock and the maximum Sharpe Ratio and minimum standard deviation in this portfolio in three conditions. In the free constraint condition, C stock has the smallest weight in both two models (-0.719 in MM and -0.411 in IM); SPX stock has the largest weight in both two models (1.003 in MM and 1.049 in IM). The maximum Sharp ratio is 0.883 in MM and 0.761 in IM; the minimum standard deviation is 0.131 in MM and 0.129 in

IM. In the condition where W>0, IBM, BAC, and C stocks all weight zero and LUV also has zero weight in MM; ADBE stock has the largest weight in both models (0.511 in MM and 0.440 in IM). The maximum Sharp ratio is 0.600 in MM and 0.584 in IM; the minimum standard deviation is 0.151 in MM and IM. In the condition where W1 = 0, C stock has the smallest weight in both two models (-1.083 in MM and -0.543 in IM); BAC stock has the largest weight 0.872 in MM while ADBE stock has the largest weight 0.685 in IM. The maximum Sharp ratio is 0.826 in MM and 0.704 in IM; the minimum standard deviation is 0.210 in MM and 0.209 in IM.

5.1 Horizontal Analysis

For horizontal analysis, the group constructs three portfolios with minimum standard deviation point, maximum Sharpe Ratio point, and effective frontier of the portfolios. The study calculates the data of the portfolio by using two models (The Full Markowitz Model (MM); The Index Model (IM)). Therefore, th group can compare the efficiency of each model and find a better model for this portfolio.

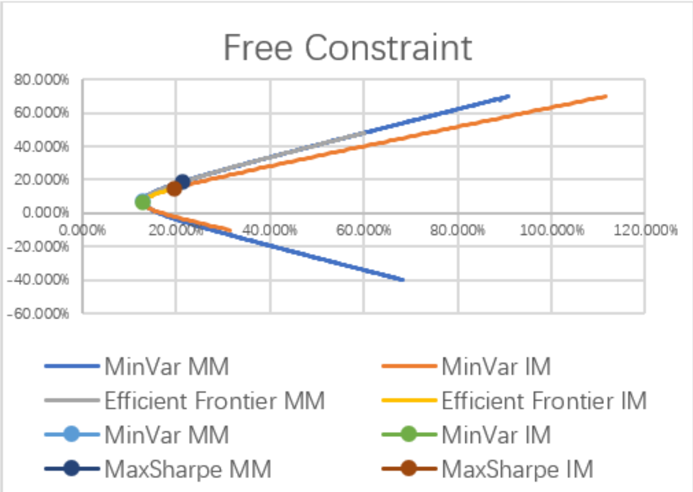

Fig. 8 Free constraint condition

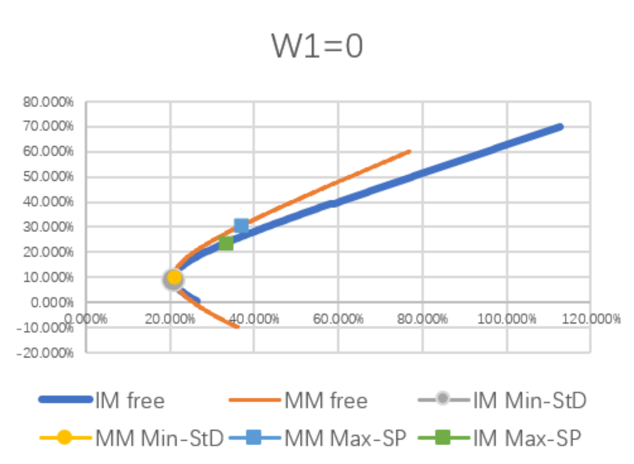

Fig. 9 W1 = 0 constraint condition

According to figure 2 (free) and figure 3 (w1 = 0), this research finds that the figure of free condition is similar to the figure of w1 = 0 condition so this research analyzes them together. Following are three findings the study finds from these two figures.

First, in these two conditions, the maximum Sharpe Ratio portfolio of the Markowitz Model has a larger risk, return, and Sharpe Ratio than the Index Model. And the reason is the following. In the portfolio, this research divides six stocks into three pairs: Technology, Financial Service (Bank), and Airline Travelling. In the portfolio, Adobe's stock price will increase as IBM's price increases, Bank of America will increase as Citi increases, and Southwest Airlines will rise as Alaska Air Group rises, because they are in the same industry. Each of the stock's unsystematic risks could be hedged by another stock in the same industry. However, in the portfolio, IM is comparing stocks with the SPX. Since this research has three pairs of stocks from different industries, the SPX is not so efficient to use to estimate the covariance between stocks and index. As a result, the beta is underestimated compared to MM, so the standard deviation and return of the portfolio estimated by IM are smaller than that by MM.

Second, this research finds when risk increase, MM is more efficient in this portfolio. This is because when the standard deviation increase, the return ratio of the two models separates more and more. It seems the MM would get more return compared to IM. This is because as standard deviation increases, the minimum variance frontier of MM and IM are moving farther away from each other. Since investors are free to long and short any stocks, they have a greater potential to achieve in their standard deviation and return. However, just as previously mentioned, IM, in this case, does not measure the beta (correlation between stock and index) effectively. As the standard deviation increases, the weights of different assets are increasing as well. Therefore, the difference between MM and IM would become larger.

Third, this research finds that the difference between models influences w1 = 0 condition the most. It can be indicated by the distance of maximum Sharpe Ratio points of w1 = 0 condition, which is largest. In this research's analysis, this is because when using the single-index model, the weight of SPX is zero, the proportion of alpha of other 6 stocks (except SPX) in the portfolio will rise faster since this case does not have an alpha of SPX to help it avoid risk. Therefore, in this case, the difference between the Max-SP points in the two models will be larger than that in the other two cases where the weight of SPX is not zero.

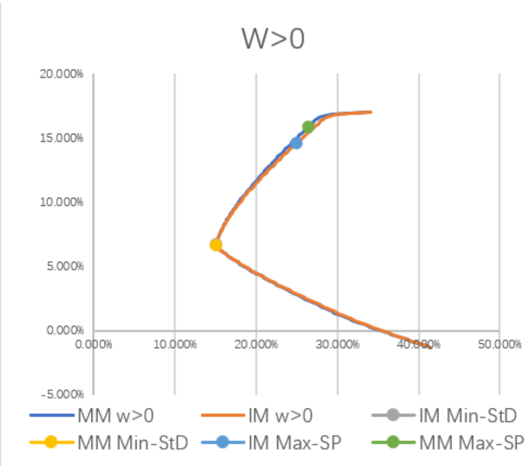

Fig. 10 W > 0 constraint condition

According to figure 4 (w > 0), the Markowitz model is also more efficient than Index Model. This circumstance has the same reason as the other two constraints have (the detail has presented in the previous paragraph).

However, the figure of w > 0 condition presents a different pattern (notice that when standard deviation increases, the two curves gradually separate from each other. But when the return ratio comes to 17%, the two curves suddenly become flatter and closer to each other and gradually overlap): it has a transition of return ratio in relatively high standard deviation circumstance. The reason is the following.

Under w >= 0 conditions, borrowing money from the bank or anywhere else is not available. Because investors are not able to hedge risk by shorting a stock, there are limitations on the portfolios' possibilities, which has been presented on the suddenly transition of return ratio. Smaller returns investors could get compared to the other two constraints.

5.2 Vertical Analysis

According to Table 1 above, the study constructs the figure of three conditions and compare them vertically.

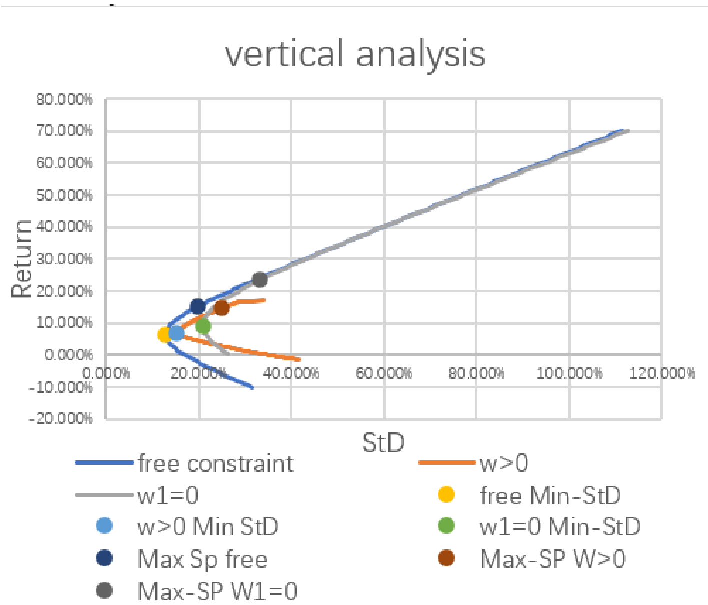

Fig. 11 W>0 vertical analysis

According to Figure 5, this research considers the differences between the three constraints and find that the two models (IM, MM) have similar result. Therefore, this research uses the IM graph to analyze the differences between these three constraints.

In this figure, this research finds the free condition is the most efficient one. Comparing free conditions with the other two conditions, this research can find the shortcoming that limits the other two conditions.

Comparing free with w > 0 condition, this research realizes shorting is beneficial to avoiding risks. Investors can use short mechanisms to hedge their funds, thereby reducing investment risks and increasing the safety of funds. Although at the approximate level of return, w >= 0 has a larger standard deviation than free constraint. Under the constraint w >= 0 condition, investors cannot borrow money from a broker or do any shorting. Therefore, the portfolio cannot hedge risk by shorting stocks. As a result, the standard deviation (risk) of a portfolio under the constraint of w > 0 conditions is riskier than that under free constraint. The lack of opportunity to short makes portfolios riskier on average.

Comparing free with w1 = 0 condition, this research can find w1 = 0 constraint has risker minimum risk point and slower slop of efficient frontier. It can indicate that the inclusion of SPX is important to avoid risk in this portfolio, which, on the contrary, means that this portfolio cannot avoid risk efficiently.

When comparing the w > 0 and w1 = 0 conditions, this research can find w1 = 0 condition is risker. This is because part of the systematic risk of this portfolio is hedged by diversifying in industries of technology, airline/travelling, and financial service. Therefore, the effect of shorting is not so effective as the SPX. As a result, this research indicates that portfolio of (w1 = 0) is riskier than w >= 0.

6. Discussion

While both of the models help investors to manage their portfolio and allocate their assets in the portfolio, the MM and IM model still varies a little from each other and has its advantages and disadvantage when adopting the method. The following section will discuss the advantages and disadvantages of these two models.

6.1 Markowitz's model

Markowitz's model requires a large number of estimates of expected returns, variances, and covariance. It considers covariance between stocks, which helps estimate non-systematic risk, and it tries to give the portfolio of the highest return on any level of risk. However, it requires a larger number of estimates. It overly relies on the past data of stocks and does not provide forecasting of the future. It also ignores the correlation between stocks and the index (market) [9].

6.2 Index Model

Index Model reduces the parameter estimation significantly, and it derives the systematic and firm-specific risk components of the total risk in the portfolio, respectively. It provides greater return with smaller risk[10]. However, Index Model simplifies the uncertainty of asset returns and simplifies it into two kinds of risks: micro and macro risks, which oversimplifies the uncertainty of the real world. It misses some factors that will affect stock returns, such as factors caused by the industry's correlation. Also, in a diversified portfolio, a single index model cannot effectively consider all socks from different industries; thus, the beta may be mismeasured. When utilizing the Single Index model asset price, some securities markets' return is correlated with some USA (SPX) securities markets.

7. Conclusion

In this paper, the groups researches the portfolios of six stocks by using Markowitz Model and Index Model. Also, the group calculates the minimum standard deviation and maximum Sharpe Ratio. Based on these two models, the group first analyzes the difference between the two models. Then, the group compares the difference of portfolio set in these two models. Moreover, the group compares the difference of each model between three constraints. The group also mention the advantages and disadvantages of these two models.

Based on the empirical data, the group has two parts of conclusion about the portfolio of the six stocks. Firstly, the group finds in this portfolio, using the Markowitz Model can almost get more return than Index Model, which means in this case, the Markowitz Model is more efficient than the Index Model. Secondly, the group find that under the free constraint, the portfolio has a higher Sharpe Ratio because the investor can use both short and SPX to help them avers risk. In the other two constraints, the effect of shorting is not so effective as the SPX.

The vertical and horizontal comparing of both IM and MM of this portfolio can conclude a precise analysis of these six stocks' portfolios. The shortcoming of this paper mainly includes two aspects. Firstly, the Markowitz Model and Index Model cannot comprehensively estimate the portfolio well. Secondly, the data of the solver and solver table in the group's analysis may not precise enough. In the future, the group will use more precise and up-day data in the group's analysis. Also, we will consider adding a Multiexponential model and more new models into our analysis to estimate the portfolio more precisely.

References

[1] Times. (n.d.). https://economictimes.indiatimes.com/definition/stock-market.

[2] Chen, J. (2021, May 19). Stock Market. Investopedia. https://www.investopedia.com/terms/s/stockmarket.asp.

[3] What Is A Portfolio? Edelweiss. (n.d). https://www.edelweiss.in/investology/introduction-to-investing-c6eaf4/what-is-an-investment-portfolio-789abc

[4] Pandey, M. (2012). APPLICATION OF MARKOWITZ MODEL IN ANALYZING RISK AND RETURN (A CASE STUDY OF BSE STOCK). Ahmedabad Chief Editor, 43(1), 1-14.

[5] Radović, M., Radukić, S., & Njegomir, V. (2018). The Application of the Markowitz's Model in Efficient Portfolio Forming on the Capital Market in the Republic of Serbia. Economic Themes, 56(1), 17-34.

[6] Ivanova, M., & Dospatliev, L. (2017). Application of Markowitz portfolio optimization on Bulgarian stock market from 2013 to 2016. International Journal of Pure and Applied Mathematics, 117(2), 291-307.

[7] Mandal, N. (2013). Sharpe's single index model and its application to construct optimal portfolio: an empirical study. Great Lake Herald, 7(1), 1-19.

[8] Nalini, R. (2014). Optimal Portfolio construction using Sharpe's Single Index Model-A study of selected stocks from BSE. International Journal of Advanced Research in Management and Social Sciences, 3(12), 72-93.

[9] Sarker, M. R. (2015). Comparison among different models in determining optimal portfolio: evidence from Dhaka Stock Exchange in Bangladesh. Journal of Business, 36(3).

[10] .Setyantho, K. S., & Wibowo, S. H. (2019). Comparison of Optimal Portfolio Performance Between Single Index Models and Markowitz Models (Case Study of Daily Return Implementation of OJK Rules Regarding Investments of State Values For Non-Bank Financial Institutions 2016-2017). Business and Entrepreneurial Review, 19(1), 43-66.